ANALYTICAL SOLUTION TO CONVECTION-RADIATION OF A CONTINUOUSLY MOVING FIN WITH TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY

by

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In this article, the simultaneous convection-radiation heat transfer of a moving fin of variable thermal conductivity is studied. The differential transformation method is applied for an analytic solution for heat transfer in fin with two different profiles. Fin profiles are rectangular and exponential. The accuracy of analytic solution is validated by comparing it with the numerical solution that is obtained by fourth-order Runge-Kutta method. The analytical and numerical results are shown for different values of the embedding parameters. Differential transformation method results show that series converge rapidly with high accuracy. The results indicate that the fin tip temperature increases when ambient temperature increases. Conversely, the fin tip temperature decreases with an increase in the Peclet number, convection-conduction and radiation-conduction parameters. It is shown that the fin tip temperature of the exponential profile is higher than the rectangular one. The results indicate that the numerical data and analytical method are in a good agreement with each other.

Key words: *differential transformation method, fin, fin tip temperature, exponential, radiation*

Introduction

In recent years, the heat transport from a moving continuous surface has attracted the attention of some researchers. This phenomenon is an important problems that occurring in a number of industrial applications. Extrusion, hot rolling, glass fiber drawing and casting are example of continuous moving surface. In industrial processes, control of cooling rate of the sheets (or the fibers) is very important to obtain a desired material structure. These types of problems have a solution substantially different from that of a boundary layer flow over a semi-infinite plate. The velocity of moving material is related to its application. For example the velocity of the material can be extremely low (few centimeters per hour) such in crystal growth or very fast (few meters per second) as in optical fiber drawing. Flow and heat transfer in boundary layer on a continuous moving surface have been studied by some researchers. The new class of boundary layer flow over a moving surface was first introduced by Sakiadis [1]. He studied the momentum transfer occurring when a flat surface continuously moves through a quiescent fluid at the constant surface velocity. Erickson *et al*. [2] developed this problem to the case with suction and blowing at the moving surface. The next researches in literature covering surface mass transfer,

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Newtonian and non-Newtonian fluid, magnetic and electric effects, different thermal boundary conditions, combined free and force convection, combined convection and radiation heat transfer, *etc*.

Cortell [3] studied heat transfer in a moving fluid over a moving surface numerically by means of a fourth-order Rung-Kutta method. Ephraim and Abraham [4] investigated the streamwise variation of the temperature of a moving sheet in the presence of moving fluid. They applied an iterative method for solving boundary layer equations. Their solution does not depend on the properties of sheet and fluid. Ming *et al.* [5] studied conjugate heat transfer from a continuous, moving flat plate numerically by employing the cubic spline collocation. They investigated effects of Prandtl number, the convection-conduction parameters and the Peclet number on the heat transfer from a continuous, moving plate. The investigation of mixed convection heat transfer along a continuously moving heated vertical plate with suction and blowing was carried out by Al-Sanea [6]. He applied the finite volume method to solve boundary layer equations. He used the published results available under special condition to validate numerical data, and the comparison indicated an excellent agreement. The buoyancy force and thermal radiation effects in magnetohydodynamics (MHD) boundary layer visco-elastic fluid flow over continuously moving surface were performed by Abel *et al.* [7]. Lee and Tsai [8] studied cooling of a continuous moving sheet of finite thickness. The effect of the buoyancy force is also taken into account. They obtained the temperature distribution along the solid-fluid interface by solving numerically a conjugate heat transfer problem that consists of heat conduction inside the sheet and induced mixed convection adjacent to the sheet surface. Other conjugate convection-conduction researches have been presented by Choudhry and Jaluria [9], and Mendez and Trevino [10], among others. The heat transfer of a moving material in a non-Newtonian fluid was first studied by Fox *et al.* [11]. They applied an exact solution for boundary layer equations. Howell *et al.* [12] studied heat transfer on a continuous moving plate in non-Newtonian power law fluid. They applied Merk-Chao series expansion to generate ordinary differential equation from the partial differential momentum and heat transfer equations in order to obtain universal velocity and temperature functions. Torabi *et al.* [13] investigated convective-radiative non-Fourier heat conduction with variable coefficients by employing homotopy perturbation method (HPM) Some of the other studies that investigated the heat transfer of a continuous moving material in power law fluid have been reported by Sahu *et al.* [14] and, Zheng and Zhang [15].

Temperature distribution for annual fins with temperature-dependent thermal conductivity was studied by Ganji *et al.* [16]. They employed HPM for solving governing equation. The effects of temperature-dependent thermal conductivity of a moving fin and added radiative component to the surface heat loss have been studied by Aziz and Khani [17]. As has been mentioned in [17], these improvements have not been pursued in the literature. They applied the homotopy analysis method (HAM) to solve governing equations. They compared the analytical and numerical results to each other and observed an excellent agreement.

The aim of this study is obtaining an analytical solution for temperature distribution of a moving fin with temperature-dependent thermal conductivity. The effect of the thermal radiation is also considered here. With the inclusion of radiation and variable thermal conductivity, three new parameters, in addition to the Peclet number and Biot number, emerge, namely a thermal conductivity parameter, a radiation-conduction parameter and an environment temperature parameter. The effect of the embedding parameters on the temperature distribution is shown in the moving material. The discrepancy of present study with Aziz's research [17] is that in this study two different profiles (rectangular and exponential profiles) are considered for moving fin. As well as the differential transformation method (DTM) is applied to solve non-linear problem analytically. To validate analytical results, the obtained DTM results are compared with numerical data that are obtained by the fourth-order Rung-Kutta method.

The concept of DTM was first introduced by Zhou [18] in 1986 and it was used to solve both linear and non-linear initial value problems in electric circuit analysis. The main benefit of this method is that it can be used directly for linear and non-linear differential equation without requiring linearization, discretization, or perturbation. The DTM has been regarded by many researchers, Rashidi and Erfani [19] used of DTM for solving Burger's equation and heat conduction problem in fin with temperature dependent thermal conductivity. Joneidi *et al.* [20] applied DTM to determine fin efficiency of convective straight fins with temperature dependent thermal conductivity. Moradi and Ahmadikia [21] applied the DTM to solve the energy equation for a fin with three different profiles and temperature-dependent thermal conductivity. The new algorithm to calculate one and 2-D differential transform of non-linear functions was presented by Chang and Chang [22, 23]. Jang [24] solved the linear and non-linear initial value problems by the projected DTM, that this method can be easily applied to the initial value problem by less computational work. The novel analytical method, namely DTM-Pade to solve MHD stagnation-point flow in porous media with heat transfer was presented by Rashidi and Erfani [25]. When there is an infinite boundary in the problem, the DTM could not to obtain the accurate solution. Because of the Pade approximation is employed to solve problem. Complementary information about this method has been presented in [26].

Fundamentals of DTM

Consider the analytic function $y(t)$ in a domain D where $t = t_i$ represent any point in it. The function $y(t)$ is represented by a power series at center t_i. The Taylor series expansion function of $y(t)$ is in the following form [20]:

$$y(t) = \sum_{j=0}^{\infty} \frac{(t - t_i)^j}{j!} \left[\frac{\mathrm{d}^j y(t)}{\mathrm{d}t^j} \right]_{t=t_i} \qquad \forall t \in D \tag{1}$$

The particular case of eq. (1) is when $t_i = 0$ and is referred to as the Maclaurin series of $y(t)$ expressed as:

$$y(t) = \sum_{j=0}^{\infty} \frac{t^j}{j!} \left[\frac{\mathrm{d}^j y(t)}{\mathrm{d}t^j} \right]_{t=0} \qquad \forall t \in D \tag{2}$$

As explained by Franco [27], differential transformation of the function $y(t)$ is defined as:

$$Y(j) = \sum_{j=0}^{\infty} \frac{H^j}{j!} \left[\frac{\mathrm{d}^j y(t)}{\mathrm{d}t^j} \right]_{t=0} \tag{3}$$

where $y(t)$ is the original function and $Y(j)$ is the transformed function. The differential spectrum of $Y(j)$ is confined within the interval $t \in [0, H]$ where H is a constant. The differential inverse transform of $Y(j)$ is defined as:

$$y(j) = \sum_{j=0}^{\infty} \left(\frac{t}{H} \right)^j Y(j) \tag{4}$$

Some of the original functions and transformed functions are shown in tab. 1. It is clear that the concept of differential transformation is the Taylor series expansion. For the solution with higher accuracy, more terms in the series in eq. (4) should be retained.

Table 1. The fundamental operations of differential transform method

Original function	Transformed function
$f(x) = \alpha g(x) \pm \beta h(x)$	$F(k) = \alpha G(k) \pm \beta H(k)$
$f(x) = g(x)h(x)$	$F(k) = \sum_{i=0}^{k} G(i)H(k-i)$
$f(x) = g(x)^n$	$F(k) = (k+1)(k+2) \ldots (k+n)G(k+n)$
$f(x) = x^n$	$F(k) = \delta(k-n) = \begin{Bmatrix} 1 & k = n \\ 0 & k \neq n \end{Bmatrix}$
$f(x) = \exp(\alpha x)$	$F(k) = \dfrac{\alpha^k}{k!}$
$f(x) = (1+x)^n$	$F(k) = \dfrac{k(k-1)\ldots(k-m-1)}{k!}$

Mathematical formulation

Consider a moving fin of the length L, with a cross-section area $A(x)$ while it moves horizontally with a constant velocity U as depicted in fig. 1. Fin surface is exposed to a convective and radiative environment at temperature T_a and the base temperature of fin is $T_b > T_a$. The local heat transfer coefficient h along the fin surface is constant and the surface of the moving fin is assumed to be gray and diffuse with constant emissivity ε. The role of radiation component

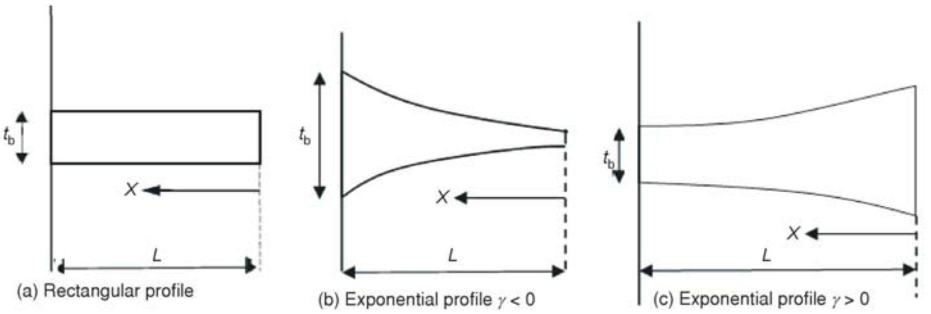

Figure 1. Schematic of different moving fin profiles

could be more sensible if the force convection is weal or absent or when only natural convection occurs. Since the material undergoing the treatment experience a large change in its temperature during thermal process, the thermal conductivity of the material could not be constant. For most materials, the thermal conductivity varies with the temperature linearly. The 1-D steady-state energy equation for the fin moving with a constant speed and losing heat by simultaneous convection and radiation can be expressed as:

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[k(T)A(x)\frac{\mathrm{d}T}{\mathrm{d}x}\right] - ph(T - T_a) - \varepsilon\sigma p(T^4 - T_a^4) - \rho c A(x) U \frac{\mathrm{d}T}{\mathrm{d}x} = 0 \quad (5)$$

where p is the periphery of the fin, T_a – the ambient temperature, ε – the emissivity, σ – the Boltzman constant, ρ – the density of the material, c – the specific heat, and $k(T)$ is defined as:

$$k(T) = k_b[1 + \lambda(T - T_a)] \tag{6}$$

where, k_b is the fin thermal conductivity at ambient temperature, and λ is a constant.

The fin profile is defined according to variation of the fin thickness along its extended length. For example, the cross-section area of the fin may vary as:

$$A(x) = bt(x) \tag{7}$$

where b is the width of the fin, and $t(x)$ – the fin thickness along the length. The $t(x)$ for different profiles can be defined as:

– for rectangular profile $\quad t(x) = t_b$

– for exponential profile $\quad t(x) = t_b \exp\left[\gamma\left(\frac{x}{L}\right)\right]$ (8)

by employing the following dimensionless parameters:

$$\theta = \frac{T}{T_b}, \quad \theta_a = \frac{T_a}{T_b}, \quad X = \frac{x}{L}, \quad N_c = \frac{hpL^2}{k_b A_b}, \quad N_r = \frac{\varepsilon\sigma pL^2 T_b^3}{k_b A_b}, \quad \alpha = \frac{k_b}{\rho c}, \quad \text{Pe} = \frac{UL}{\alpha} \tag{9}$$

where A_b is the base area, N_c – the convection-conduction parameter (more popularly known as the Biot number), N_r – the radiation-conduction parameter, α – the thermal diffusivity of fin and Pe – the Peclet number which represent the dimensionless speed of the moving fin and Pe = 0 represents a stationary fin. Thus, the energy equation for two profiles are reduces to:

– the rectangular profile

$$[1 + a(\theta - \theta_a)]\frac{d^2\theta}{dX^2} + a\left(\frac{d\theta}{dX}\right)^2 - N_c(\theta - \theta_a) - N_r(\theta^4 - \theta_a^4) - \text{Pe}\frac{d\theta}{dX} = 0 \tag{10}$$

– the exponential profile

$$\begin{gathered}[1 + a(\theta - \theta_a)]\left(\gamma\frac{d\theta}{dX} + \frac{d^2\theta}{dX^2}\right) + \exp(\gamma X)a\left(\frac{d\theta}{dX}\right)^2 - \\ -N_c(\theta - \theta_a) - N_r(\theta^4 - \theta_a^4) - \exp(\gamma X)\text{Pe}\frac{d\theta}{dX} = 0\end{gathered} \tag{11}$$

where $a = \lambda T_b$ in which T_b is the base temperature and fin tip is insulated. Therefore, boundary conditions for this problem are defined as:

$$X = 0 \quad \frac{d\theta}{dX} = 0 \tag{12}$$

$$X = 1 \quad \theta = 1 \tag{13}$$

Solution by differential transformation method

The 1-D transform of eqs. (10), and (11) considered by using the related definition in tab. 1, we have:

– rectangular profile

$$\begin{gathered}(j+1)(j+2)\Theta(j+2)(1 - a\theta_a) + a\sum_{i=0}^{j}\Theta(i)(j-i+1)(1-i+2)\Theta(j-i+2) + \\ +a\sum_{i=0}^{j}(i+1)\Theta(i+1)(1-i+1)\Theta(j-i+1) - N_c[\Theta(j) - \theta_a\eta(j)] - \\ -N_r[\sum_{i=0}^{j}\sum_{s=0}^{j-i}\sum_{z=0}^{j-i-s}\Theta(i)\Theta(s)\Theta(z)\Theta(j-i-s-z) - \theta_a^4\eta(j)] - \text{Pe}(j+1)\Theta(j+1) = 0\end{gathered} \tag{14}$$

– exponential profile

$$(1-a\theta_a)\sum_{i=0}^{j}\frac{\gamma^i}{i!}(j-i+1)(j-i+2)\Theta(j-i+2)+$$
$$+a\sum_{s=0}^{j}\sum_{i=0}^{j-s}\frac{\gamma^s}{s!}(j-i-s+1)(j-i-s+2)\Theta(i)\Theta(j-i-s+2)+$$
$$+(1-a\theta_a)\gamma\sum_{i=0}^{j}\frac{\gamma^i}{i!}(j-i+1)\Theta(j-i+1)+a\gamma\sum_{s=0}^{j}\sum_{i=0}^{j-s}\frac{\gamma^s}{s!}(j-i-s+1)\Theta(i)\Theta(j-i-s+1)+$$
$$+a\sum_{s=0}^{j}\sum_{i=0}^{j-s}\frac{\gamma^s}{s!}(i+1)(j-i-s+1)\Theta(i+1)\Theta(j-i-s+1)-N_\mathrm{c}[\Theta(j)-\theta_a\eta(j)]-$$
$$-N_\mathrm{r}[\sum_{i=0}^{j}\sum_{s=0}^{j-i}\sum_{z=0}^{j-i-s}\Theta(i)\Theta(s)\Theta(z)\Theta(j-i-s-z)-\theta_a^4\eta(j)]-$$
$$-\mathrm{Pe}\sum_{i=0}^{j}\frac{\gamma^i}{i!}(j-i+1)\Theta(j-i+1)=0 \tag{15}$$

In the above equations $\Theta(j)$ is transformed function of $\theta(X)$. The transformed boundary condition takes the form:

$$\Theta(1)=0 \tag{16}$$

$$\sum_{i=0}^{\infty}\Theta(i)=1 \tag{17}$$

Supposing $\Theta(0)=\beta$ and using eqs. (16) and (17), another value of $\Theta(i)$ for two profiles can be calculated. The value of can be calculated using the eq. (17). Thus end up having the following:

– rectangular profile

$$\Theta(2)=\frac{N_\mathrm{e}(\beta-\theta_\mathrm{a})+N_\mathrm{r}(\beta^4-\theta_\mathrm{a}^4)}{2(1+a\beta-a\theta_\mathrm{a})}$$
$$\Theta(3)=\frac{\frac{\mathrm{Pe}}{3}[N_\mathrm{c}(\beta-\theta_\mathrm{a})+N_\mathrm{r}(\beta^4-\theta_\mathrm{a}^4)]}{2(1+a\beta-a\theta_\mathrm{a})^2} \tag{18}$$
$$\Theta(4)=\frac{\frac{1}{24}[N_\mathrm{c}(\beta-\theta_\mathrm{a})+N_\mathrm{r}(\beta^4-\theta_\mathrm{a}^4)]\left\{\begin{array}{l}2[N_\mathrm{c}+2\beta^3N_\mathrm{r})+\\+\dfrac{2\mathrm{Pe}^2-6a[N_\mathrm{c}(\beta-\theta_\mathrm{a})+N_r(\beta^4-\theta_\mathrm{a}^4)]}{1+a\beta-a\theta_\mathrm{a}}\end{array}\right\}}{2(1+a\beta-a\theta_\mathrm{a})^2}$$
$$\vdots$$

– exponential profile

$$\Theta(2)=\frac{N_\mathrm{c}(\beta-\theta_\mathrm{a})+N_\mathrm{r}(\beta^4-\theta_\mathrm{a}^4)}{2(1+a\beta-a\theta_\mathrm{a})}$$
$$\Theta(3)=\frac{(\beta-\theta_\mathrm{a})[\mathrm{Pe}-2\gamma(1+a\beta)+2a\gamma\,\theta_\mathrm{a}][N_\mathrm{c}+N_\mathrm{r}(\beta^3+\beta^2\theta_\mathrm{a}+\beta\theta_\mathrm{a}^2+\theta_\mathrm{a}^3)]}{6(1+a\beta-a\theta_a)^4} \tag{19}$$
$$\vdots$$

By substituting eqs. (18) in eq. (4), for $H=1$, we can obtain the closed form of the solution:

– rectangular profile

$$\theta(X)=\beta+\frac{N_{\mathrm{c}}(\beta-\theta_{\mathrm{a}})+N_{\mathrm{r}}(\beta^4-\theta_{\mathrm{a}}^4)}{2(1+a\beta-a\theta_{\mathrm{a}})}X^2+\frac{\frac{\mathrm{Pe}}{3}[N_{\mathrm{c}}(\beta-\theta_{\mathrm{a}})+N_{\mathrm{r}}(\beta^4-\theta_{\mathrm{a}}^4)]}{2(1+a\beta-a\theta_{\mathrm{a}})^2}X^3+$$
$$+\frac{\frac{1}{24}[N_{\mathrm{c}}(\beta-\theta_{\mathrm{a}})+N_{\mathrm{r}}(\beta^4-\theta_{\mathrm{a}}^4)\left\{\begin{aligned}&2(N_{\mathrm{c}}+2\beta^3N_{\mathrm{r}})+\\&+\frac{2\mathrm{Pe}^2-6a[N_{\mathrm{c}}(\beta-\theta_{\mathrm{a}})+N_{\mathrm{r}}(\beta^4-\theta_{\mathrm{a}}^4)}{(1+a\beta-a\theta_{\mathrm{a}})}\end{aligned}\right\}}{2(1+a\beta-a\theta_{\mathrm{a}})^2}X^4+\dots \tag{20}$$

In order to obtain the value β we used eq. (17). Then we will have:

$$\theta(1)=\beta+\frac{N_{\mathrm{c}}(\beta-\theta_{\mathrm{a}})+N_{\mathrm{r}}(\beta^4-\theta_{\mathrm{a}}^4)}{2(1+a\beta-a\theta_{\mathrm{a}})}+\frac{\frac{\mathrm{Pe}}{3}[N_{\mathrm{c}}(\beta-\theta_{\mathrm{a}})+N_{\mathrm{r}}(\beta^4-\theta_{\mathrm{a}}^4)]}{2(1+a\beta-a\theta_{\mathrm{a}})^2}+$$
$$+\frac{\frac{1}{24}[N_{\mathrm{c}}(\beta-\theta_{\mathrm{a}})+N_{\mathrm{r}}(\beta^4-\theta_{\mathrm{a}}^4)]\left\{\begin{aligned}&2(N_{\mathrm{c}}+2\beta^3N_{\mathrm{r}})+\\&+\frac{2\mathrm{Pe}^2-6a[N_{\mathrm{c}}(\beta-\theta_{\mathrm{a}})+N_{\mathrm{r}}(\beta^4-\theta_{\mathrm{a}}^4)}{1+a\beta-a\theta_{\mathrm{a}}}\end{aligned}\right\}}{2(1+a\beta-a\theta_{\mathrm{a}})^2}+\dots=1 \tag{21}$$

Solving eq. (21) by MATHEMATICA software, gives the value of β. For the exponential profile the same process is used to obtain the value of β and temperature distribution.

Results and discussions

The analytical results are shown for 50 terms of the final power series here. The effect of thermal conductivity parameter a on the temperature distribution for both rectangular and exponential ($\gamma = 1$) profiles is shown in fig. 2 for $N_{\mathrm{c}} = 4$, $N_2 = 4$, Pe = 3, and $\theta_{\mathrm{a}} = 0.2$. The bottom line ($a = 0$) represent the constant thermal conductivity for the rectangular profile. As shown in fig. 2, with an increase in the thermal conductivity parameter a, the fin tip temperature increases as well. The fin tip temperature for the exponential profile is higher than the rectangular one. For validating DTM results, the analytical solution is compared with the numerical solution which is obtained by the fourth-order Rung- -Kutta scheme. Figures 3 and 4 show that how the temperature distributions of fin are affected by the changes in the convection-conduction parameter for rectangular and exponential ($\gamma = 1$) profiles, respectively, when $N_{\mathrm{r}} = 0.25$, $a = 1$, Pe = 2 and $\theta_{\mathrm{a}} = 0.5$. As depicted in figs. 3 and 4, when the convection-conduction parameter increases, the losing heat of the fin by convection gets stronger, the cooling becomes more effective, thus the temperature of fin decrease.

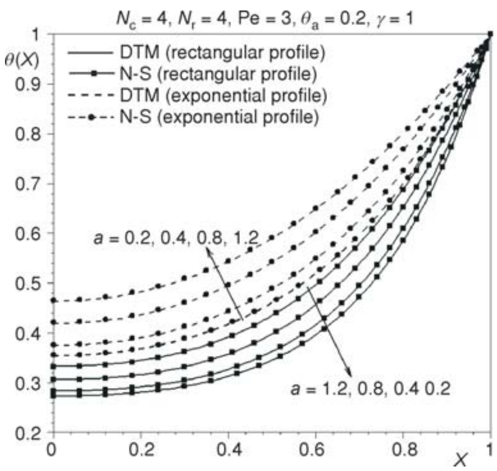

Figure 2. Temperature profile for the rectangular and exponential ($\gamma = 1$) profiles for different values of *a* when: $N_{\mathrm{c}} = 4$, $N_{\mathrm{r}} = 4$, Pe = 3, and $\theta_{\mathrm{a}} = 0.2$

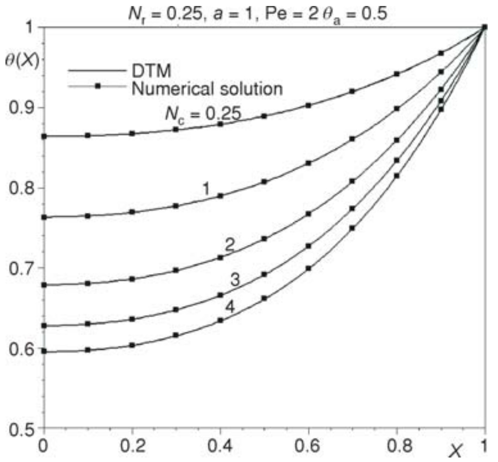

Figure 3. Temperature distribution of the rectangular profile for different values of N_c when: N_r = 0.25, a = 1, Pe = 2, and θ_a = 0.5

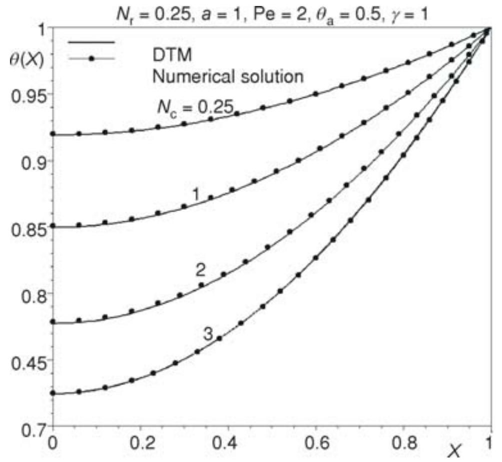

Figure 4. Temperature distribution of the exponential profile (γ = 1) for different values of N_c when: N_r = 0.25, a = 1, Pe = 2, and θ_a = 0.5

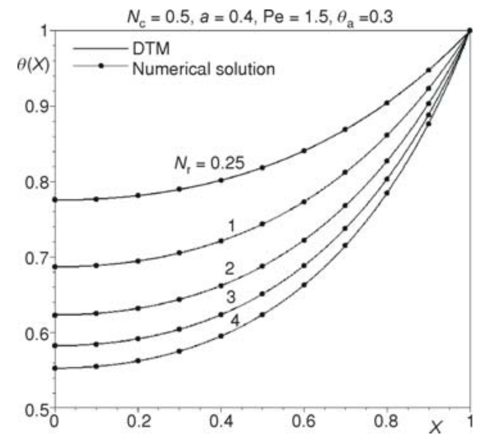

Figure 5. Temperature distribution of the rectangular profile for different values of N_r when: N_c = 0.5, a = 0.4, Pe = 1.5, and θ_a =0.3

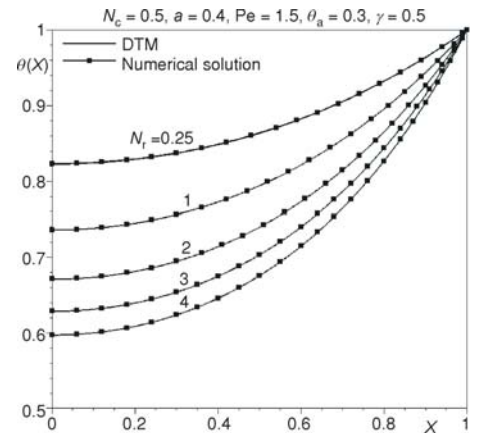

Figure 6. Temperature distribution of the exponential profile (γ = 0.5) for different values of N_r when: N_c = 0.5, a = 0.4, Pe = 1.5, and θ_a

The effect of radiation-conduction parameter on the temperature profiles for rectangular and exponential (γ = 0.5) profiles are shown in figs. 5 and 6, respectively, with the rest of the parameters fixed at N_c = 0.25, a = 0.4, Pe = 1.5, and θ_a = 0.3. With an increase in the radiation-conduction parameter as the convection-conduction parameter, the radiative transfer becomes stronger, thus as shown in figs. 3 and 4, the fin temperature decreases. Figure 7 depicts the temperature profiles for both rectangular and exponential (γ = 0.5) profiles when the ambient temperature is 0.2, 0.4, 0.6, and 0.8. The other parameters are fixed at N_c = 4, N_r = 4, a = 0.2, and Pe = 3. As shown in fig. 7, when the ambient temperature increases, the temperature difference between the fin and ambient temperature becomes shorter. Consequently, the fin temperature increases. As illustrated, the obtained fin temperature for the exponential profile is higher

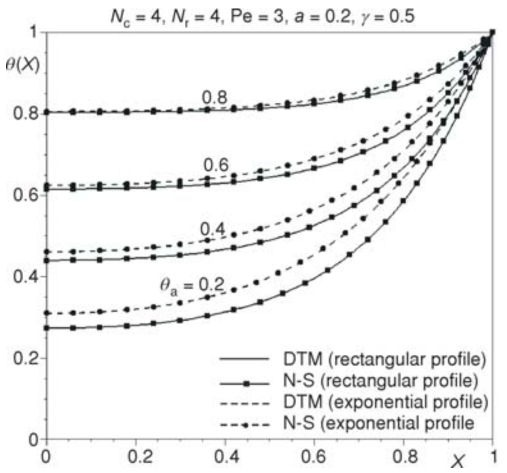

Figure 7. Temperature distribution for the rectangular and exponential (γ = 0.5) profiles for different values of θ_a when: N_c = 0.5, N_r = 4, a = 0.4, and Pe = 3

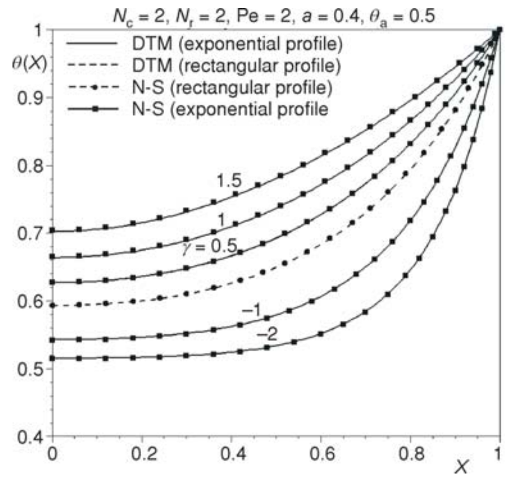

Figure 8. Temperature distribution for different values of exponential parameter when: N_c = 2, N_r = 2, a = 0.4, Pe = 2, and θ_a = 0.5

than in the rectangular one. While the discrepancy between rectangular and exponential profiles becomes shorter for the larger values of ambient temperature (see fig. 7). The effect of the exponential parameter on the temperature profile is illustrated in fig. 8 when $N_c = 2$, $N_r = 2$, $a = 0.4$, Pe = 2, and $\theta_a = 0.5$. As the exponential parameter becomes larger, the fin temperature increases. The results indicate that for positive values of the exponential parameter, the fin temperature of exponential profile is larger the rectangular one. While for the negative values of the exponential parameter, this result is reverse. The effect of the Peclet number (dimensionless speed) on the temperature distribution for both rectangular and exponential (γ = 0.3) profiles is shown in fig. 9 with the remaining parameters fixed at $N_c = 0.25$, $N_r = 1$, $a = 0.6$, and $\theta_a = 0.6$.

With an increase in the Peclet number, the losing heat form fin surface becomes stronger, thus the fin temperature decreases. The comparison between DTM and numerical results for both profiles when $N_c = 1.5$, $N_r = 0.5$, $a = 0.5$, and Pe = 1.5 is shown in tab. 2. In all figures and tab. 2, the agreement between analytical and numerical results is observed that it confirms the accuracy of the differential transformation method to solve non-linear boundary value problems.

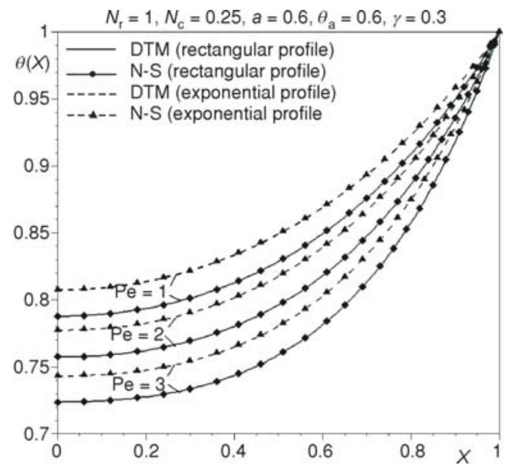

Figure 9. Temperature distribution for the rectangular and exponential (γ= 0.3) profiles for different values of Peclet number when: N_c = 0.25, N_r = 1, a = 0.6, and θ_a = 0.6

Conclusions

In this study, the differential transformation method was applied to solve simultaneous con-

Table 2. Comparison between analytical and numerical results for both the rectangular and exponential profile when: N_c = 1.5, N_r = 0.5, *Pe* = 1.5, *a* = 0.5, and θ_a =0.3

X	Rectangular profile		Exponential profile $\gamma = 1$		Exponential profile $\gamma = -1$	
	$\theta(X)$ DTM	$\theta(X)$ N-S	$\theta(X)$ DTM	$\theta(X)$ N-S	$\theta(X)$ DTM	$\theta(X)$ N-S
0	0.6087817	0.6087816	0.731148	0.7328734	0.480268	0.4786148
0.1	0.6111722	0.611172	0.73430151	0.7360088	0.4817751	0.4801234
0.2	0.6188068	0.6188067	0.74348468	0.7451388	0.4870606	0.4854151
0.3	0.6324888	0.6324887	0.75834274	0.7599093	0.4975721	0.4959404
0.4	0.6531764	0.6531763	0.77858166	0.7800266	0.5152683	0.5136639
0.5	0.6819939	0.6819938	0.80394739	0.8052367	0.5427909	0.5412371
0.6	0.7202402	0.7202402	0.83421062	0.8353101	0.583679	0.5822145
0.7	0.7693942	0.7693942	0.8691558	0.8700314	0.6426316	0.64132
0.8	0.8311216	0.8311216	0.90857383	0.9091914	0.7258297	0.7247716
0.9	0.9072892	0.9072893	0.95225757	0.9525833	0.8413691	0.8407204
1	1	1	1	1	1	1

vection and radiation heat transfer problem in a continuously moving fin with temperature thermal conductivity. The rectangular and exponential profiles were considered for a moving fin. This method has been applied for the linear and non-linear differential equations. This method is an infinite power-series form and has high accuracy and fast convergence. To validate the analytical results, DTM results are compared with numerical data obtained using the fourth order Runge-Kutta method. The results illustrate how the temperature distributions in the moving fin are affected by the changes in the embedding parameters. The results indicate that the fin temperature increases with an increase in the exponential parameter as well as the fin temperature of exponential profile for the positive values of exponential parameter is larger than the rectangular one. In general, DTM has a good approximate analytical solution for the linear and non-linear engineering problems without any assumption and linearization.

Nomenclature

$A(x)$ – fin cross-section, [m^2]
a – thermal expansion coefficient, [K^{-1}]
b – width of the fin
c – specific heat, [Jkg^{-1}K^{-1}]
H – constant
h – heat transfer coefficient, [Wm^{-2}K^{-1}]
k – thermal conductivity, [Wm^{-1}K^{-1}]
L – fin length, [m]
N_c – convection-conduction fin parameter (= hpL^2/K_bA_b), [–]
N_r – radiation-conduction fin parameter (= $\varepsilon\sigma pL^2T_b^3/k_bA_b$), [–]
p – periphery of the fin cross-section, [m]
Pe – Peclet number (= UL/α) , [–]
T – temperature, [K]
t – fin thickness, [m]
U – velocity of fin, [ms^{-1}]
X – non-dimensional space co-ordinate
x – dimensional space co-ordinate, [m]
Y – transformed function
$y(t)$ – original analytic function

Greek symbols

α – thermal diffusivity (= $k_b/\rho c$), [m^{2}s^{-1}]
γ – exponential parameter
ε – emissivity
λ – dimensional constant, [K^{-1}]
Θ – transformed temperature
θ – dimensionless temperature
ρ – density, [kgm^{-3}]
σ – Stefan-Boltzmann constant, [Wm^{2}K^{-4}]

Subscripts

a – ambient property
b – fin base

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