HEAT TRANSFER COMPARISON BETWEEN A VERTICAL RECTANGULAR CAVITY AND AN ISOSCELES RIGHT-ANGLED TRIANGULAR CAVITY OF EQUAL CROSS-SECTIONAL AREA

by

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This paper addresses the heat transfer performance of natural convection flows in three different (but related) cavities in the form of: a square, isosceles right-angled triangle, and vertical rectangle with aspect ratio 2:1. The isosceles right-angled triangular cavity is derived from a square cavity when cut in half diagonally, whereas the vertical rectangular cavity is derived from a square cavity when cut in half vertically. In the three cavities, the left vertical wall is the common wall heated. The buoyant air flow is characterized by height-based Rayleigh numbers ranging from a conduction-dominant to up to $10^6$ for the laminar natural convection regime. Employing the finite volume method, the velocity and temperature fields as well as the mean convective coefficients evaluated at the common heated vertical wall are numerically determined for the isosceles right-angled triangular cavity. For this cavity, flow streamlines and temperature contours are presented in graphical form and some numerical results are validated against published experimental measurements. A one-to-one comparison for the heat transfer performance of the three interconnected cavities is reported in tabulated form.

Key words: natural convection, laminar regime, square cavity, isosceles right-angled triangular cavity, 2:1 vertical rectangular cavity, heat transfer performance

Introduction

A vast collection of studies encompassing theoretical analyses, numerical computations and experimental measurements for natural convection flows of single-phase fluids across 2-D stationary vertical, rectangular cavities has been disclosed in handbook chapters written by Raithby et al. [1], Charmchi et al. [2], and Jaluria [3].

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In particular, the specialized literature is superabundant with regards to natural convection in horizontal-placed right triangular cavities. In this configuration, there are two situations of interest: (1) a hot base, a cold inclined side and thermally insulated vertical side and (2) a cold base, a hot inclined side and thermally insulated vertical side. This topic is relevant to quantify the heat transfer characteristics of house and building attics within the framework of HVAC industries and representative publications are those of Asan et al. [4], Haese et al. [5], Ridouane et al. [6, 7] and some references cited therein. With regards to the results in [4, 5], it should be recognized that they are of limited utility because of the supposition that a plane of symmetry. This plane of symmetry exists for low flow and temperature values that correspond to low Rayleigh numbers. On the contrary, the flow and temperature results divulged in [6, 7] fully conform to the physics of the problem. Certainly, they are more accurate because the tricky idealization of the plane of symmetry was not invoked.

To the author’s knowledge, the specialized literature is scarce for natural convection in vertical-placed right-angled triangular cavities. This specific layout of cavities finds application in contemporary electronic packaging because of space and/or weight constraints [8, 9].

In general, when buoyant fluid motion occurs in confined spaces like cavities, regardless of their shapes, the augmentation of natural convection heat transfer becomes a difficult task because of the low fluid velocities imparted by the acting gravitational forces. Owing to this adverse effect, it is of fundamental and practical interest to explore other suitable cavity shapes that promote the needed augmentation of natural convection in cavities. In general, the design engineer has to resort to experience, intuition and experimentation in order to improve the heat transfer performance of natural convection cavities. Thereby, the present paper is centered in studying the heat transfer performance of three cavities: the isosceles right-angled triangular cavity and the 2:1 vertical rectangular cavity; both are derived from the square cavity. The finite volume method is the vehicle used to determine the velocity and temperature fields induced by the buoyant air in the isosceles right-angled triangular cavity under the influence of low, moderate and large height-based Rayleigh numbers. Pertinent heat transfer information about the square and the 2:1 vertical rectangular cavities is taken from the specialized literature.

**Physical system and mathematical formulation**

A sketch of a stationary square cavity heated at the left vertical wall and cooled at the right vertical wall with the top and bottom walls being thermally insulated is unnecessary. In the context of natural convection cavities, Frederick [10] conducted a numerical study with air in several horizontal and vertical rectangular cavities with lateral heating/cooling at the two vertical walls. Selecting three different Rayleigh numbers for each aspect ratio $Ar = H/W$ ($H$ is the height and $W$ is the base) comprised between 1 and 2, this author found that the heat transfer across the collection of cavities tested attained a maximum at different Rayleigh numbers. This discovery is equivalent to saying that such optimal cavity size lies between the square cavity and a vertical rectangular cavity twice the size of the square cavity. Motivated by the outcome of this work, to search for superior packaging, we decided to move further and explore two possibilities for cutting a square cavity in half in two different ways. Firstly, the square cavity was cut diagonally retaining the upper isosceles right-angled triangular cavity in fig. 1(a) with the diagonal wall cold. Secondly, the square cavity was cut vertically retaining the left vertical rectangular cavity in fig. 1(b) owing an aspect ratio $Ar = 2:1$ with the right wall cold. While keeping the square cavity as the baseline case, the principal idea of the present study is to carry a parametric heat/fluid flow study of the two derived cavities sharing
the same cross-sectional area. In the three cases, the gravitational acceleration points downward. The dimension perpendicular to the plane of the trio of cavities is assumed to be long compared to other two dimensions, so that the laminar thermo-buoyant flow movement is predominantly 2-D. To avoid restrictions in the operational temperatures, the confined air is treated as a non-Boussinesqian fluid. Accordingly, the mathematical formulation written in Cartesian tensor notation \((j = 1, 2)\) is:

\[
\begin{align*}
- \text{mass} & \quad \frac{\partial (p u_j)}{\partial x_j} = 0 \\
- \text{momentum} & \quad \rho u_j \frac{\partial u_j}{\partial x_j} = \frac{\partial}{\partial x_j} \left[ \mu \left( \frac{\partial u_j}{\partial x_j} + \frac{\partial u_j}{\partial x_j} \right) \right] + X_j \\
- \text{energy} & \quad \rho c_v u_j \frac{\partial T}{\partial x_j} = \frac{\partial}{\partial x_j} \left( k \frac{\partial T}{\partial x_j} \right) \\
- \text{ideal gas equation of state} & \quad p = \rho RT
\end{align*}
\]

In eq. (2) \(X_j\) stands for the gravitational body force, being equal to \(X_1 = 0\) in the horizontal direction and \(X_2 = -g(\rho - \rho_{\text{ref}})\) in the vertical direction. Herein, the reference density \(\rho_{\text{ref}}\) is evaluated at a reference temperature \(T_{\text{ref}} = (T_H + T_C)/2\).

The velocity boundary conditions are connected to: (a) solid, impermeable walls and (b) no slip air occurs at the bounding walls. For the temperature boundary conditions, a prescribed hot temperature at the left vertical walls, a prescribed cold temperature at the inclined wall and the right vertical walls in addition to the temperature gradient equal to zero at all horizontal wall(s).

The velocity and temperature fields of the circulatory air \(u(x, y), v(x, y), T(x, y)\) are obtained numerically. Based on the definition of the stream function \(\psi(x, y)\), the velocity field \(u(x, y)\), and \(v(x, y)\) are post-processed first to determine the flow streamlines:

\[
u = -\frac{\partial \psi}{\partial x}, \quad v = \frac{\partial \psi}{\partial y}
\]

and the companion temperature contours. Second, the local wall heat flux \(q_w(y)\) is found by specifying Fourier’s law at the hot vertical wall, \((0, y)\), where the thermal conductivity \(k\) of the air is evaluated at the reference temperature \(T_{\text{ref}} = (T_H + T_C)/2\). Third, the computation of the mean wall heat flux \(\overline{q_w}\) is carried out by way of the mean value of the \(q_w(y)\) function along the hot vertical wall:

![Figure 1. (a) The isosceles right-angled triangular cavity, (b) the 2:1 vertical rectangular cavity](image-url)
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\[
\overline{q_w} = \frac{1}{H} \int_0^H q_w(y)dy \tag{6}
\]

Fourth, the mean convective coefficient \(\overline{h}\) on the hot vertical wall defined as:

\[
\overline{h} = \frac{q_w}{T_H - T_C} \tag{7}
\]
gives way to the equivalent mean Nusselt number:

\[
\overline{Nu}_H = \frac{\overline{h}H}{k} = \frac{q_w}{T_H - T_C} \frac{H}{k} \tag{8}
\]

**Computational procedure**

There is no need to redo the numerical calculations for the square cavity and the 2:1 rectangular cavity because there is a superabundance of heat transfer information for both in the specialized literature [1-3]. In view of this, the numerical computations are carried out for the isosceles right triangular cavity exclusively.

For a square and the 2:1 vertical rectangular cavities, the highly acclaimed correlation equation for the mean Nusselt number \(\overline{Nu}_H\) developed by Berkosky et al. [11] is:

\[
\overline{Nu}_H = 0.18 \left( \frac{Pr}{0.2 + Pr} \cdot Ra_H^{0.29} \right) \left( \frac{H}{W} \right)^{0.13} \quad \text{for} \quad \begin{cases} 
1 < \frac{H}{W} < 2 \\
10^{-3} < Pr < 10^5 \\
Ra_H \left( \frac{H}{W} \right)^{-3} > 10^3 
\end{cases} \tag{9}
\]

For the case of air or pure gases with \(Pr = 0.71\), the above correlation equation simplifies to:

\[
\overline{Nu}_H = 0.17 \cdot Ra_H^{0.29} \left( \frac{H}{W} \right)^{0.13} \quad \text{for} \quad \begin{cases} 
1 < \frac{H}{W} < 2 \\
Ra_H \left( \frac{H}{W} \right)^{-3} > 1,282 
\end{cases} \tag{10}
\]

The numerical computations for the isosceles right-angled triangular cavity are performed with the finite volume code FLUENT 6.1 [12]. The computational domain being coincident with the physical domain was created and meshed with the grid generation software Gambit 2.0® [12]. In eqs. (1)-(4), the discretization of the convective term is accomplished by the second order accurate scheme QUICK, while the pressure-velocity coupling is handled with the SIMPLE scheme [13].

As estimations for natural convection cavity flows are obtained at increasingly higher Rayleigh numbers, it is logical to question the physical reality behind the numerical solutions that are generated under the premises of laminar regime. Yet as the impressed wall-to-wall temperature difference \(T_H - T_C\) rises, all laminar convective flows become turbulent at sufficiently high
Rayleigh numbers (Ra) and many of them turn oscillatory over a certain range of Ra. The oscillatory regime usually is manifested somewhere in between the laminar and turbulent regimes. Le Quere et al. [14] investigated the validity of steady solutions by way of analyzing laminar convective flows in a square cavity with top and bottom insulated walls. Using a time-dependent finite-difference code, these authors found that the onset of oscillations usually occurs at a Ra that ranges between $2 \cdot 10^5$ and $2.2 \cdot 10^5$. This solid recommendation will be adopted in this work too for one half of the square cavity. Basak et al. [15] analyzed the heat flow patterns in cavities using Bejan’s heatline concept. The key parameters for our study are the Prandtl number, Rayleigh number, and Nusselt number. The Ra has been varied from $10^5$ to $10^6$. For low Ra, it is found that the heatlines are smooth and perfectly normal to the isotherms indicating the dominance of conduction. But as Ra increases, flow slowly becomes convection dominant. It is also observed that multiple secondary circulations are formed for fluids with low Pr where these features are absent in higher Pr fluids. Multiple circulation cells for smaller Pr also correspond multiple cells of heatlines which illustrate less thermal transport from hot wall. On the other hand, the dense heatlines at bottom wall display enhanced heat transport for larger Pr.

Fixing $T_H = 313$ K and $T_C = 287$ K, the cavity height (the characteristic length) was varied to obtain different values of the Rayleigh number inside the interval $(10^3-10^5)$. Care was taken to increase the element density in vulnerable areas of the isosceles right-angled triangular cavity, such as near the solid walls where high velocity and temperature gradients would occur. Based on a sequence of numerical experiments, various grid sizes having between 30,000 up to 90,000 triangular elements were tried and examined. The optimal grid was found with 65,000 triangular elements carrying an error within 1%. This grid layout rendered ultra reliable results for the velocity and temperature fields $u(x, y)$, $v(x, y)$, and $T(x, y)$ for all values of Ra. Also, global convergence was guaranteed by controlling the residuals of eqs. (1)-(4) to be less than $10^{-5}$. After satisfactory convergence of the velocity and temperature fields was attained, we proceeded to calculate the streamlines, isotherms, and mean wall heat flux at the vertical heated wall, $\overline{Q}_w$. Needless to say, $\overline{Q}_w$ is the ultimate quantity of interest for purposes of engineering analysis and design of cavities.

![Figure 2](image)

**Figure 2. Comparison between the predicted and measured dimensionless temperatures in the isosceles right-angled triangular cavity**

Experimental validation

A search of the specialized literature revealed no experimental results for the size of the isosceles right-angled triangular cavity under study here. Luckily, experimental temperature measurements for a slender right-angled triangular cavity with apex angle $\alpha = 15^\circ$ are available in Elicer-Cortes et al. [16]. Figure 2 illustrates a reasonable parity between the estimated and the measured air temperature profiles at three different relative heights $y/H = 0.1, 0.58$, and 0.99. The lowermost curve for $y/H = 0.1$ indicates that the numerical predictions overlap perfectly with the experimental measurements. From thermal physics, it may also be inferred that in this lower corner region, the air is basically remains motionless and the transfer of heat occurs by conduction. The numerical temperatures slightly overpredict the experimental ob-
servations at the other relative heights $y/H = 0.58$ and $0.99$. The largest discrepancy occurs for the uppermost curve representative of the relative height $y/H = 0.99$. This location is very close to the horizontal insulated wall where the upward airflow slows down after turning the upper left corner but still moves horizontally with a vigorous velocity. Overall, the figure reaffirms that the agreement between the numerical and experimental temperatures of the air flows at the three locations is acceptable.

**Discussion of results**

From the framework of thermal physics, knowledge of the departure from conduction and the onset of natural convection is of special interest. In the case of the isosceles right-angled triangular cavity, this phenomenon is quantified by an approximate critical Rayleigh number $Ra_{HC} = 8 \times 10^3$, which is roughly one order of magnitude higher than $Ra_{HC} = 10^5$ for the counterpart square cavity.

When examining fig. 3, we observed contour plots of stream functions and temperatures for the isosceles right-angled triangle cavity characterized by a low $Ra_H = 10^3$. From the stream function plots, it can be seen that the configuration contains a single rotating vortex. The direction of the vortex rotation can be determined by finding the sign of the gradient for the stream function in the $x$-direction and also remembering that the velocity in the $y$-direction is opposite in sign to the stream function gradient. The vertical velocity is positive along the hot vertical wall and negative along the cold inclined wall and the vortex is rotating in a clockwise direction.

Next, fig. 4 portrays the stream function and temperature contour plots for a high $Ra_H = 10^6$. In here, the presence of the strongest vortex is observable, which is in sharp contrast to the weak rotation strength illustrated in fig. 3. This specific vortex moves towards the zone with high horizontal thermal gradient (downwards) when $Ra_H$ is increased.

![Figure 3. Stream functions and isotherms for an isosceles right-angled triangular cavity characterized by $Ra_H = 10^6$ (color image see on our web site)](image)

Table 1 presents a comparison of the heat transfer performance for the three cavities under study, all influenced by various temperature differentials contained indirectly in the relative large $Ra_H$ interval: $10^3 \leq Ra_H \leq 10^6$. The baseline case is obviously the square cavity.
Figure 4. Stream functions and isotherms for an isosceles right-angled triangular cavity characterized by $Ra_H = 10^6$ (color image see on our website).

Table 1. Comparison of the heat transfer performance between a square cavity, the 2:1 vertical rectangular cavity, and the isosceles right-angled triangular cavity

<table>
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<tr>
<td>$10^3$</td>
<td>1.26</td>
<td>1.38</td>
<td>9.52</td>
<td>4.23</td>
<td>235.71</td>
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<td>2.36</td>
<td>17.41</td>
<td>4.35</td>
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<td>$10^4$</td>
<td>2.46</td>
<td>2.89</td>
<td>17.48</td>
<td>4.80</td>
<td>95.12</td>
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<td>$5 \times 10^4$</td>
<td>3.92</td>
<td>4.61</td>
<td>17.60</td>
<td>5.95</td>
<td>51.79</td>
</tr>
<tr>
<td>$10^5$</td>
<td>4.79</td>
<td>5.64</td>
<td>17.75</td>
<td>6.69</td>
<td>39.67</td>
</tr>
<tr>
<td>$5 \times 10^5$</td>
<td>7.64</td>
<td>8.99</td>
<td>17.67</td>
<td>9.13</td>
<td>19.50</td>
</tr>
<tr>
<td>$10^6$</td>
<td>9.34</td>
<td>10.99</td>
<td>17.67</td>
<td>10.70</td>
<td>14.56</td>
</tr>
</tbody>
</table>

It is evident that for the natural convection effectiveness, the 2:1 vertical rectangular cavity outperforms the square cavity by a margin of approximately 17.5% for all $Ra_H < 10^6$. There is one exception for the low $Ra_H = 10^3$ where this margin descends to 9.52%. The superiority manifested by the isosceles right triangular cavity over the square cavity starts with a remarkable 235.71% at a low $Ra_H = 10^3$ and manifests a gradual decreasing behavior with increments in $Ra_H$. That is, the improvement is still high with 95.12% for $Ra_H = 10^4$ and for $Ra_H = 10^5$ comes down to 39.67%. This beneficial behavior can be explained in two parts. That is, (1) the stagnant core region in the isosceles right triangular cavity is smaller than in the 2:1 vertical rectangular cavity and (2) the ascending hot flow turns two 90° angles in the 2:1 vertical rectangular cavity whereas the flow turns one 90° degree angle and one 45° degree angle. Furthermore, a notable exception corresponds to the large $Ra_H = 10^5$ owing that the betterment is just 14.56%, as compared to 17.67% for the 2:1 vertical rectangular cavity.

A non-linear regression analysis of the $Nu_H$ vs $Ra_H$ data for the isosceles right triangular cavity was performed with Minitab [17]. The baseline involves the correlation equation for the square cavity:
\[ \bar{\text{Nu}}_H = 0.17 \text{Ra}_{H}^{0.29} \]  
\[ (11a) \]
given in eq. (10). Correspondingly, two avenues are explored. A first outcome in additive form is:
\[ \bar{\text{Nu}}_H = 0.17 \text{Ra}_{H}^{0.29} + 6.18 \text{Ra}_{H}^{-0.11} \]  
\[ (11b) \]
with R-square = 99.8% and maximum error 7.43% at Ra\_H = 5,000. A second outcome in direct form is:
\[ \bar{\text{Nu}}_H = 4.54 + 18.6 \frac{\text{Ra}_{H}}{10^6} - 12.8 \left( \frac{\text{Ra}_{H}}{10^6} \right)^2 \]  
\[ (11c) \]
with R-square = 98.9% and maximum error 6.32% at Ra\_H = 10,000. The two new correlation equations are valid for the moderate Ra\_H-interval \(10^3 \leq \text{Ra}_{H} \leq 10^6\).

When the magnitude of the mean Nusselt number \( \bar{\text{Nu}}_H \) is known as a function of Ra\_H, the heat transfer rate \( Q \) can be determined from “Newton’s law of cooling”:
\[ Q = hA_s(T_H - T_C) = k \bar{\text{Nu}}_H A_s(T_H - T_C) \]  
\[ (12) \]
For the case of air with \( k = 0.026 \text{ W/mK} \), the heat transfer rate per unit depth is:
\[ Q' = 0.026 \bar{\text{Nu}}_H H (T_H - T_C) \]  
\[ (13) \]
where \( \text{Nu}_H \) is taken from either eqs. (11b) or (11c).

**Conclusions**

For the isosceles right-angled triangular cavity with a cold wall \( \sqrt{2} H \) larger than the cold wall of the square and 2:1 vertical rectangular cavities relatively, it is evident that high fluid velocities exist. The heat transfer activity is higher in the bottom corner between the hot and cold walls, which occurs by conduction. In conclusion, it has been demonstrated that for purposes of heat transfer enhancement, the best is achieved by the isosceles right-angled triangular cavity up to Ra\_H = 10^6 where both \( \bar{\text{Nu}}_H \) equalize. From fluid physics, the onset of natural convection in the isosceles right-angled triangular cavity is dictated by Ra\_HC = 8 \times 10^3.

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**Nomenclature**

- \( Ar \) – aspect ratio of cavity, \( (= H/W) \)
- \( A_s \) – surface area, [m\(^2\)]
- \( c_p \) – specific heat at constant pressure, [Jkg\(^{-1}\)K\(^{-1}\)]
- \( \text{Gr}_H \) – Grashof number, \( [= g\beta\rho\beta'((T_H - T_C) H)]\)
- \( g \) – acceleration of gravity, [m\(^{-2}\)]
- \( H \) – height of cavity and characteristic length, [m]
- \( \bar{h} \) – mean convective coefficient, [Wm\(^{-2}\)K\(^{-1}\)]
- \( k \) – thermal conductivity, [Wm\(^{-1}\)K\(^{-1}\)]
- \( \text{Nu}_H \) – mean Nusselt number, \( (= \bar{h} H/k) \)
- \( \text{Pr} \) – Prandtl number, \( (= \mu c_p k) \)
- \( p \) – pressure, [Pa]
- \( q_w \) – wall heat flux, [Wm\(^{-2}\)]
- \( q_{\text{m}} \) – meanwall heat flux, [Wm\(^{-2}\)]
- \( R \) – gas constant, [Jkg\(^{-1}\)K\(^{-1}\)]
Greek symbols

\[ \alpha \] – apex angle formed between the vertical and inclined wall
\[ \beta \] – coefficient of volumetric thermal expansion, \([K^{-1}]\)
\[ \theta \] – dimensionless temperature,
\[ (T - T_c)/(T_h - T_c) \]
\[ \mu \] – viscosity, \([kgm^{-1}s^{-1}]\)
\[ \rho \] – density, \([kgm^{-3}]\)
\[ \psi \] – steam function, \( u = \partial \psi / \partial y, v = -\partial \psi / \partial x \)

References

[12] www.fluent.com
[17] www.minitab.com

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