NUMERICAL SIMULATION OF SUDDEN-EXPANSION PARTICLE-LADEN FLOWS USING THE EULERIAN LAGRANGIAN APPROACH

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A Lagrangian-Eulerian model for the dispersion of solid particles in sudden-expansion flows is reported and validated. The fluid was calculated based on the Eulerian approach by solving the Navier-Stokes equations. A Lagrangian model is also applied, using a Runge-Kutta method to obtain the particle trajectories. The effect of fluid turbulence upon particle dispersion is taken into consideration through a statistical model. The predicted axial mean velocity and turbulent kinetic energy of both phases agree well with experimental data reported by Sommerfield.

Keywords: *Eulerian-Lagrangian model, gas-particles, sudden-expansion*

1. Introduction

Two-phase flows can be found in several industrial processes involving e.g. transport conveying, separation of solid particles and pulverized-coal combustion.

Nowadays, two categorical approaches for predicting tow-phase flows are Eulerian and Lagrangian. In the Eulerian approach the two phases are considered to be separate interpenetrating continua, and separate equations of motion are solved for each phase. Recent models of this type are those of Elghobashi et al. [1], Lun [2], Simonin [3]. This method may be preferably used for dense two-phase flows, for example, in fluidized beds or two-phase flows with phase transition, e.g. from bubbly flow to mist flow. In the Lagrangian approach the dispersed phase is treated by solving Lagrangian equations for the trajectories of a statistically significant sample of individual particles, while the liquid phase is treated as a continuum in the Eulerian approach. The two-way coupling between both phases is also accounted, Berlemeont et al. [4,5], Sommerfeld [6,7]. The Eulerian-Lagrangian approach allows an easy implementation of physical effects occurring on the scale of the particle size

as, for example, particle-particle interactions and particle-wall collisions. The key element of the Eulerian-Lagrangian approach is how it takes account of the effects of turbulent fluctuations on particle, as well as the effects of particles on turbulence properties of the liquid phase.

Historically, turbulent two-phase free jets have been the subject of many studies but, until very recently it has been difficult to find in the literature a well-documented numerical study of a two-phase turbulent sudden expansion jet. For an improved understanding of the characteristics of sudden expansion particle-laden flows are necessary.

A Eulerian-Lagrangian model was used to solve the governing equations of particle and liquid phase. The Eulerian framework was used for the liquid phase, whereas the Lagrangian approach was used for the particle phase. The steady-state equations of conservation of mass and momentum were used for the liquid phase, and the effect of turbulence on the flow-field was included via the standard k-ε model. The particle equation of motion included the drag force. Turbulence dispersion effect on the particles was simulated by statistical model. The effects of particles on the flow were modelled by appropriate source terms in the momentum equation.

This paper is organized as follows. In section 2, the mathematical model is discussed. In section 3, the numerical procedure and boundary conditions are presented. Finally, the numerical results which the predicted axial mean velocity and turbulent kinetic energy of both phases are compared with experimental data of Sommerfield [8].

2. Mathematical model

This section describes the mathematical model for turbulent liquid-particle flows assuming that the particulate phase is diluted, so that inter-particle effects are neglected. It is also assumed that the mean flow is steady and the material properties of the phases are constant.

The liquid phase has been calculated based on the Euler approach. The particle phase was treated following the Lagrangian approach, which means that the parameters of every particle are functions of time. For the coupling of phases, the PSI-CELL method of Crowe et al. [9] was chosen. By this method all influences of the dispersed phases on the continuous phase are accounted for through source terms in momentum equation.

2.1. Gas-phase flow model

The motion of the fluid is described by the continuity and the Navier-Stokes equations. Turbulence is modelled by the standard model Patanakar [10]. According to the experiment, the sudden-expansion particle-laden flows are considered to be steady axisymmetrical turbulent liquid-particle flows, with these assumptions, the corresponding governing equations for the liquid phase then are:

Continuity

$$\frac{\partial(\rho U_i)}{\partial x_i} = 0 \tag{1}$$

Navier-Stokes

$$\frac{\partial}{\partial x_j}\left[\rho U_i U_j - \mu\frac{\partial U_i}{\partial x_j}\right] = -\frac{\partial P}{\partial x_i} - \frac{\partial\left(\overline{\rho u_i u_j}\right)}{\partial x_j} + S_{u_i}^{p} \tag{2}$$

In these equations U_i denotes the Cartesian velocity components, P represent the fluid pressure, ρ and μ the density and viscosity of the fluid, respectively.

The turbulence was represented here by the k-ε model, which implies the need to solve two additional equations, namely.

turbulent kinetic energy

$$\frac{\partial}{\partial x_j}\left[\rho U_j k - \frac{\mu_{eff}}{\sigma_k}\frac{\partial k}{\partial x_j}\right] = P_k - \rho\varepsilon \tag{3}$$

turbulent dissipation

$$\frac{\partial}{\partial x_j}\left[\rho U_j \varepsilon - \frac{\mu_{eff}}{\sigma_\varepsilon}\frac{\partial \varepsilon}{\partial x_j}\right] = \frac{\varepsilon}{k}\left(C_1 P_k - C_2\rho\varepsilon\right) \tag{4}$$

$$\mu_{eff} = \mu + \mu_t \qquad P_k = \mu_{eff}\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}$$

where μ_t the turbulent viscosity is given by $\mu_t = \dfrac{C_\mu \rho k^2}{\varepsilon}$

For the k-ε model, the following standard coefficients are used see table I.

The interaction between particles and the liquid phase yields source terms in the governing equation for conservation of momentum is considered. The standard expression for the momentum equation source terms due to the particles has been used. It is obtained by time and ensemble averaging for each control volume in the form :

$$S_{u_i}^{p} = \frac{1}{V_{cv}} \sum_n m_n \left(u_i^{out} - u_i^{in} \right) - g_i \left(1 - \frac{\rho}{\rho_p} \right) \Delta t \tag{5}$$

Here V_{CV} is the control volume, m is the mass of an individual particle and n is the number of real particles.

2.2. Particle equation of motion

The equation of particle motion than can be written as follows:

$$m_p \frac{du_{p,i}}{dt} = \frac{3 \rho C_D m_p}{4 D p \rho_p} \left(u_i - u_{P,i} \right) \left| u_i - u_{p,i} \right| + m_p g_i \left(1 - \frac{\rho}{\rho_p} \right) \tag{6}$$

$$\frac{dx_{p,i}}{dt} = u_{p,i} \tag{7}$$

here $x_{p,i}$ are the co-ordinates of the particle position, $u_{p,i}$ are the velocity components, D_p is the particle diameter and ρ_p is the particle density.

The term on the right-hand side of eq. (6) is the drag force. Drag force is always present and is generally the dominating force for particle motion in most regions of the flow. Here C_D is the drag coefficient, which varies with particle Reynolds number, the drag coefficient for the solid particles is given as:

$$C_D = \begin{cases} \frac{24}{\mathrm{Re}_p} \left(1 + 0.15 \mathrm{Re}_p^{0.687} \right) & \mathrm{Re_p} \le 1000 \\ 0.44 & \mathrm{Re_p} > 1000 \end{cases} \tag{8}$$

The particle Reynolds number is defined as:

$$\mathrm{Re}_p = \frac{D_p \left| u_i - u_{p,i} \right| \rho}{\mu} \tag{9}$$

The effect of turbulence on the particle motion is modelled in the present work by a statistical model. The instantaneous liquid velocity along the particle trajectory is sampled from a Gaussian velocity distribution, with the equal RMS value u'=v'=2/3k in all two Cartesian co-ordinate directions to simulate isotropic turbulence. The instantaneous fluid velocity is assumed to influence the particle

motion during a given time period, called the interaction time, before a new fluctuation component is sampled from the Gaussian distribution function.

In the present model, the simulation of the interaction time of a particle with the individual turbulent eddies is limited by the lifetime of the turbulent eddy, T_e, or the transit time, T_r, for the particle to traverse the eddy, i.e.

$$\Delta t = \min(T_e, T_r) \tag{10}$$

where $T_e = 0.2\frac{k}{\varepsilon}$ and the eddy length scale $l_e = C_\mu^{0.75}\,\kappa^{1.5}/\varepsilon$. The transit time T_r can be determined from the linearized form of the particle momentum equation, given by Gosman and Ioannides [11]:

$$T_r = -\tau_P Ln\left(1 - \frac{l_e}{\tau_p \left|u_i - u_{p,i}\right|}\right) \tag{11}$$

where τ_P is the particle relaxation time, defined as:

$$\tau_p = \frac{4}{3}\rho_p D_p \Big/ \left(\rho C_D \left|u_i - u_{p,i}\right|\right). \tag{12}$$

The Lagrangian approach can follow only a moderate a number of particles. The real number of particles in the flow domain, however, is very large. For this reason, the special term parcel of particles was defined. Parcels represent a large bulk of particles with the same size, mass, velocity and position. The model is arranged such that every parcel represents the same mass of a disperse phase.

3. Numerical procedure and boundary conditions

The boundary conditions of the liquid phase are specified as table 2. for the particle phase, velocity and turbulent kinetic energy at the inlet are given as the same as the liquid phase. Particles are introduced into the flow at 25 different inlet positions between r=0 m and r=12.75 10^{-3}m. The resulting set of equations is solved by using a finite volume discretisation scheme and applying an iterative solution procedure based on the **SIMPLE** algorithm. This code was extended by introducing the additional source terms to account for the presence of particles.

A solution for the liquid field assuming no particles is initially obtained, and the prediction of the particle motion is carried out. Therefore Eq. (6) and (7) are solved by using a standard 4^{th} order Runge-Kutta scheme. The source terms are predicted simultaneously during trajectory calculation. The liquid field is then recomputed with the contribution of particle source terms. The detail of numerical scheme sees Mergheni et al. [12, 13].

4. Results and discussion

In this section, this simulations result for liquid and particle velocities and turbulence kinetic energy are presented. This simulations were performed for a 2D downward fully developed sudden-expansion, which is D1= 25.5 10^{-3}m diameter, D2=51 10^{-3}m diameter large and 1.0 m long. In order to obtain a better definition and, possibly a deeper physical understanding of the flow field of a sudden-expansion jet configuration, the radial profiles at various axial stations (x=9, 50, 100, 150, 200, and 300 mm) are presented one at the inlet of the sudden expansion geometry, three within the recirculation zone and two in the re-development zone.

Figure (2.a) and (3.a) show the presence of recalculating flow regions in the first two calculation sections for both phases. Comparing the experimental and numerical results at x=9mm of the mean velocities of fluid and particle, it can be observed the mean velocities are not well predicted. But figure (2.b) and (3.b) show that the simulated mean particle and fluid velocities are in favourable agreement with the experimental data.

For the turbulent kinetic energy of both phases is shown in figures 4 and 5. For the near exit region figures (4.a) and (5.a) the turbulent kinetic increase radially towards the shear layer at r=0.125 10^{-3}m , whereas the other stations (x=100, 150 and 200 mm) in the centre of the initial region the gas turbulence increases with increasing distances to the nozzle for the fluid and particles. The reason for increased turbulence is high radial diffusive turbulence from the shear layer into the centre of the flow.

In general, the numerical predictions of both phases are in good agreement with Sommerfield measurements especially for axial mean velocities of both phases. The difference between the measurement and prediction of radial velocity is caused by the turbulent model of liquid phase. Also, the small magnitude of the radial velocity in the experimental results may carry some error. There could be two reasons for the difference between measurements and prediction of turbulent kinetic energy of both phases. One is due to the assumed inlet conditions, as there are no experimental data available. The other is due to the prediction ability of the k-ε model for liquid phase in sudden expansion flow. Figures 4 and 5 show that, despite these two factors, numerical results of turbulent kinetic energy of both phases are in good agreement with measurements in the downstream region where the effect of the inlet conditions become weak and the k-ε is suitable for a channel flow.

5. Conclusion

Using the Eulerian-Lagrangian model, predicted results of velocity and turbulent kinetic energy of fluid and solid particles of sudden-expansion in a downward fully developed channel flow is represented. A Eulerian-Lagrangian model was used to solve the coupled governing equations of particle-laden flows. The steady-state equations of conservation of mass and momentum were used for the fluid, and the effect of turbulence on the flow-field was included via the standard k-ε model. The particle equation of motion included the drag force. Turbulence dispersion effect on the particles

was simulated as a continuous Gaussian random field. The effect of particles on the fluid was considered by inclusion of appropriate source terms in the momentum equation. The predicted axial mean velocity and turbulent kinetic energy of both phases agree well with experimental of Sommerfield, it is concluded that the Eulerian-Lagrangian model has been successfully applied in predicting sudden-expansion particles-laden flows.

Nomenclature

C_μ, C_1, C_2 coefficients of the turbulence model

C_D drag coefficient

D center jet diameter (mm)

D_p particle diameter (μm)

κ kinetic energy of turbulence (m^2s^{-2})

l_e eddy size (m)

r coordinate radial (m)

m_p particle mass

Re_p particle Reynolds number

S source term

t_r residence time of the particle (s)

t_e turbulent eddy lifetime (s)

u axial mean velocity (ms^{-1})

x coordinate axial (m)

V control volume (m^3)

Δt interaction time of particle (s)

Greek letters

Φ particle-loading ratio

ρ gas density (kgm^{-3})

ρ_p particle density (kgm^{-3})

τ_p particle time (s)

ε kinetic energy dissipation rate (m^2s^{-3})

μ dynamic viscosity ($kgm^{-1}\ s^{-1}$)

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CAPTIONS

TABLE CAPTIONS

Table. I. Turbulence model constants

Table II. Flow parameters

Table III. Boundary conditions for the liquid phase

FIGURE CAPTIONS

Figure 1. The jet flow configuration

Figure 2. axial mean velocity of the liquid phase

Figure 3. axial mean velocity of the particle phase

Figure 4. turbulent kinetic energy of the liquid phase.

Figure 5. turbulent kinetic energy of the particle phase.

Constant	C_μ	C_1	C_2	σ_κ	σ_ε
value	0.09	1.44	1.92	1.0	1.3

Table I : Turbulence model constants

liquid phase	**particle phase**
density ρ =830kg/.m^3	diameter D_p=450 μm
kinetic viscosity ν=5.205 cst	density ρ_p 2500kg/m^3
inlet velocity Uin=6.021 m/s	mass flux fp=0.213 kg/s
mass flux f =2.29 kg/s	

Table II. Flow parameters

at the inlet	u_{in}=6.021; v_{in}=0; k_{in}=0.006(u^2+v^2); $\varepsilon_{in} = \frac{\kappa_{in}^{3/2} C_\mu^{3/4}}{D/2}$
at the wall	no-slip conditions for velocity and the wall-function approximations for near-wall grid nodes
along the axis	v=0 and $\frac{\partial \Phi}{\partial x} = 0, \Phi = u, p, k, \varepsilon$
at the exit	full developed flow conditions

Table III: Boundary conditions for the liquid phase

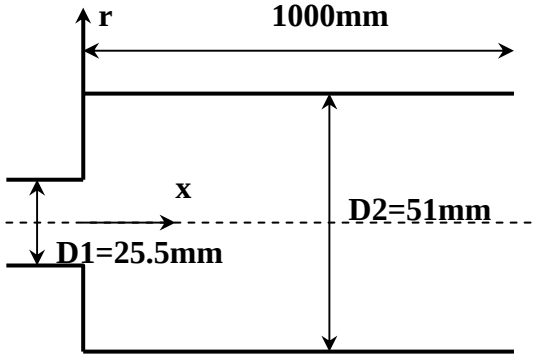

Fig. 1: The jet flow configuration of Sommerfield.

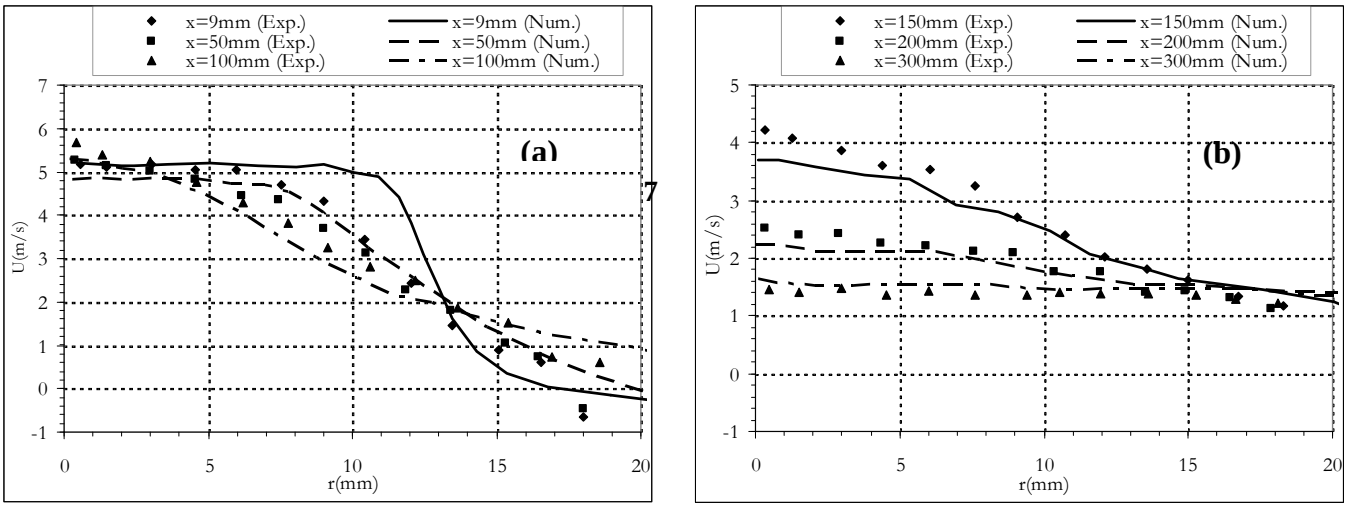

Figure 2. Axial mean velocity of the liquid phase

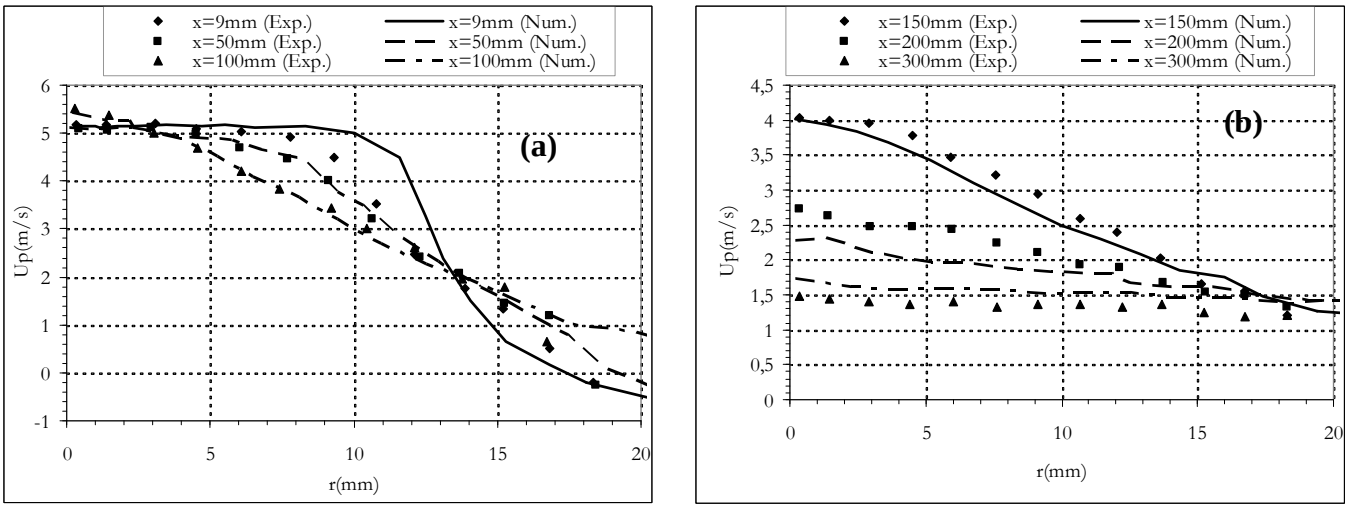

Figure 3. Axial mean velocity of the particle phase

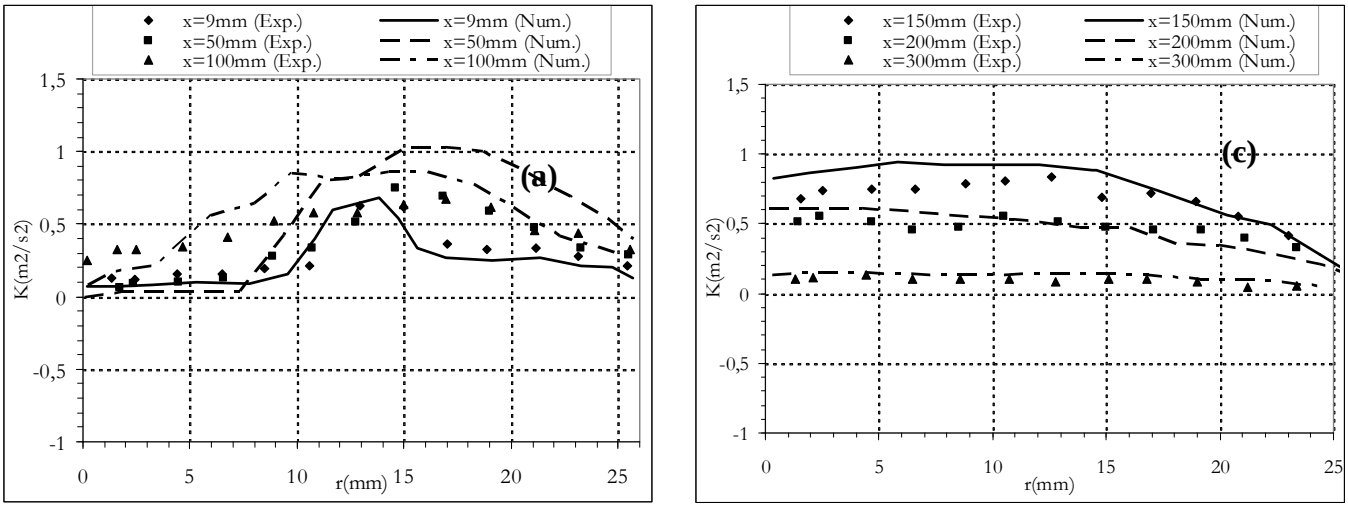

Figure 4. Turbulent kinetic energy of the liquid phase.

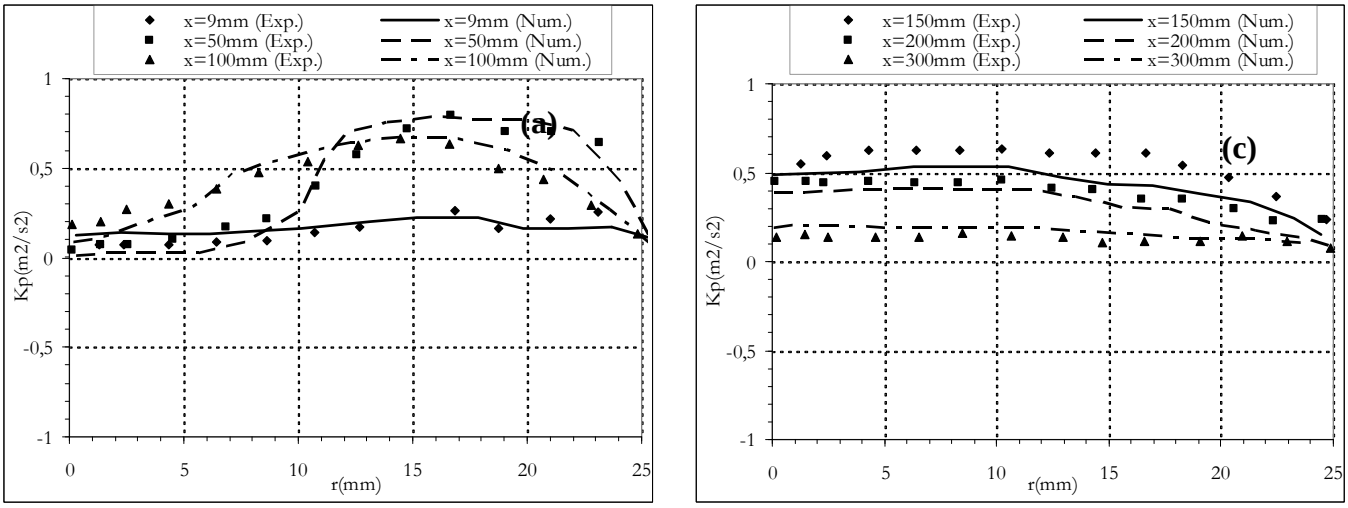

Figure 5. Turbulent kinetic energy of the particle phase.