

THERMAL CONVECTION OF DUSTY COMPRESSIBLE RIVLIN-ERICKSEN VISCOELASTIC FLUID WITH HALL CURRENTS

by

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An investigation is made on the effect of Hall currents and suspended particles on the hydromagnetic stability of a compressible, electrically conducting Rivlin-Ericksen elastico-viscous fluid. The perturbation equations are analyzed in terms of normal modes after linearizing the relevant set of hydromagnetic equations. A dispersion relation governing the effects of viscoelasticity, magnetic field, Hall currents, compressibility and suspended particles is derived. For the stationary convection Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid due to the vanishing of the viscoelastic parameter. Compressibility and magnetic field are found to have a stabilizing effect on the system whereas Hall currents and suspended particles hasten the onset of thermal instability. These analytic results are confirmed numerically and the effects of various parameters on the stability parameter are depicted graphically. The critical Rayleigh numbers and the wavenumbers of the associated disturbances for the onset of instability as stationary convection are obtained and the behavior of various parameters on critical thermal Rayleigh numbers has been depicted graphically. It has been observed that oscillatory modes are introduced due to the presence of viscoelasticity, suspended particles and Hall currents which were not existing in the absence of these parameters.

Key words: Rivlin-Ericksen fluid, suspended particles, compressibility, thermal instability, Hall currents.

1. Introduction

Chandrasekhar [1] in his celebrated monograph discussed in detail the theoretical and experimental results of the onset of thermal instability (Bénard convection) under varying assumptions of hydrodynamics and hydromagnetics for viscous/inviscid fluids. If an electric field is applied at right angles to the magnetic field, the whole current will not flow along the electric field. This tendency of the electric current to flow across an electric field in the presence of magnetic field is called Hall effect. The Hall current is important in flows of laboratory plasmas as well as in geophysical and astrophysical situations. Sherman and Sutton [2] have considered the effect of Hall currents on the efficiency of a magneto-fluid dynamic (MHD) generator while Gupta [3] studied the effect of Hall currents on the thermal instability of electrically conducting fluid in the presence of uniform vertical magnetic field. In another study, Sharma and Gupta [4] discussed the effect of Hall currents and finite Larmor radius on thermosolutal instability of a rotating plasma and established the destabilizing influence of Hall currents. Chandra [5] observed a contradiction between the theory and experiment for the onset of convection in fluids heated from below. He performed the experiment in an air layer and

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found that the instability depended on the depth of the layer. A Bénard-type cellular convection with the fluid descending at the cell centre was observed when the predicted gradients were imposed for layers deeper than 10 mm. A convection which was different in character from that in deeper layers occurred at much lower gradients than predicted if the layer depth was less than 7 mm, and called this motion, “Columnar instability”. Scanlon and Segel [6] investigated some of the continuum effects of particles on Bénard convection and found that a critical Rayleigh number was reduced solely because the heat capacity of the pure gas was supplemented by that of the particles. The effect of suspended particles was thus found to destabilize the layer. Sharma et al. [7] considered the effect of suspended particles on the onset of Bénard convection in hydromagnetics and confirmed its destabilizing role while Palaniswamy and Purushotham [8] studied the stability of shear flow of stratified fluids with fine dust and have found that the effect of fine dust is to increase the region of instability. Later on, Sharma and Gupta [9] investigated the effect of Hall currents and suspended particles on thermal instability of compressible fluids saturating a porous medium.

For compressible fluids, the equations governing the system become quite complicated. Spiegel and Veronis [10] simplified the set of equations governing the flow of compressible fluids assuming that the depth of the fluid layer is much smaller than the scale height as defined by them and motions of infinitesimal amplitude are considered. Under these assumptions, the flow equations for compressible fluids are the same as for incompressible fluids except that the static temperature gradient β is replaced by its excess over the adiabatic $(\beta - g/C_p)$, C_p being specific heat of the fluid at constant pressure. Thermal instability problem in the presence of compressibility for varying assumptions of rotation, magnetic field, finite Larmor radius and Hall currents for Newtonian fluids has been considered by Sharma [11], Sharma and Gupta [12] and Sharma and Sunil [13]. In all the above studies, fluid has been considered to be Newtonian. There is growing importance of non-Newtonian viscoelastic fluids in geophysical fluid dynamics, chemical technology and petroleum industry. Such flows have particular relevance in the extrusion of polymer sheets, glass blowing, manufacturing plastic films, crystal growing, hot rolling and many others. There are some viscoelastic fluids which are characterized by Maxwell’s constitutive relations and some by Oldroyd’s [14] constitutive relations. Bhatia and Steiner [15] studied the problem of thermal instability of a Maxwellian viscoelastic fluid in the presence of rotation while thermal instability of an Oldroydian viscoelastic fluid acted on by a uniform rotation has been studied by Sharma [16]. Another important class of elastico-viscous fluids is given by Rivlin-Ericksen [17]. Rivlin and Ericksen [17] in mid fifties have proposed a theoretical model for such elastico-viscous fluids. Such polymers are used in agriculture, communication appliances and in biomedical applications. Some more investigations on thermal instability of Rivlin-Ericksen fluid in the presence of magnetic field, rotation, finite Larmor radius and variable gravity have been reported by Sharma and Kumar [18], [19], Prakash and Kumar [20] and Kumar et al. [21].

Recently, Gupta and Sharma [22], [23], Gupta and Kumar [24], Kolsi et al. [25] and Savić et al. [26] studied thermal/thermosolutal convection problems but for non-dusty viscoelastic fluids with rotation, magnetic field and Hall currents. Motivated by the fact that knowledge regarding fluid and dust particle mixture is not commensurate with their industrial and scientific importance and the importance of Hall currents in geophysical and astrophysical situations in addition to the flow of laboratory plasmas we have investigated the combined effect of Hall currents and suspended/dust particles on a compressible Rivlin-Ericksen fluid-layer. This problem may be considered as an extension of the earlier work of Gupta and Aggarwal [27] and has not been studied so far to the very best of our knowledge.

2. Mathematical formulation of the problem

We have considered an infinite, horizontal, compressible electrically conducting Rivlin-Ericksen fluid permeated with suspended/dust particles, bounded by the planes $z = 0$ and $z = d$, as shown in fig.1. This layer is heated from below so that temperature at bottom (at $z = 0$) and

at the upper layer (at $z = d$) is T_0 and T_d respectively and a uniform temperature gradient $\beta (= dT / dz)$ is maintained. A uniform vertical magnetic field intensity $\vec{H} = (0, 0, H)$ and gravity force $\vec{g} = (0, 0, -g)$ pervade the system.

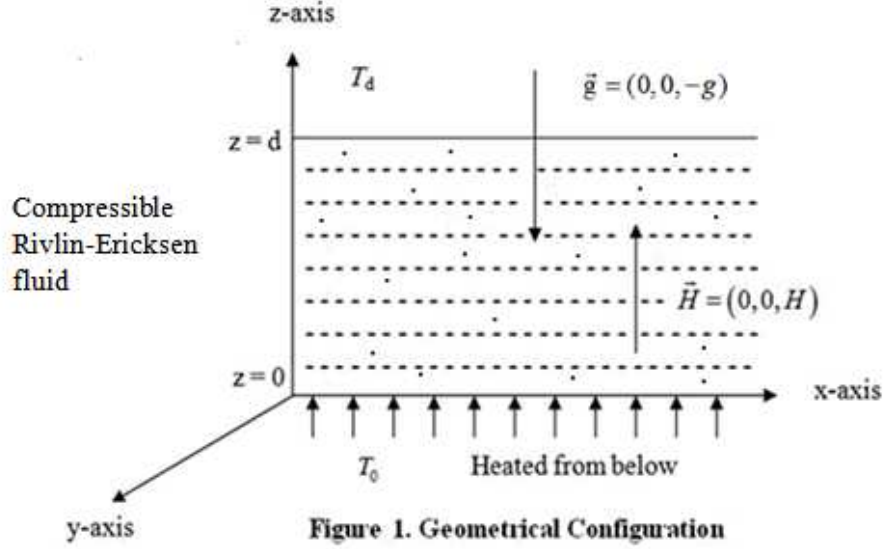


Figure 1. Geometrical Configuration

Let $\rho, \mu, \mu', p, \vec{v} = (u, v, w)$ denote, respectively, the density, viscosity, viscoelasticity, pressure and velocity of the pure fluid, $\vec{v}_d = (l, r, s), N(\vec{x}, t)$ denote the velocity and number density of the suspended particles, μ_e the magnetic permeability, $\vec{x} = (x, y, z)$. The equations of motion and continuity relevant to the problem are (Chandrasekhar [1], Rivlin-Ericksen [17])

$$\rho \left[\frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla) \vec{v} \right] = -\nabla p + \rho \vec{g} + \left(\mu + \mu' \frac{\partial}{\partial t} \right) \nabla^2 \vec{v} + \frac{\mu_e}{4\pi} (\nabla \times \vec{H}) \times \vec{H} + K' N (\vec{v}_d - \vec{v}), \quad (1)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \quad (2)$$

where $K' = 6\pi\mu\eta'\eta'$ being the particle radius, is the Stokes' drag coefficient. Assuming uniform particle size, spherical shape and small relative velocities between the fluid and particles, the presence of particles adds an extra force term, in the equations of motion (1), proportional to the velocity difference between particles and fluid. In the equations of motion for the particles there will also be an extra force term equal in magnitude but opposite in sign because the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid. Interparticle reactions are ignored because the distances between the particles are assumed to be quite large compared with their diameter. The effects due to pressure, gravity and magnetic field on the particles are small and so ignored. If mN is the mass of the particles per unit volume, then the equations of motion and continuity for the particles are

$$mN \left[\frac{\partial \vec{v}_d}{\partial t} + (\vec{v}_d \cdot \nabla) \vec{v}_d \right] = K' N (\vec{v} - \vec{v}_d), \quad (3)$$

$$\frac{\partial N}{\partial t} + \nabla \cdot (N \vec{v}_d) = 0. \quad (4)$$

Let C_f , C_{pt} , T and q denote, respectively, the heat capacity of the pure fluid, the heat capacity of particles, the temperature and the “effective thermal conductivity” of the pure fluid. Assuming that the particles and the fluid are in thermal equilibrium, the equation of heat conduction gives

$$\rho C_f \left(\frac{\partial}{\partial t} + \vec{v} \cdot \nabla \right) T + m N C_{pt} \left(\frac{\partial}{\partial t} + \vec{v}_d \cdot \nabla \right) T = q \nabla^2 T. \quad (5)$$

From Maxwell's equations in the presence of Hall currents, we have

$$\frac{\partial \vec{H}}{\partial t} = \nabla \times (\vec{v} \times \vec{H}) + \eta \nabla^2 \vec{H} - \frac{1}{4\pi N' e} \nabla \times [(\nabla \times \vec{H}) \times \vec{H}], \quad (6)$$

$$\nabla \cdot \vec{H} = 0, \quad (7)$$

where η , N' and e denote, respectively, the resistivity, the electron number density and the charge of an electron. The state variables pressure, density and temperature are expressed in the form (Spiegel and Veronis [10])

$$f(x, y, z, t) = f_m + f_0(z) + f'(x, y, z, t), \quad (8)$$

where f_m stands for constant space distribution of f , f_0 is the variation in the absence of motion and $f'(x, y, z, t)$ is the fluctuation resulting from motion. For the basic state of the system with a uniform particle distribution, we have

$$p = p(z), \rho = \rho(z), T = T(z), \vec{v} = (0, 0, 0), \vec{H} = (0, 0, H), \vec{v}_d = (0, 0, 0) \text{ and } N = N_0 = \text{constant}. \quad (9)$$

Following Spiegel and Veronis [10], we have

$$p(z) = p_m - g \int_0^z (\rho_m + \rho_0) dz,$$

$$\rho(z) = \rho_m [1 - \alpha_m (T - T_0) + K_m (p - p_m)],$$

$$T(z) = -\beta z + T_0,$$

$$\alpha_m = - \left(\frac{1}{\rho} \frac{\partial \rho}{\partial T} \right)_m (= \alpha, \text{ say}),$$

(10)

$$K_m = \left(\frac{1}{\rho} \frac{\partial \rho}{\partial p} \right)_m,$$

where p_m and ρ_m stand for a constant space distribution of p and ρ ; and ρ_0 and T_0 stand for the density and temperature of the fluid at the lower boundary.

3. Perturbation equations

Let us consider a small perturbation on steady state solution and let $\delta p, \delta \rho, \theta$,

$\vec{v} = (u, v, w)$, $\vec{v}_d = (l, r, s)$, $\vec{h} = (h_x, h_y, h_z)$ and N denote the perturbations in fluid pressure,

density, temperature, fluid velocity, particle velocity, magnetic field intensity \vec{H} and particle number density N_0 respectively. Then the linearized hydromagnetic perturbation equations of the fluid-particle layer are

$$\frac{\partial \vec{v}}{\partial t} = -\frac{1}{\rho_m}(\nabla \delta p) + \bar{g} \frac{\rho}{\rho_m} + \left(v + v' \frac{\partial}{\partial t} \right) \nabla^2 \vec{v} + \frac{\mu_e}{4\pi\rho_m} (\nabla \times \vec{h}) \times \vec{H} + \frac{K'N_0}{\rho_m} (\vec{v}_d - \vec{v}), \quad (11)$$

$$\nabla \cdot \vec{v} = 0, \quad (12)$$

$$mN_0 \frac{\partial \vec{v}_d}{\partial t} = K'N_0 (\vec{v} - \vec{v}_d), \quad (13)$$

$$\frac{\partial M_d}{\partial t} + \nabla \cdot \vec{v}_d = 0, \quad (14)$$

$$\nabla \cdot \vec{h} = 0, \quad (15)$$

$$\frac{\partial \vec{h}}{\partial t} = (\vec{H} \cdot \nabla) \vec{v} + \eta \nabla^2 \vec{h} - \frac{1}{4\pi N'e} \nabla \times [(\nabla \times \vec{h}) \times \vec{H}], \quad (16)$$

$$(1 + h_d) \frac{\partial \theta}{\partial t} = \left(\beta - \frac{g}{C_p} \right) (w + h_d s) + \kappa \nabla^2 \theta, \quad (17)$$

where $\alpha_m = \frac{1}{T_m} = \alpha$, say, $v = \frac{\mu}{\rho_m}$, $v' = \frac{\mu'}{\rho_m}$, $\kappa = \frac{q}{\rho_m C_f}$ and g/C_p , v , v' and κ stand for the adiabatic gradient, kinematic viscosity, kinematic viscoelasticity and thermal diffusivity respectively. Also, $M_d = N/N_0$ and $h_d = \frac{mN_0 C_{pt}}{\rho_m C_f}$. In eq. (17), the static temperature gradient

β is replaced by its excess over the adiabatic $(\beta - g/C_p)$, C_p being specific heat of the fluid at constant pressure following assumptions and results for compressible fluids given by Spiegel and Veronis [10].

Eliminating \vec{v}_d between eqs. (11) - (13) and rewriting the above set of eqs., we have

$$\begin{aligned} & \left(\frac{m}{K'} \frac{\partial}{\partial t} + 1 \right) \left[\frac{\partial}{\partial t} \nabla^2 w + g\alpha \left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} \right) - \frac{\mu_e H}{4\pi\rho_m} \frac{\partial}{\partial z} \nabla^2 h_z \right] + \\ & + \frac{mN_0}{\rho_m} \frac{\partial}{\partial t} \nabla^2 w = \left(\frac{m}{K'} \frac{\partial}{\partial t} + 1 \right) \left(v + v' \frac{\partial}{\partial t} \right) \nabla^4 w, \end{aligned} \quad (18)$$

$$\left(\frac{m}{K'} \frac{\partial}{\partial t} + 1 \right) \frac{\partial \zeta}{\partial t} = \frac{\mu_e H}{4\pi\rho_m} \left(\frac{m}{K'} \frac{\partial}{\partial t} + 1 \right) \frac{\partial \xi}{\partial z} - \frac{m}{K'} \frac{\partial}{\partial t} \left(\frac{K'N_0 \zeta}{\rho_m} \right) + \left(\frac{m}{K'} \frac{\partial}{\partial t} + 1 \right) \left(v + v' \frac{\partial}{\partial t} \right) \nabla^2 \zeta, \quad (19)$$

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2 \right) h_z = H \frac{\partial w}{\partial z} - \frac{H}{4\pi N'e} \frac{\partial \xi}{\partial z}, \quad (20)$$

$$\left(\frac{\partial}{\partial t} - \eta \nabla^2\right) \xi = H \frac{\partial \xi}{\partial z} + \frac{H}{4\pi N'e} \frac{\partial}{\partial z} (\nabla^2 h_z), \quad (21)$$

$$\left(\frac{m}{K'} \frac{\partial}{\partial t} + 1\right) \left[(1 + h_d) \frac{\partial}{\partial t} - \kappa \nabla^2 \right] \theta = \beta \left(\frac{G-1}{G} \right) \left(\frac{m}{K'} \frac{\partial}{\partial t} + H_d \right) w, \quad (22)$$

where $\xi = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z-component of vorticity, $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is the z-component of current density and $G = \left(\frac{C_p}{g} \right) \beta$.

4. Dispersion relation

Analyzing the disturbances into normal modes, we assume that the perturbation quantities are of the form

$$[w, h_z, \theta, \xi, \zeta] = [W(z), K(z), \Theta(z), Z(z), X(z)] \exp(ik_x x + ik_y y + nt), \quad (23)$$

where k_x and k_y are the wavenumbers along x and y directions and resultant wavenumber is given by $k = (k_x^2 + k_y^2)^{1/2}$ and n is the growth rate.

Using expression (23) and the following non-dimensionalized parameters

$$a = kd, \quad \sigma = \frac{nd^2}{\nu}, \quad \text{Pr}_1 = \frac{\nu}{\kappa}, \quad \text{Pr}_2 = \frac{\nu}{\eta}, \quad F = \frac{\nu'}{d^2}, \quad H_d = 1 + h_d, \quad f = \frac{mN_0}{\rho_m}, \quad \tau = \frac{m\kappa}{K'd^2}, \quad (24)$$

$$x^* = \frac{x}{d}, \quad y^* = \frac{y}{d}, \quad z^* = \frac{z}{d}, \quad D = \frac{d}{dz^*},$$

eqs. (18)-(22) are modified to

$$(1 + \text{Pr}_1 \sigma \tau) (D^2 - a^2) \left[(1 + \sigma F) (D^2 - a^2) - \sigma \right] W - f \sigma (D^2 - a^2) W + (1 + \text{Pr}_1 \sigma \tau) \frac{\mu_e H d}{4\pi \rho_m \nu} (D^2 - a^2) DK = (1 + \text{Pr}_1 \sigma \tau) \frac{g a^2 d^2 \alpha}{\nu} \Theta, \quad (25)$$

$$(1 + \text{Pr}_1 \sigma \tau) \left[(1 + \sigma F) (D^2 - a^2) - \sigma \right] Z - f \sigma Z = - \frac{\mu_e H d}{4\pi \rho_m \nu} (1 + \text{Pr}_1 \sigma \tau) DX, \quad (26)$$

$$(1 + \text{Pr}_1 \sigma \tau) [D^2 - a^2 - H_d \text{Pr}_1 \sigma] \Theta + \frac{\beta d^2}{\kappa} \left(\frac{G-1}{G} \right) (H_d + \text{Pr}_1 \sigma \tau) W = 0, \quad (27)$$

$$[D^2 - a^2 - \text{Pr}_2 \sigma] K + \left(\frac{H d}{\eta} \right) DW - \frac{H d}{4\pi N' e \eta} DX = 0 \quad (28)$$

$$[D^2 - a^2 - \text{Pr}_2 \sigma]X + \left(\frac{Hd}{\eta}\right)DZ + \frac{H}{4\pi N'e\eta d}D(D^2 - a^2)K = 0. \quad (29)$$

Considering the case of two free boundaries in which the medium adjoining the fluid is perfectly conducting and temperatures at the boundaries are kept fixed. The relevant boundary conditions are following (Chandrasekhar [1])

$$\begin{aligned} W = D^2W = 0, \quad DZ = 0, \quad \Theta = 0 \quad \text{at } z = 0 \text{ and } 1, \\ K = 0, \quad \text{on perfectly conducting boundaries} \end{aligned} \quad (30)$$

and h_x, h_y, h_z are continuous. Since the components of magnetic field intensity are continuous and the tangential components are zero outside the fluid, we have

$$DK = 0, \quad \text{on the boundaries.} \quad (31)$$

Using the above boundary conditions (30) and (31), it can be shown that all the even order derivatives of W must vanish for $z = 0$ and 1 . Hence, the proper solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \quad (32)$$

where W_0 is a constant. Eliminating Θ, X, Z and K between eqs. (25)-(29), we obtain

$$\begin{aligned} R_1 = & \left(\frac{G}{G-1}\right) \left\langle (1+x) \left(1 + i \text{Pr}_1 \sigma_1 \pi^2 \tau\right) \left[(1+x) (1 + i \sigma_1 F \pi^2) + i \sigma_1 \right] + i (1+x) f \sigma_1 + \right. \\ & + \left\{ Q_1 \left(1 + i \text{Pr}_1 \sigma_1 \pi^2 \tau\right) (1+x) \left[(1+x + i \sigma_1 \text{Pr}_2) \left\{ (1 + i \text{Pr}_1 \sigma_1 \pi^2 \tau) \left[(1+x) (1 + i F \sigma_1 \pi^2) + i \sigma_1 \right] + i f \sigma_1 \right\} + \right. \right. \\ & Q_1 \left(1 + i \text{Pr}_1 \sigma_1 \pi^2 \tau\right) \left. \right] \left[\left\{ M(1+x) + (1+x + i \sigma_1 \text{Pr}_2)^2 \right\} \left\{ (1 + i \text{Pr}_1 \sigma_1 \pi^2 \tau) \left[(1+x) (1 + i F \sigma_1 \pi^2) + i \sigma_1 \right] \right. \right. \right. \\ & \left. \left. + i f \sigma_1 \right\} + Q_1 \left(1 + i \text{Pr}_1 \sigma_1 \pi^2 \tau\right) (1+x + i \sigma_1 \text{Pr}_2) \right]^{-1} \left. \right\rangle \frac{(1+x + i H_d \text{Pr}_1 \sigma_1)}{x (H_d + i \text{Pr}_1 \sigma_1 \pi^2 \tau)}, \end{aligned} \quad (33)$$

where

$$\begin{aligned} R_1 = \frac{g \alpha \beta d^4}{\nu \kappa \pi^4}, \quad Q_1 = \frac{\mu_e H^2 d^2}{4\pi \rho_m \nu \eta \pi^2}, \quad M = \left(\frac{H}{4\pi N'e\eta}\right)^2, \\ x = \frac{a^2}{\pi^2} \text{ (square of the scaled wavenumber) and } i \sigma_1 = \frac{\sigma}{\pi^2}. \end{aligned}$$

But for the sake of convenience, we will be using the term ‘wavenumber’ instead of ‘square of the scaled wavenumber’ for x hereafter.

Equation (33) is the required dispersion relation including the effects of Hall currents and compressibility on the thermal instability of Rivlin-Ericksen fluid permeated with dust particles. The dispersion relation can be reduced to the one derived by Sharma and Aggarwal [28] under the following conditions

1. The factor corresponding to Hall currents ($M = 0$) is reduced to zero.
2. The factor corresponding to viscoelasticity of the fluid F is negative as Walters’ (Model B’) fluid has been considered therein.

5. Case of stationary convection

When the instability sets in the form of stationary convection, the marginal state will be

characterized by $\sigma_1 = 0$. Putting $\sigma_1 = 0$, the dispersion relation (33) reduces to

$$R_1 = \left(\frac{G}{G-1} \right) \left[(1+x)^2 + \frac{Q_1 \{ (1+x)^2 + Q_1 \}}{(1+x)(M+1+x) + Q_1} \right] \left(\frac{1+x}{xH_d} \right), \quad (34)$$

which expresses the modified Rayleigh number R_1 as a function of dimensionless wave number x and the parameters Q_1, G, M and H_d . We thus find that for stationary convection the viscoelastic parameter F vanishes with σ_1 and the Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid. For incompressible $\left(\frac{G}{G-1} = 1 \right)$, non-dusty fluid $(H_d = 1 + h_d = 1)$, the above expression for R_1 reduces to

$$R_1 = \left(\frac{1+x}{x} \right) \left[(1+x)^2 + \frac{Q_1 \{ (1+x)^2 + Q_1 \}}{(1+x)(M+1+x) + Q_1} \right], \quad (35)$$

the one derived by Sharma et al. [29]. Further for $M = 0$, Rayleigh number R_1 reduces to

$$R_1 = \left(\frac{G}{G-1} \right) \left[(1+x)^2 + Q_1 \right] \left(\frac{1+x}{xH_d} \right), \quad (36)$$

the expression derived by Sharma and Aggarwal [28]. Let the non-dimensional number G accounting for compressibility effect is kept as fixed, then we get

$$\overline{R_c} = \left(\frac{G}{G-1} \right) R_c, \quad (37)$$

where R_c and $\overline{R_c}$ denote, respectively, the critical Rayleigh numbers in the absence and presence of compressibility. Thus, the effect of compressibility is to postpone the onset of thermal instability. The cases $G < 1$ and $G = 1$ correspond to negative and infinite value of Rayleigh number which are not relevant in the present study. Hence, compressibility has a stabilizing effect on the thermal convection problem under consideration.

To investigate the effects of suspended particles, magnetic field and Hall currents, we examine the natures of $\frac{dR_1}{dH_d}$, $\frac{dR_1}{dQ_1}$ and $\frac{dR_1}{dM}$ analytically. To investigate the effect of suspended particles, from eq. (34), we obtain

$$\frac{dR_1}{dH_d} = - \left(\frac{G}{G-1} \right) \left(\frac{1+x}{xH_d^2} \right) \left[(1+x)^2 + \frac{Q_1 \{ (1+x)^2 + Q_1 \}}{(1+x)(M+1+x) + Q_1} \right], \quad (38)$$

the negative sign implies that the effect of suspended particles is to destabilize the system. Figure 2 confirms this result numerically as is clear from various curves since R_1 decreases as H_d increases for the permissible range of values of various parameters. This result is in agreement with the result of Sharma and Gupta [9] for Newtonian fluids.

To analyze the effect of magnetic field, eq. (34) yields

$$\frac{dR_1}{dQ_1} = \left(\frac{G}{G-1} \right) \left(\frac{1+x}{xH_d} \right) \left[\frac{(1+x)(M+1+x) \{ (1+x)^2 + 2Q_1 \} + Q_1^2}{\{ (1+x)(M+1+x) + Q_1 \}^2} \right], \quad (39)$$

which shows the usual stabilizing effect of magnetic field on thermal convection for Rivlin-Ericksen viscoelastic fluid in the presence of dust particles as well. Numerically, as shown in fig. 3, R_1 is plotted against x for various values of $Q_1 = 100, 150, 200, 250, 300$. This stabilizing effect of magnetic field is in good agreement with earlier works of Sharma et al. [29] and Kumar et al. [30].

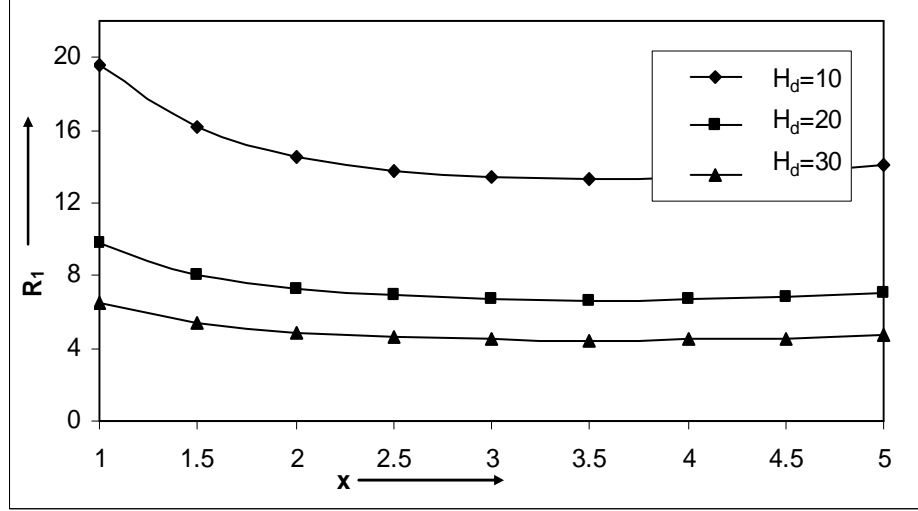


Figure 2. Variation of Rayleigh number R_1 with wavenumber x for fixed $G=10$, $Q_1=100$, $M=10$ and for various values of $H_d = 10, 20$ and 30 .

Expression for observing the effect of Hall currents is obtained as

$$\frac{dR_1}{dM} = - \left(\frac{G}{G-1} \right) \left(\frac{(1+x)^2}{xH_d} \right) \left[\frac{Q_1 \{ (1+x)^2 + Q_1 \}}{\{ (1+x)(M+1+x) + Q_1 \}^2} \right], \quad (40)$$

which reflects the destabilizing influence of Hall currents on thermal instability of Rivlin-Ericksen fluid in the presence of compressibility and suspended particles. Also in fig. 4, R_1 decreases with the increase in M which confirms the result numerically.

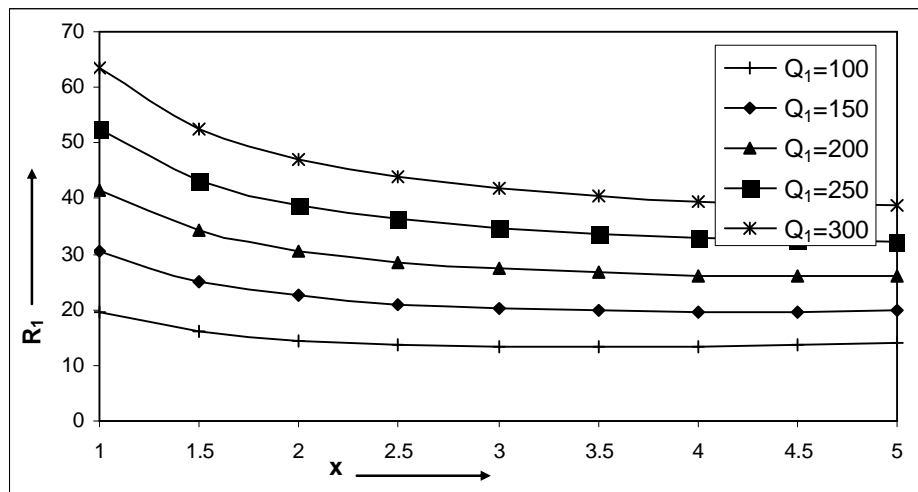


Figure 3. Variation of Rayleigh number R_1 with wavenumber x for fixed $G=10$, $H_d = 10$, $M=10$ and for various values of $Q_1 = 100, 150, 200, 250$ and 300 .

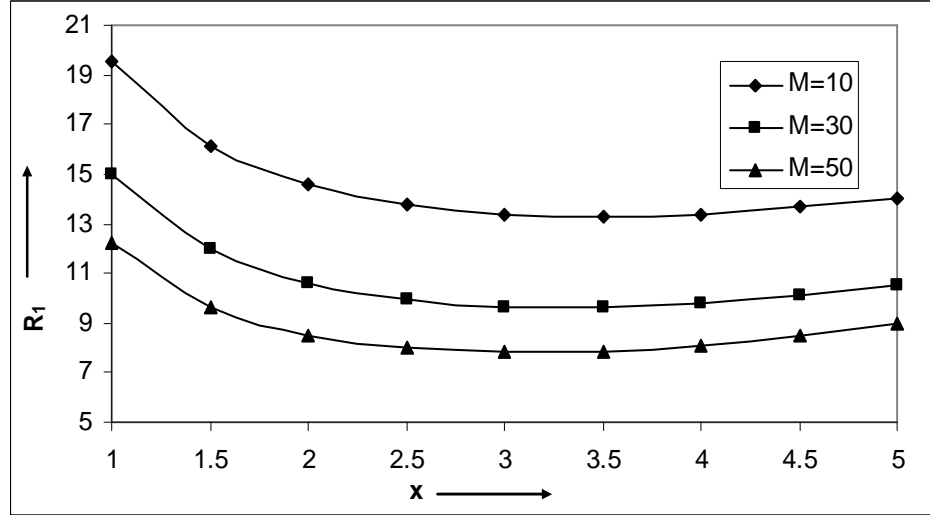


Figure 4. Variation of Rayleigh number R_1 with wavenumber x for fixed $G=10$, $Q_1=100$, $H_d=10$ and for various values of $M=10, 30$ and 50 .

As a function of x , R_1 given by eq. (34) attains its extremal value when

$$(1+x)^4 (2x-1)(1+x+M)^2 + (1+x)^2 (3x-2)Q_1^2 + 2(1+x)^3 (4x-1)(1+x+M)Q_1 - (1-x)^2 (1+x+M)Q_1^2 - (1+x)^4 (1+2x+M)Q_1 = 0. \quad (41)$$

This is to find out critical Rayleigh number R_c and the associated critical wavenumber x_c for various values of the parameters Q_1 , M and H_d . However, rather than evaluating R_c from eq. (41), it is more convenient to evaluate R as a function of x in accordance with eq. (34) for various values of H_d , Q_1 and M as depicted in figs. 2-4 and locate the minimum numerically. The critical numbers listed in Tables 1-3 and illustrated in figs. 5-7 are obtained in this fashion. It is clear from fig. 5 that the critical Rayleigh number R_c increases with the increase in magnetic field parameter Q_1 for fixed value of suspended particle parameter H_d . Also, the various curves for different values of H_d indicate the destabilizing influence of suspended particles parameter as R_c decreases with the increase in H_d . Thus, magnetic field has a stabilizing effect on the system whereas the effect of suspended particles is destabilizing.

Table 1. The critical Rayleigh numbers and the wavenumbers of the associated disturbances for the onset of instability as stationary convection for $G=10$, $M=10$ and for various values of Q_1 and H_d .

Q_1	$H_d=10$		$H_d=20$		$H_d=30$		$H_d=50$	
	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
100	3.5	13.29	3.5	6.64	3.5	4.43	3.5	2.66
200	4.5	26.03	4.5	13.02	4.5	8.68	4.5	5.21
300	5.0	38.74	5.0	19.37	5.0	12.91	5.0	7.75
500	5.0	64.76	5.0	32.38	5.0	21.59	5.0	12.95
1000	5.0	130.83	5.0	65.42	5.0	43.61	5.0	26.17

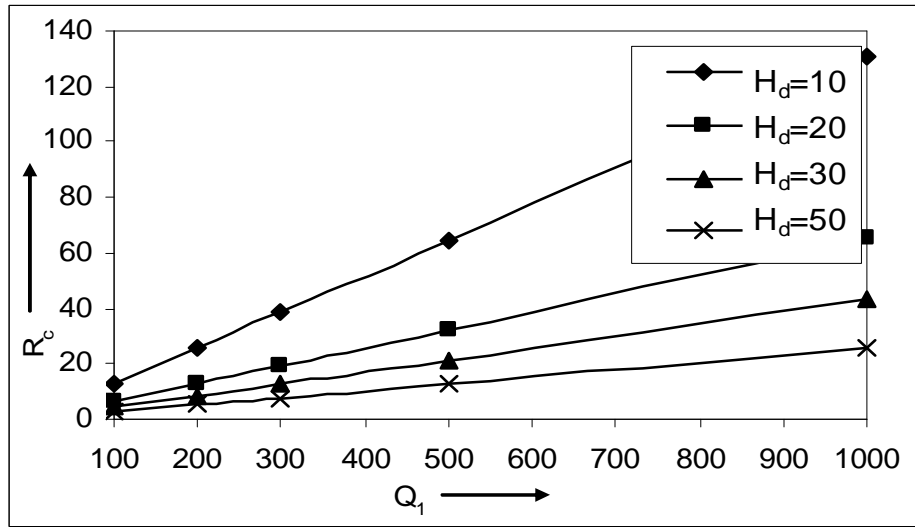


Figure 5. Variation of critical Rayleigh number R_c with magnetic field parameter Q_1 for fixed $G=10$, $M=10$ and for various values of $H_d=10, 20, 30$ and 50 .

Table 2. The critical Rayleigh numbers and the wavenumbers of the associated disturbances for the onset of instability as stationary convection for $G=10$, $M=10$ and for various values of Q_1 and H_d .

	$Q_1=100$		$Q_1=200$		$Q_1=300$		$Q_1=500$		$Q_1=1000$	
H_d	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
10	3.5	13.29	4.5	26.03	5.0	38.74	5.0	64.76	5.0	130.83
20	3.5	6.64	4.5	13.02	5.0	19.37	5.0	32.38	5.0	65.42
30	3.5	4.43	4.5	8.68	5.0	12.91	5.0	21.59	5.0	43.61
50	3.5	2.66	4.5	5.21	5.0	7.75	5.0	12.95	5.0	26.17

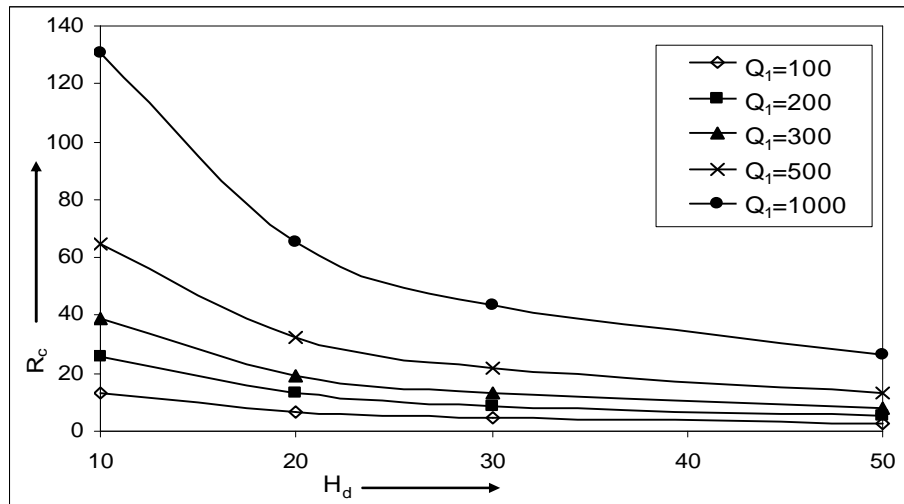


Figure 6. Variation of critical Rayleigh number R_c with suspended particle factor H_d for fixed $G=10$, $M=10$ and for various values of $Q_1=100, 200, 300, 500$ and 1000 .

In fig. 6, R_c is plotted against H_d for various values of Q_1 . For fixed Q_1 , R_c decreases with increase in H_d and R_c increases with the increase in Q_1 (as is clear from curves for different

values of Q_1). This confirms the destabilizing effect of suspended particles and stabilizing influence of magnetic field as obtained earlier. In fig. 7, variation of R_c with the Hall current parameter M is investigated for different values of Q_1 . The critical Rayleigh number R_c decreases with the increase in M confirming the destabilizing influence of Hall currents. The different curves for various values of Q_1 confirm the stabilizing effect of magnetic field as R_c increases with the increase in Q_1 .

Table 3. The critical Rayleigh numbers and the wavenumbers of the associated disturbances for the onset of instability as stationary convection for $G = 10$, $H_d = 10$ and for various values of Q_1 and M .

	$Q_1=100$		$Q_1=200$		$Q_1=300$		$Q_1=500$		$Q_1=1000$	
M	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c	x_c	R_c
10	3.5	13.29	4.5	26.03	5.0	38.74	5.0	64.76	5.0	130.83
30	3.5	9.62	5.0	19.93	5.0	30.85	5.0	54.71	5.0	118.4
50	3.0	7.81	4.5	16.49	5.0	25.93	5.0	47.54	5.0	108.19
100	2.5	5.68	4.0	12.09	5.0	19.16	5.0	36.26	5.0	89.23

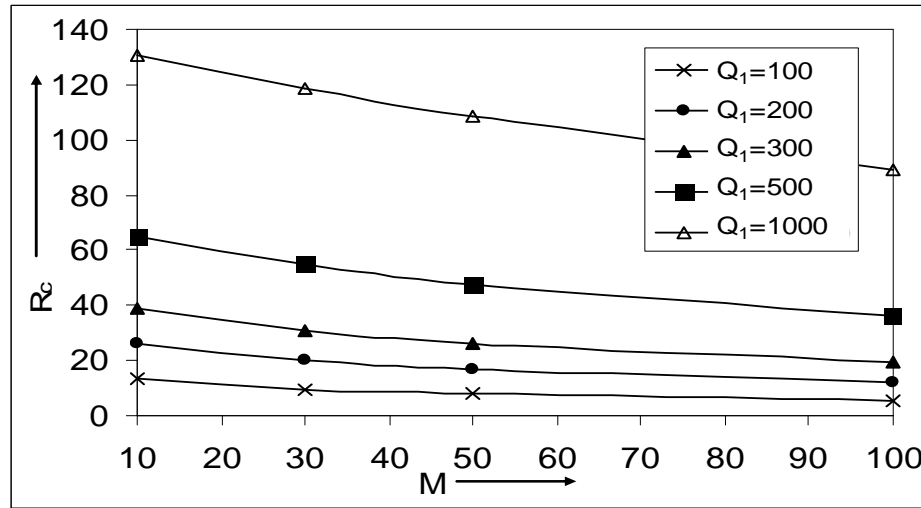


Figure 7. Variation of critical Rayleigh number R_c with Hall current parameter M for fixed $G=10$, $H_d=10$ and for various values of $Q_1 = 100, 200, 300, 500$ and 1000 .

6. Stability of the system and oscillatory modes

Here we examine the possibility of oscillatory modes, if any, on the Rivlin-Ericksen elasto-viscous fluid. Multiplying eq. (25) by W^* , the complex conjugate of W and making use of eqs. (26)-(29) together with the boundary conditions (30) and (31), we obtain

$$\begin{aligned}
 & (1 + \text{Pr}_1 \sigma \tau)(1 + F \sigma)I_1 + (1 + f + \text{Pr}_1 \sigma \tau)\sigma I_2 + \frac{\mu_e \eta}{4\pi \rho_m \nu}(1 + \text{Pr}_1 \sigma \tau)[I_3 + \text{Pr}_2 \sigma^* I_4] + \\
 & \frac{\mu_e \eta d^2}{4\pi \rho_m \nu}[I_7 + \text{Pr}_2 \sigma I_8] + d^2(1 + \sigma^* F)I_9 + \sigma^* d^2 \left(1 + \frac{f}{1 + \text{Pr}_1 \sigma^* \tau}\right)I_{10} - \\
 & \left(\frac{G}{G-1}\right) \frac{g \alpha \kappa a^2}{\nu \beta} \frac{(1 + \text{Pr}_1 \sigma \tau)(1 + \text{Pr}_1 \sigma^* \tau)}{(H_d + \text{Pr}_1 \sigma^* \tau)}[I_5 + H_d \text{Pr}_1 \sigma^* I_6] = 0,
 \end{aligned} \tag{42}$$

where,

$$I_1 = \int_0^1 \left(|D^2 W|^2 + 2a^2 |DW|^2 + a^4 |W|^2 \right) dz,$$

$$I_2 = \int_0^1 \left(|DW|^2 + a^2 |W|^2 \right) dz,$$

$$I_3 = \int_0^1 \left(|D^2 K|^2 + 2a^2 |DK|^2 + a^4 |K|^2 \right) dz,$$

$$I_4 = \int_0^1 \left(|DK|^2 + a^2 |K|^2 \right) dz, \quad I_5 = \int_0^1 \left(|D\Theta|^2 + a^2 |\Theta|^2 \right) dz,$$

$$I_6 = \int_0^1 \left(|\Theta|^2 \right) dz, \quad I_7 = \int_0^1 \left(|DX|^2 + a^2 |X|^2 \right) dz,$$

$$I_8 = \int_0^1 \left(|X|^2 \right) dz, \quad I_9 = \int_0^1 \left(|DZ|^2 + a^2 |Z|^2 \right) dz, \quad I_{10} = \int_0^1 \left(|Z|^2 \right) dz,$$

where integrals I_1, I_2, \dots, I_{10} are all positive definite.

Putting $\sigma = \sigma_r + i\sigma_i$ and equating real and imaginary parts of eq. (42), we get

$$\begin{aligned} & \left(1 + \text{Pr}_1 \tau \sigma_r + F \sigma_r + F \text{Pr}_1 \tau \sigma_r^2 + F \text{Pr}_1 \tau \sigma_i^2 \right) I_1 + \left[\sigma_r (1 + f + \text{Pr}_1 \tau \sigma_r) - \text{Pr}_1 \tau \sigma_i^2 \right] I_2 + \\ & \frac{\mu_e \eta}{4\pi \rho_m \nu} \left[(1 + \text{Pr}_1 \tau \sigma_r) I_3 + \left\{ \text{Pr}_2 \sigma_r (1 + \text{Pr}_1 \tau \sigma_r) + \text{Pr}_1 \text{Pr}_2 \tau \sigma_i^2 \right\} I_4 \right] + \frac{\mu_e \eta d^2}{4\pi \rho_m \nu} [I_7 + \text{Pr}_2 \sigma_r I_8] + \\ & + d^2 (1 + \sigma_r F) I_9 + \left\{ \sigma_r + \frac{\left[f \sigma_r (1 + \text{Pr}_1 \tau \sigma_r) + f \text{Pr}_1 \tau \sigma_i^2 \right]}{(1 + \text{Pr}_1 \tau \sigma_r)^2 + \text{Pr}_1^2 \tau^2 \sigma_i^2} \right\} d^2 I_{10} - \frac{g \alpha \kappa a^2}{\nu \beta} \left(\frac{G}{G-1} \right) \times \\ & \left\{ \frac{(1 + \text{Pr}_1 \tau \sigma_r)^2 + \text{Pr}_1^2 \tau^2 \sigma_i^2}{(H_d + \text{Pr}_1 \tau \sigma_r)^2 + \text{Pr}_1^2 \tau^2 \sigma_i^2} \right\} \left[(H_d + \text{Pr}_1 \tau \sigma_r) (I_5 + H_d \text{Pr}_1 \sigma_r I_6) + \text{Pr}_1^2 \tau H_d \sigma_i^2 I_6 \right] = 0, \end{aligned} \quad (43)$$

$$\begin{aligned} & i\sigma_i \left[\left\{ \text{Pr}_1 \tau + F \text{Pr}_1 \tau \sigma_r + F (1 + \text{Pr}_1 \tau \sigma_r) \right\} I_1 + (1 + f + 2 \text{Pr}_1 \tau \sigma_r) I_2 + \frac{\mu_e \eta}{4\pi \rho_m \nu} \{ \text{Pr}_1 \tau I_3 - \text{Pr}_2 I_4 \} + \right. \\ & \left. + \frac{\mu_e \eta d^2}{4\pi \rho_m \nu} \text{Pr}_2 I_8 - d^2 F I_9 - \left\{ 1 - \frac{\left[f \text{Pr}_1 \tau \sigma_r - f (1 + \text{Pr}_1 \tau \sigma_r) \right]}{(1 + \text{Pr}_1 \tau \sigma_r)^2 + \text{Pr}_1^2 \tau^2 \sigma_i^2} \right\} d^2 I_{10} + \right. \\ & \left. + \frac{g \alpha \kappa a^2}{\nu \beta} \left(\frac{G}{G-1} \right) \left\{ \frac{(1 + \text{Pr}_1 \tau \sigma_r)^2 + \text{Pr}_1^2 \tau^2 \sigma_i^2}{(H_d + \text{Pr}_1 \tau \sigma_r)^2 + \text{Pr}_1^2 \tau^2 \sigma_i^2} \right\} (-\text{Pr}_1 \tau I_5 + \text{Pr}_1 H_d^2 I_6) \right] = 0. \end{aligned} \quad (44)$$

It implies from eq. (43) that σ_r may be positive or negative which means that the system may be unstable or stable. Also, from eq. (44) σ_i may be zero or non-zero, meaning thereby that the modes may be non-oscillatory or oscillatory. In the absence of suspended particles, magnetic field and viscoelasticity, eq. (44) reduces to

$$i\sigma_i \left[I_2 + \frac{g\alpha\kappa a^2}{\nu\beta} \left(\frac{G}{G-1} \right) \text{Pr}_1 I_6 \right] = 0. \quad (45)$$

If $G > 1$ or $(C_p\beta/g) > 1$, then the coefficient of σ_i in eq. (45) is positive definite and hence implies that $\sigma_i = 0$. Thus oscillatory modes are not allowed and the principle of exchange of stabilities is satisfied in the absence of suspended particles, magnetic field and viscoelasticity. Therefore, oscillatory modes are introduced due to the presence of suspended particles alongwith viscoelasticity and magnetic field (hence Hall currents).

7. Concluding remarks

Combined effect of various parameters that is magnetic field, compressibility, Hall currents and suspended particles has been investigated on thermal instability of a Rivlin-Ericksen fluid. The principal conclusions are the following:

1. For stationary convection Rivlin-Ericksen fluid behaves like an ordinary Newtonian fluid due to the vanishing of the viscoelastic parameter.
2. From eq. (37), it is clear that the effect of compressibility is to postpone the onset of instability.
3. To investigate the effects of suspended particles, magnetic field and Hall currents, we examined the expressions dR_1/dH_d , dR_1/dQ_1 and dR_1/dM analytically. Magnetic field postpones the onset of instability whereas suspended particles and Hall currents are found to hasten the same. These analytic results are re-examined numerically for permissible range of values of various parameters and figs. (2)-(4) support these results graphically.
4. In Tables I – III, the critical thermal Rayleigh numbers and the associated wavenumbers are listed for different values of Q_1 , H_d and M respectively. Figures (5)-(7) show that the critical Rayleigh number R_c increases with the increase in Q_1 and decreases with the increase in H_d and M . Thus magnetic field stabilizes the system whereas the effect of suspended particles and Hall currents is to destabilize the system.
5. The oscillatory modes are introduced due to the presence of viscoelasticity, Hall currents and suspended particles. In the absence of these effects, the principle of exchange of stabilities is found to hold good.

Nomenclature

C_p	specific heat of the fluid at constant pressure, $[\text{J kg}^{-1}\text{K}^{-1}]$
C_{pt}	heat capacity of particles, $[\text{J kg}^{-1}\text{K}^{-1}]$
C_f	heat capacity of the fluid, $[\text{J kg}^{-1}\text{K}^{-1}]$
d	depth of fluid layer, $[\text{m}]$
e	charge of an electron, $[\text{C}]$
F	dimensionless kinematic viscoelasticity, $[-]$
$\vec{g} = (0, 0, -g)$	acceleration due to gravity, $[\text{ms}^{-2}]$
$\vec{h} = (h_x, h_y, h_z)$	perturbation in magnetic field $\vec{H}(0, 0, H)$, $[\text{G}]$
$\vec{H} = (0, 0, H)$	magnetic field intensity vector having components $(0, 0, H)$, $[\text{G}]$
$k = (k_x^2 + k_y^2)^{1/2}$	wavenumber of the disturbance, $[\text{m}^{-1}]$
$K' = 6\pi\mu\eta'$	Stokes' drag coefficient, $[\text{kgs}^{-1}]$
k_x, k_y	wavenumbers in x and y directions respectively, $[\text{m}^{-1}]$
M	dimensionless Hall current parameter, $[-]$
n	growth rate of the disturbance, $[\text{s}^{-1}]$
N_0	particle number density, $[\text{m}^{-3}]$

N	perturbation in suspended particle number density, [m^{-3}]
N'	electron number density, [m^{-3}]
p	fluid pressure, [Pa]
Pr_1	thermal Prandtl number, [-]
Pr_2	magnetic Prandtl number, [-]
q	effective thermal conductivity of the pure fluid, [$\text{Wm}^{-1}\text{K}^{-1}$]
Q_1	dimensionless Chandrasekhar number, [-]
R_1	dimensionless Rayleigh number, [-]
R_c	critical Rayleigh number, [-]
T	temperature, [K]
$\vec{v} = (u, v, w)$	fluid velocity vector having components (u, v, w) , [ms^{-1}]
$\vec{v}_d = (l, r, s)$	velocity of suspended particles, [ms^{-1}]
(x, y, z)	x, y, z directions
x	square of scaled wavenumber, [-]
x_c	critical wavenumber, [-]

Greek Letters

α	thermal coefficient of expansion, [K^{-1}]
$\beta (= dT / dz)$	temperature gradient, [Km^{-1}]
∂	curly operator, [-]
∇	del operator, [-]
δ	perturbation in the respective physical quantity, [-]
η'	particle radius, [m]
η	resistivity, [m^2s^{-1}]
θ	perturbation in temperature, [K]
κ	thermal diffusivity, [m^2s^{-1}]
μ	viscosity of the fluid, [$\text{kg m}^{-1}\text{s}^{-1}$]
μ'	viscoelasticity of the fluid, [$\text{kg m}^{-1}\text{s}^{-1}$]
μ_e	magnetic permeability, [H m^{-1}]
ν	kinematic viscosity, [m^2s^{-1}]
ν'	kinematic viscoelasticity, [m^2s^{-1}]
ρ	density of the fluid, [kgm^{-3}]

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