

## MODELING OF THE CONJUGATE RADIATION AND CONDUCTION PROBLEM IN A 3-D COMPLEX MULTI-BURNER FURNACE

by

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*Radiation is a major component of heat transfer in the modeling of furnaces. In this study, coupled radiative and conductive heat transfer problems are analyzed in complex geometries with inhomogeneous and anisotropic scattering participating media. A 3-D model is developed using combination of the discrete ordinates method and blocked-off-region procedure. The finite volume method has been adopted to solve the energy equation and the radiative source term in the energy equation is computed from intensities field. The accuracy of radiative conductive model is verified by comparison with benchmark solutions from the literature. As an example of engineering problems, radiative-conductive heat transfer in a furnace model with gray, inhomogeneous, and anisotropic scattering media is numerically studied. The distributions of temperature and heat flux in the furnace are analyzed for different thermoradiative parameters such as conduction-radiation parameter, scattering albedo, and anisotropic scattering coefficient. The numerical algorithm described is found to be fast and reliable for studying combined conductive and radiative heat transfer in 3-D irregular geometries.*

Key words: 3-D complex geometries, radiation, conduction, blocked-off method, discrete ordinates method

### Introduction

Growing of energy consumption and generation of combustion pollutants causes the necessity of developing of the thermal performances in the combustion systems. This purpose can be obtained by improving mathematical models to design combustion systems that have enhanced energy efficiency with reduced pollutants production. Radiation heat transfer is an essential mode of heat transfer in high temperatures industrial systems such as combustion chambers and furnaces. So, it is important to develop mathematical models with high strength to analyze thermal radiation especially in case of combustion systems with coupled heat transfer modes and in enclosures with complex geometries [1].

Recently, considerable attention has been paid to the development of accurate and efficient methods for handling the radiation heat transfer. Among other methods, the discrete ordinates method (DOM) has received increasingly more attention because of its capability to join

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with other algorithms such as common control volume algorithms used in fluid dynamics. The method of discrete ordinates DOM is a straightforward method to solve radiative transfer problems. It offers a good compromise between accuracy and computational requirements. In particular, the DOM was originally formulated by Chandrasekar in 1950 [2], and has been deeply studied by Carlson *et al.* [3] and by Fiveland *et al.* [4, 5].

Among the few earlier works on radiation problems with inhomogeneous participating media, Henson *et al.* [6] studied a comparison of the Monte Carlo and the discrete transfer method in 3-D inhomogeneous scattering media. Guo *et al.* [7] developed the REM<sup>2</sup> method to investigate radiative heat transfer in inhomogeneous, non-gray and anisotropically scattering media. The discrete ordinates method has been reformulated to include effects of weak inhomogeneity in 3-D mediums by Galinsky [8]. Zorzano *et al.* [9] considered the atmospheric radiation transfer with discrete ordinates method in plane-parallel problems with strongly non-homogeneous media. Formulation of the FTn finite volume method using the blocked-off-region technique in 3-D complex inhomogeneous participating media was presented by Guedri *et al.* [10].

In order to avoid the complexity of treating non-orthogonal grids, it is suitable to formulate a procedure to model irregular geometries using Cartesian coordinates formulation. The blocked-off method which was previously applied in the computational fluid dynamics (CFD) problems [11] is a procedure that can model complex geometries using Cartesian regular mesh. This method can be used to treat both curved and inclined surfaces. It was firstly used in 2-D radiative transfer problems by Chai *et al.* [12, 13]. They found very promising results for different 2-D problems. Coelho *et al.* [14] used this method with the FVM and the DOM to predict radiative heat transfer in enclosures containing obstacles of very small thickness (baffles). The block-off method is used in the work of Guedri *et al.* [10] for inhomogeneous gray participating media and Borjini *et al.* [15] in another work used the same formulation and geometry with non-gray sooting media. Comparison between three types of boundary treatments, the spatial multiblock, blocked-off and embedded boundary has been studied by Byun *et al.* [16] for different cases of 2-D complex enclosures.

Many researchers have studied problems with combined conduction-radiation heat transfer. Tan [17] used the product integration method for solving combined conduction-radiation problem in square enclosure with isothermal walls. The DOM found its application in the work of Kim *et al.* [18] in the case of coupled radiative and conductive heat transfer in rectangular enclosures. Rouse [19] and Rouse *et al.* [20] used the control-volume finite element method (CVFEM) for the solution of combined mode of heat transfer in 2-D cavities. 3-D complex enclosures with coupled radiative and conductive heat transfer was studied in the work of Guedri *et al.* [21] using the finite volume method in gray absorbing-emitting and isotropically scattering medium. They found satisfactory solutions with comparison to reference data. Recently, Amiri *et al.* [22] analyzed the problem of combined conduction and radiation heat transfer in 2-D irregular geometries by using DOM and blocked-off method with both temperature and heat flux boundary conditions. Chaabane *et al.* [23] proposed a hybrid solver based on the lattice Boltzmann method (LBM) and the CVFEM for solving 2-D transient conduction and radiation heat transfer problems.

Although, there is a fast growth of research activity in this heat transfer area, but to the best of authors' knowledge, the effects of both radiation and conduction heat transfer on 3-D irregular enclosures with inhomogeneous and anisotropically scattering media and with the flame effects has not been investigated.

Therefore, in the present study, a 3-D blocked-off-region procedure was offered to model combined conductive and radiative heat transfer in multi-burner furnace with complex geometries, and inhomogeneous participating media. The energy equation is solved using the finite volume method and the radiative source term in the energy equation is computed from intensities field. The standard solution of DOM has been adopted to solve the radiative transfer equation in an absorbing-emitting, and anisotropic scattering medium. The radiative conductive model is validated by comparison with the well-documented results in literature. Furthermore, as a case of engineering applications this formulation is applied to analyze the effect of the main thermoradiative parameters on the temperature and radiative flux distributions in a 3-D multi-burner furnace with inhomogeneous participating media.

### Mathematical and numerical formulations

#### *Energy and radiative transfer equations*

The energy equation for coupled radiation–conduction heat transfer of an absorbing, emitting and scattering media under steady state condition with constant thermal conductivity and heat generation is:

$$k\nabla^2 T - \nabla q_r + \dot{q}''' = 0 \quad (1)$$

where  $\dot{q}'''$  ( $\text{Wm}^{-3}$ ) is the heat generation per unit volume. To obtain the temperature distribution in the medium by solving eq. (1), it is necessary to relate  $\nabla q_r$  to the temperature distribution within the medium. One approach is to obtain  $\nabla q_r$  directly by considering the local radiative interaction with a differential volume in the medium. The local divergence of the radiative flux is related to the local intensities by:

$$\nabla q_r = \kappa [4\pi I_b(r) - \int_{4\pi} I(r, \Omega) d\Omega] \quad (2)$$

To obtain the radiation intensity field and  $\nabla q_r$ , it is necessary to solve of the radiative transfer equation (RTE). This equation for an absorbing, emitting, and scattering gray medium can be written as in Modest [24]:

$$(\Omega \nabla) I(r, \Omega) = -\beta I(r, \Omega) + \kappa I_b(r) + \frac{\sigma_s}{4\pi} \int_{4\pi} I(r, \Omega') \Phi(\Omega, \Omega') d\Omega' \quad (3)$$

where  $I(r, \Omega)$  is the radiation intensity in position  $r$ , and in the direction  $\Omega$ ,  $I_b(r)$  – the radiation intensity of the black body in the position  $r$  and at the temperature of the medium,  $\kappa$  and  $\sigma_s$  are the gray medium absorption and scattering coefficients, respectively,  $\beta = (\kappa + \sigma_s)$  is the extinction coefficient, and  $\Phi(\Omega, \Omega')$  – the scattering phase function for radiation from incident direction  $\Omega'$  to scattered direction  $\Omega$ , and the integration is in the incident direction. For linear-anisotropic scattering, phase function is:

$$\Phi(\Omega, \Omega') = 1 + A_1(\Omega, \Omega') \quad (4)$$

in which  $-1 \leq A_1 \leq 1$  is an asymmetry factor. Values of parameter  $A_1$  are +1, 0, or –1 whether scattering is forward, isotropic or backward, respectively.

For diffusely reflecting surfaces, the radiative boundary condition for eq. (3) is:

$$I(r, \Omega) = \varepsilon I_b(r) + \frac{\rho}{\pi} \int_{\pi} I(r, \Omega') |n \cdot \Omega'| d\Omega' \quad (5)$$

where  $r$  belongs to the boundary surface and eq. (7) applies for  $n\Omega > 0$ ,  $I(r, \Omega)$  is the radiation intensity leaving the surface at the boundary condition,  $\varepsilon$  – the surface emissivity,  $\rho$  – the surface reflectivity, and  $n$  – the unit vector normal to the boundary surface.

In the DOM, the equation of radiation transfer is substituted by a set of  $M$  discrete equations for a finite number of directions  $\Omega_m$ , and each integral is substituted by a quadrature series of the form:

$$(\Omega_m \cdot \nabla)I(r, \Omega_m) = -\beta I(r, \Omega_m) + \kappa I_b(r) + \frac{\sigma_s}{4\pi} \sum_{k=1}^M w_k I(r, \Omega_k) \Phi(\Omega_m, \Omega_k) \quad (6)$$

subject to the boundary conditions:

$$I(r, \Omega_m) = \varepsilon I_b(r) + \frac{\rho}{\pi} \sum_{n\Omega_k < 0} w_k I(r, \Omega_k) |n\Omega_k| \quad (7)$$

where  $w_k$  are the quadrature weights. This angular approximation transforms the original equation into a set of coupled differential equations [24].

#### *Method of solution of radiative-conductive model*

To solve the coupled radiation-conduction problem, the eqs. (1) and (2) are converted to non-dimensional form:

$$\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2} + \frac{\partial^2 \theta}{\partial Z^2} + \dot{Q}''' = S_r \quad (8)$$

$$S_r = \frac{\tau^2(1-\omega)}{N_{cr}} \left[ \theta^4(r) - \frac{1}{4} \sum_{k=1}^M w_k \bar{I}(r, \Omega_k) \right] \quad (9)$$

with employing a reference length,  $L_*$ , a reference absolute temperature,  $T_*$ , and the following dimensionless variables:

$$\theta = \frac{T}{T_*}, \quad X = \frac{x}{L_*}, \quad Y = \frac{y}{L_*}, \quad Z = \frac{z}{L_*}, \quad \tau = \beta L_*, \quad (10)$$

$$\bar{I} = \frac{I}{\sigma T_*^4}, \quad N_{cr} = \frac{k\beta}{4\sigma T_*^3}, \quad Q = \frac{q}{\sigma T_*^4}, \quad \dot{Q}''' = \frac{\dot{q}''' L_*^2}{k T_*}$$

where  $N_{cr}$  is the conduction-radiation parameter (the Stark number, [19]).

The procedure of the numerical calculations starts by assuming that the radiative source term is zero and the energy equation is solved to find the temperature field. The line by line TDMA algorithm is used to quickly bring the information from all boundaries to the interior. Then the RTE is solved using DOM. The finite volume technique is used to discretize the spatial part of the RTE and an iterative procedure is employed to determine the intensity radiation field. The discretization details and solution procedure can be found in [24]. The employed angular quadrature set is the  $S_N$ . Consequently, the radiative source term is calculated and the energy equation is solved including the source term only once for the global iterative process to permit faster convergence.

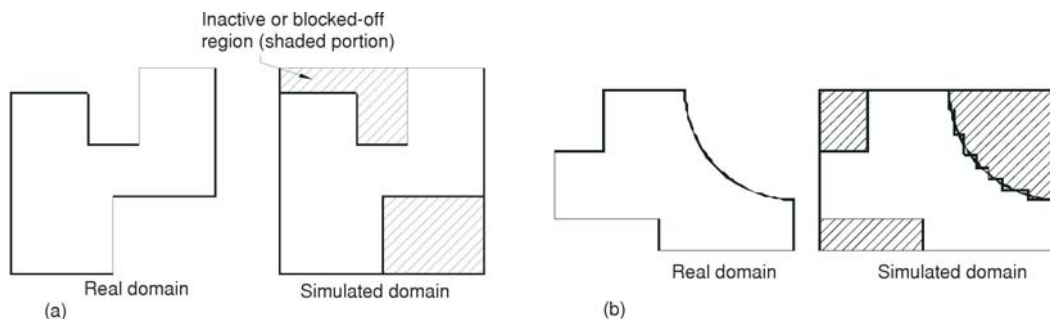
The iterative process continues until achieving convergence of the intensities and the temperature field. The convergence of the solution was evaluated using a convergence criterion taken as the error in the intensity field in RTE solution and the error in the temperature field for each node  $p$ , respectively, by the following criteria:

$$\begin{aligned} \text{error } I &= \max. \left| \frac{\bar{I}_p^n - \bar{I}_p^{n-1}}{\bar{I}_p^n} \right| \leq 10^{-6} \\ \text{error } T &= \max. \left| \frac{\theta_p^n - \theta_p^{n-1}}{\theta_p^n} \right| \leq 10^{-6} \end{aligned} \quad (11)$$

*The blocked-off method*

Irregular enclosures usually model using body-fitted mesh which grid lines are not necessarily orthogonal to each other. This kind of mesh demands extra complication in computations. In this work a treatment for irregular geometries in Cartesian co-ordinates has been presented that called the block-off region procedure.

The philosophy of this procedure consists on drawing nominal domains around given physical domains, so the region is divided into two parts: active and inactive or blocked-off regions. With this method, we can use the same rectangular (cubic in 3-D enclosures) algorithms to handle curved or inclined boundaries. Two sample 2-D geometries (for better observation) are presented in fig. 1 to show how they are treated to simulate from a rectangular geometry. In these figures, shaded portion is called inactive or blocked-off region. The remaining portion is the active region which is the real domain of interest. Curved boundaries appear to be a stair-stepped mesh as shown in fig. 1(b).



**Figure 1. Sample irregular geometries**

In order to distinguish active region from inactive one in the blocked-off method, an additional source term is introduced [10, 12, 13]:

$$S_{r,bloc} = S_{rC} + S_{rP}I \quad (12)$$

and added to the right part of eq. (6).

For a given real black boundary, in a cell in the inactive region, the additional source term becomes  $(S_{rC}, S_{rP}) = (MI_b, -M)$  (where  $M$  is a larger number). This additional source term forces the nodal intensity of the inactive control volumes to  $I_p = I_b$ . In the active region, the additional source term becomes  $(S_{rC}, S_{rP}) = (0, 0)$ .

For the case of real gray boundaries, we distinguish between emitted and reflected intensities. In the inactive cells the additional source terms is equal at  $(\epsilon MI_b, -M)$  for the emitted

part [10]. Thus, the nodal intensity of the inactive control volumes becomes  $I_p = \varepsilon I_b$ . The reflected part is added directly into the active region calculation of the control volumes adjacent to the real boundaries (active cell in direct contact with an inactive cell). The additional source terms for the reflected part becomes:

$$(S_{rC}, S_{rP}) = C \frac{\rho}{\pi} \sum_{n, \Omega_k < 0} w_k I(r, \Omega_k) |n\Omega_k|_0 \quad (13)$$

where  $C$  is a coefficient that is obtained from the discretized formulation of RTE. This coefficient is determined such that the intensity at the real boundaries satisfies eq. (7).

For the energy equation, the blocked-off-region technique consists on introducing a source term in the discretized energy equation,

$$S_{c,bloc} = S_{cC} + S_{cP} T_P \quad (14)$$

where for all active control volume, the additional source term becomes  $(S_{cC}, S_{cP}) = (0, 0)$ . But in the inactive region, those coefficients are given by  $(S_{cC}, S_{cP}) = (MT_b, -M)$  [21].

**Validation**

To show the validity and the accuracy of the current method, different test problems are analyzed and comparisons are made with the available results in literature.

*Test problem 1 (the blocked-off method validation)*

For the first test case an L-shaped enclosure is considered containing an emitting-absorbing medium at a temperature of 1000 K (see fig. 2), where the walls are black at 500 K. The employed grid includes  $12 \times 12 \times 5$  uniform control volumes, with  $8 \times 8 \times 5$  control volumes in the block-off region. This test case is investigated in the works of Sakami et al. [25] (a new 3-D algorithm based on the DOM and using an  $S_4$  quadrature with the exponential spatial discretization scheme applied to a grid include 2000 tetrahedra) and Joseph et al. [26] (standard DOM applied to a non-orthogonal structured grid with the  $S_4$  angular quadrature and the step differencing scheme).

Effect of the absorption coefficient of the medium on the wall heat flux exited from the AA' line (marked on fig. 2) is displayed in fig. 3. These results were obtained by using of  $S_4$  quadrature with

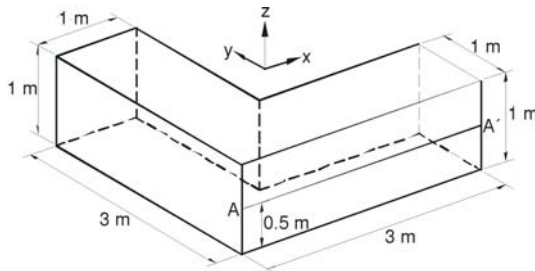


Figure 2. The geometry of test problem 1

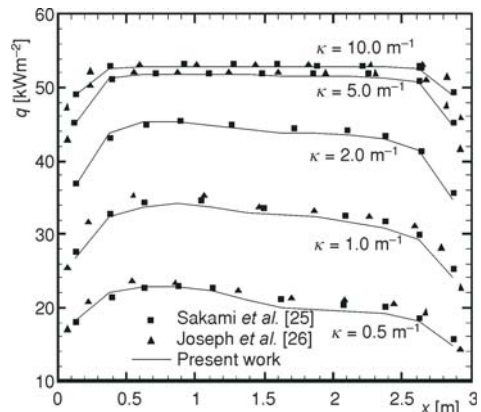


Figure 3. Net heat flux along line AA' of the L-shaped enclosure (fig. 2), comparison with the results of Sakami et al., and Joseph et al., ( $\omega = 0, \varepsilon = 1.0$ )

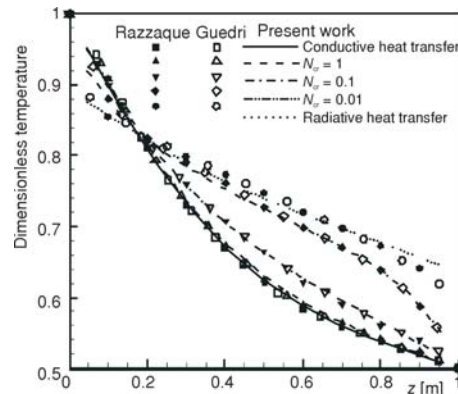


the step scheme. Comparison between the present results with those obtained in Sakami *et al.* [25] and Joseph *et al.* [26] shows an appropriate consistency. It can be found from this figure that the effect of the left-hand branch of the L-shaped becomes less important as the absorption coefficient increases.

*Test problem 2 (conduction-radiation validation)*

In this section, coupled conduction-radiation problem is analyzed in a 3-D rectangular enclosure with participating media. The dimensions of this tested geometry are  $L_x \times L_y \times L_z = 1 \times 10 \times 1 \text{ m}^3$ . The bottom wall ( $z=0$ ) is hot at dimensionless temperature  $\theta = 1$  and other walls are cold at dimensionless temperature  $\theta = 0.5$ . The media in cavity is gray and the boundaries are black surfaces. The medium is further assumed to absorb and emit radiation with  $\kappa = 1 \text{ m}^{-1}$ , but not scatter radiant energy,  $\omega = 0$ . This test problem has been solved in the work of Guedri *et al.* [21]. They used finite volume method to model combined conductive and radiative heat transfer. In the present work  $21 \times 21 \times 11$  control volumes and  $S_4$  angular quadrature is used.

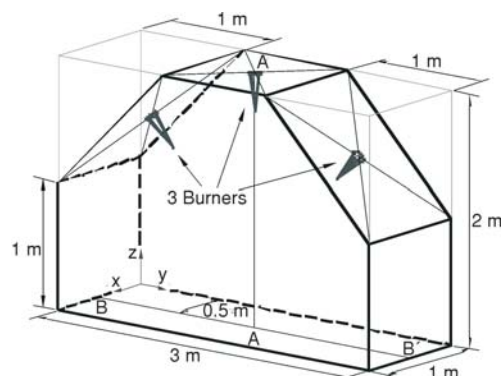
The effect of conduction-radiation parameter  $N_{cr}$  on the dimensionless temperature at the symmetry line ( $x = L_x/2, y = L_y/2$ ) is presented in fig. 4. A finite volume solution is also given for the case of pure conductive heat transfer. As  $N_{cr}$  decreases, radiation plays a more significant role than conduction. Therefore, as  $N_{cr}$  decreases, a steeper temperature gradient is formed at both end walls (right and left surfaces) and the medium temperature inside increases as shown in fig. 4. The present results show comparable accuracy with those presented by Guedri *et al.* [21] and the finite element method solutions in the work of Razzaque *et al.* [27].



**Figure 4. Dimensionless temperature profiles at line  $x = L_x/2, y = L_y/2$  for various conduction-radiation parameters  $N_{cr}$ , ( $\omega = 0, \tau = 1$ )**

**Results and discussion**

In this section, as an example of an application of engineering interest, the block-off region procedure is applied to a 3-D furnace. Figure 5 shows the schematic and dimensions of this 3-D furnace enclosure with its simulated domain. The furnace has two incline boundary planes on top-right and top-left and three burners that are sited on the middle of these two incline planes and horizontal top plane as shown in fig. 5. Each burner exit is simulated with a cone that radius of its base is equal to and height equal to 30 cm.

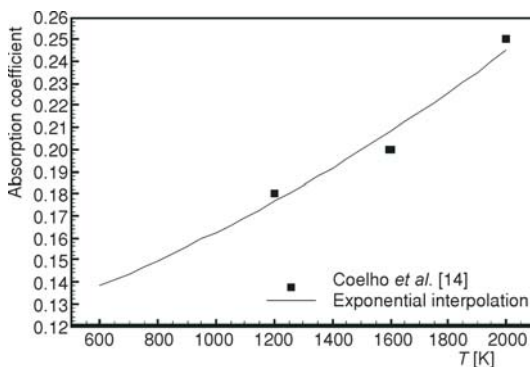


**Figure 5. Schematic of a 3-D furnace enclosure with simulated domain**

It should be mentioned that this model is not realistic. We have adopted this model to assess the applicability of the present method in analyzing radiative transfer in complex configurations with inhomogeneous and linear-anisotropic scattering media. The calculations have been carried out for several cases with combined conduction-radiation situation to investigate the effects of conduction-radiation parameter  $N_{cr}$  and scattering albedo  $\omega$  on thermal behavior of the system. Also, anisotropic scattering effects has been analyzed by using the different values of the scattering phase function coefficient  $A_1$ .

The medium is considered as gray and all furnace walls are diffuse with emissivity of 0.8. The temperature of walls is  $T = 600$  K which is also considered as the reference temperature. Since the present paper has been focused on combined radiative and conductive heat transfer modeling, the combustion process at the each burner is modeled as a uniform heat generation zone that takes place in flame cone areas shown in fig. 5. The immediate advantage of this approach is that it does not need to render fully detailed modeling of the combustion process.

The media in furnace enclosure, especially because of combustion process, consists of different gases and the temperature of the domain has intensive variation that makes variable absorption coefficient at the different points of the domain. So, considering homogeneous enclosure makes significant errors in the calculations. Therefore in this study, inhomogeneous participating media with temperature dependent absorption coefficients for the furnace enclosure is considered.



**Figure 6. Absorption coefficient vs. temperature obtained using exponential interpolation on the data of Coelho *et al.* [14] work**

In order to achieve this purpose, the data of Coelho *et al.* [14] work is used. In their work, radiation analysis in a combustion chamber of a utility boiler was investigated with three fixed different temperatures and absorption coefficients for various areas of enclosure. Because, in our work combined radiation and conduction heat transfer is analyzed and the temperature of furnace enclosure is not fixed, so a continuous distribution of absorption coefficient vs. temperature is needed. Therefore, as shown in fig. 6, an exponential interpolation of the data of Coelho *et al.* [14] work is obtained.

For using these temperature dependent absorption coefficients, first an initial solution with averaged absorption coefficient for entire computation domain is achieved. Then absorption coefficients of the domain are corrected based on the obtained temperature field and another solution is performed. This iterative process continues until achieving convergence.

For using these temperature dependent absorption coefficients, first an initial solution with averaged absorption coefficient for entire computation domain is achieved.

To check whether the grid resolution used is adequate, a grid independent study is carried out for the furnace with  $N_{cr} = 0.01$  and  $\omega = 0$  at four spatial grid sizes, *i. e.*,  $11 \times 31 \times 21$ ,  $15 \times 45 \times 31$ ,  $21 \times 61 \times 41$ , and  $31 \times 91 \times 61$ . The calculated thermal energy out going from the bottom wall of the furnace for the four grid sizes are 315.06 W, 235.488 W, 205.681 W, and 199.475 W, respectively. The percentage of difference between the first and second grid values is 25.2%, between the second and third grid values is 12.6% and between the third and fourth grid values is 3.0%. Using these results, a spatial grid size of  $21 \times 61 \times 41$  is chosen as optimum for the present problem. For the angular resolution,  $S_{12}$  angular quadrature is selected. Grid independence study shows that the results did not fully achieve grid independence. However, it has



been checked that further refinement of the grids does not affect the qualitative conclusions of this study but greatly increases the computing time [28].

In the first test case, the medium is considered to be non-scattering. Figure 7 shows the dimensionless temperature along the  $z$ -direction at the line  $AA'$  of the furnace enclosure (marked on fig. 5) for various values of the conduction-radiation parameter  $N_{cr}$  while other parameters are fixed. For better observation, only the upper half of the line  $AA'$  has been shown ( $1 \leq z \leq 2$ ). It is obvious that when  $N_{cr}$  increases the conductive heat transfer mode becomes dominant until it reaches infinity that in this point, the energy equation will transform into purely conduction heat transfer.

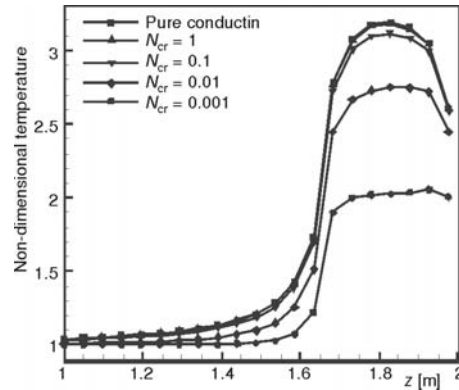


Figure 7. Dimensionless temperature profiles at line  $AA'$  for various conduction-radiation parameters  $N_{cr}$ , ( $\omega = 0$ )

As shown in fig. 7, when  $N_{cr}$  is unity, the temperature profile is not greatly affected by radiation when compared to the case of pure conduction. This suggests that the magnitude of the conduction-radiation parameter should be less than unity to require a radiation heat transfer calculation ( $N_{cr} \leq 1$ ). In an opposing manner, when  $N_{cr}$  decreases, radiative heat transfer becomes predominant. It is seen from this figure that the maximum temperature is occurred in the flame area.

In order to better studying the effect of conduction-radiation parameter  $N_{cr}$  on temperature profiles, isothermal contours at the plane  $x = L_x/2$  for four various have been shown in fig. 8. The effect of three burners' flames on temperature field has clearly appeared in this figure. It can be seen that when  $N_{cr}$  gradually decreases and radiative heat transfer becomes dominant mode, isotherms concentrate around the flames that cause sharp slope in the temperature profiles.

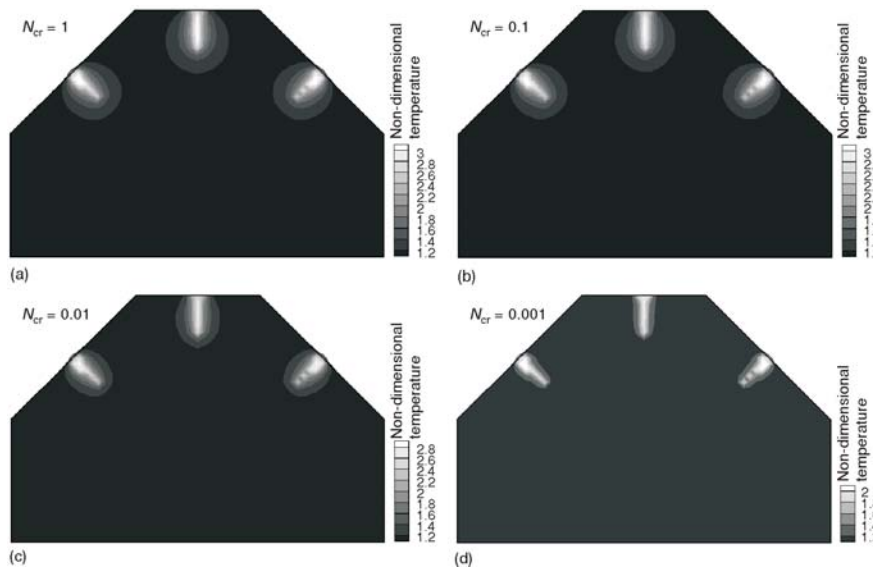


Figure 8. Isotherms at the plane  $x = L_x/2$  for various conduction-radiation parameters  $N_{cr}$ , ( $\omega = 0$ )

The non-dimensional total heat flux and the fractional radiative heat flux along the line  $BB'$  of the furnace enclosure (marked in fig. 5) for different values of  $N_{cr}$  are shown in fig. 9. The total heat flux is the summation of radiative heat flux and conductive heat flux. Also, the fractional radiative heat flux is the ratio of radiative heat flux to the total heat flux. Because the furnace is symmetric along the plane  $y = L_y/2$ , the results is shown for half of line  $BB'$  ( $0 \leq y \leq 1.5$ ). Figure 9(a) shows that the total flux becomes clearly much higher as  $N_{cr}$  increases and more uniform as  $N_{cr}$  decreases. According to fig. 9(b), the fractional radiative heat flux is seen to increase when  $N_{cr}$  decreases. Figure 9(b) truly reveals earlier described sentence that when decreases, radiative heat transfer becomes predominant.

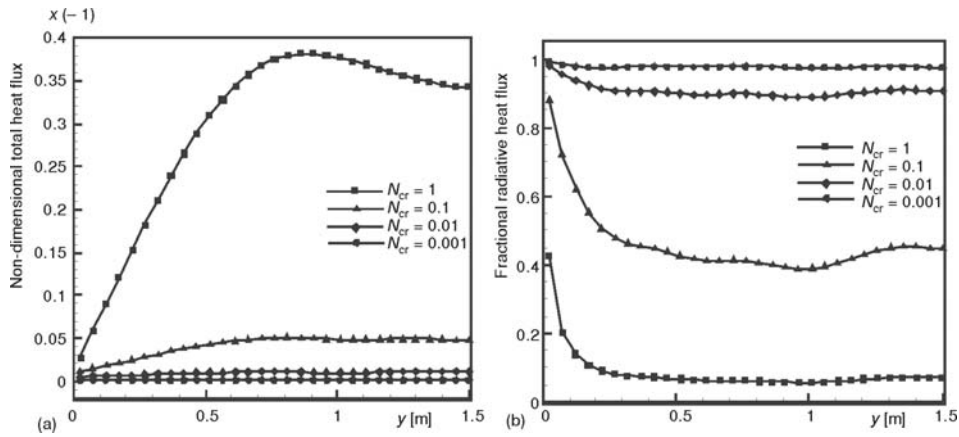


Figure 9. (a) Non-dimensional total heat flux; (b) fractional radiative heat flux at line  $BB'$  for various conduction-radiation parameters  $N_{cr}$ , ( $\omega = 0$ )

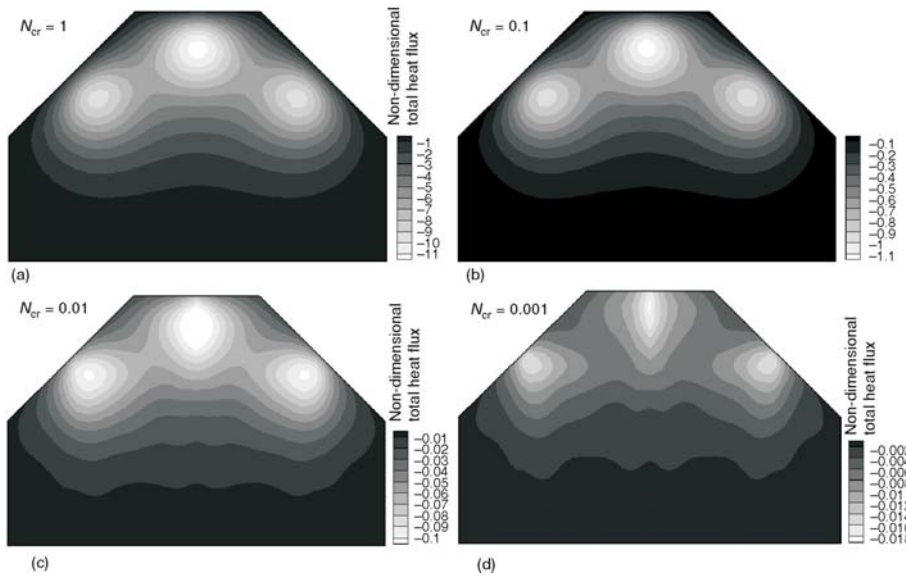


Figure 10. Non-dimensional total heat flux contours at the front plane for various conduction-radiation parameters  $N_{cr}$ , ( $\omega = 0$ )

Again, for better observing, the non-dimensional total heat flux contours at the front plane ( $x = L_x$ ) and bottom plane ( $z = 0$ ) for various conduction-radiation parameter  $N_{cr}$  are shown in figs. 10 and 11, respectively. It is seen from fig. 10 that when  $N_{cr}$  decreases and radiative heat transfer becomes dominant, heat flux contours concentrate around the flames of burners and disturbance in heat flux distribution progressively grows.

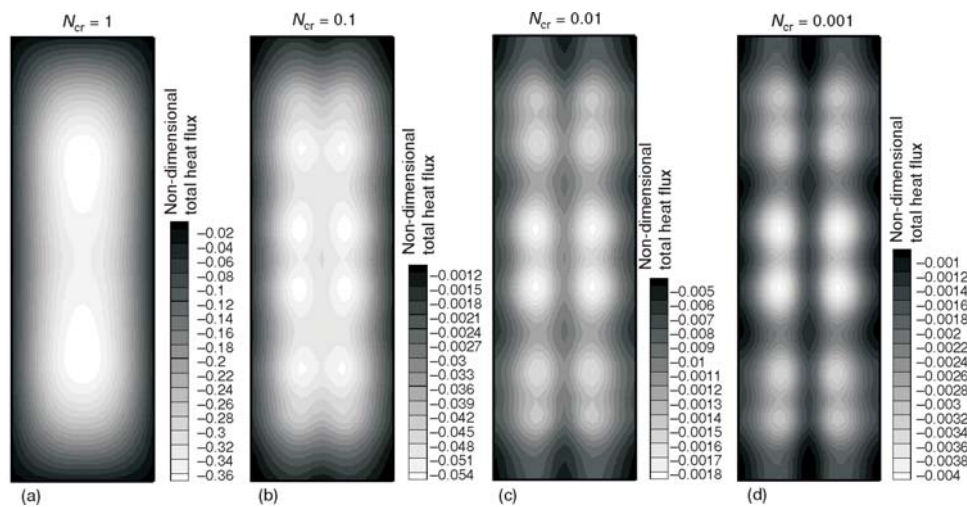


Figure 11. Non-dimensional total heat flux contours at the bottom plane for various conduction-radiation parameters  $N_{cr}$ , ( $\omega = 0$ )

This disturbance is better seen in fig. 11. In this figure is also observed that when radiation becomes dominant (small  $N_{cr}$ ), radiation intensity that is reflected from other furnace walls causes multiple projections of flames on bottom plane. For example, it is seen from fig. 11(d) that when  $N_{cr} = 0.001$  any burners have four projections on bottom plane.

Figure 12 shows the dimensionless total flux and the fractional radiative heat flux at the line  $BB'$  for various scattering albedo values with  $N_{cr} = 0.01$  and isotropic scattering ( $A_1 = 0$ ).

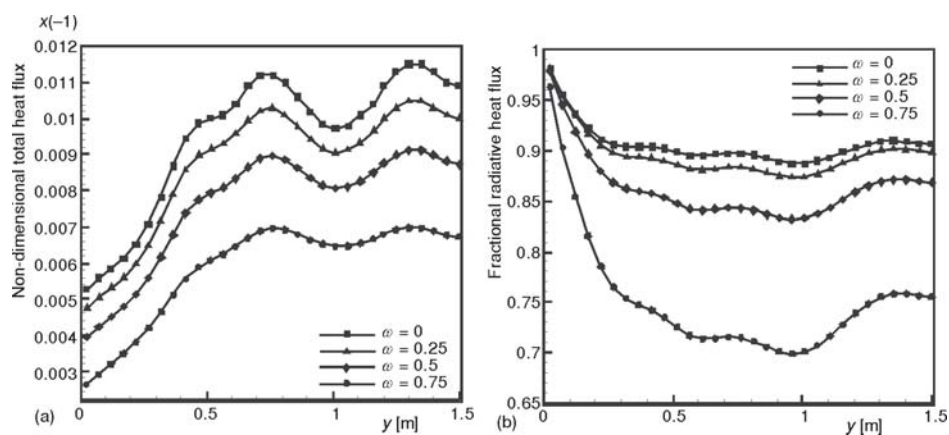


Figure 12. (a) Non-dimensional total heat flux; (b) fractional radiative heat flux at line  $BB'$  for various scattering albedos  $\omega$ , ( $N_{cr} = 0.01$ ,  $A_1 = 0$ )

According to fig. 12(a) it is seen that the dimensionless total heat flux decreases when increases. Such event occurs for the fractional radiative heat flux (fig. 12(b)).

In the following, we fixed the conduction-radiation parameter at  $N_{cr} = 0.01$  and the scattering albedo at  $\omega = 0.5$ . The dimensionless total heat flux and the fractional radiative heat flux along the line  $BB'$  are shown in fig. 13 for various values of the scattering phase function coefficient  $A_1$  to analysis the effect of linear-anisotropic scattering. It can be seen from this figure that the total heat flux and the fractional radiative heat flux are higher in forward scattering situation and lower in backward scattering. But it is obvious that in general they are approximately insensitive to this parameter. So, it can be concluded that linear-anisotropic scattering does not have much effect on thermal behavior of system.

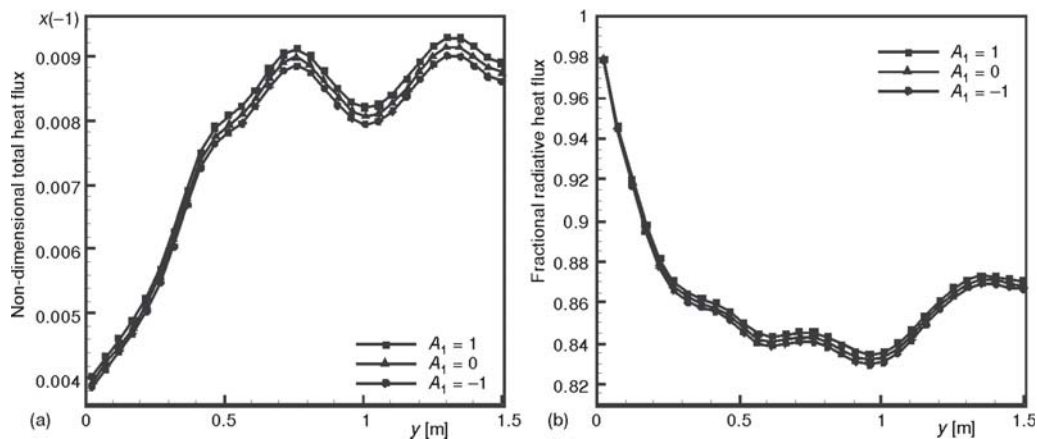


Figure 13. (a) Non-dimensional total heat flux; (b) fractional radiative heat flux at line  $BB'$  for various scattering phase function coefficient  $A_1$ , ( $N_{cr} = 0.01$ ,  $\omega = 0.5$ )

## Conclusions

An original 3-D combination of the DOM and the blocked-off-region technique was developed for analysis of combined conduction-radiation heat transfer problems. The blocked-off method that is used in this work is simple and has the major advantage of using the same rectangular (cubic in 3-D enclosures) algorithm for all types of geometries and can be utilized coupled with common control volume algorithms used in fluid dynamics.

In the first part of this work, this approach was applied to 3-D complex enclosures with diffuse reflective surfaces and containing gray absorbing-emitting and scattering medium. Results show that both wall heat flux and temperature distributions predictions are satisfactory when compared to benchmarked results. This consistency confirms that this method is a good general way for studying combined conductive-radiative heat transfer in 3-D irregular geometries. It is evident that for curved or inclined boundaries, a fine or a non-uniform grid is needed.

In the second part, a multi-burner furnace enclosure as an engineering case study of combined conductive-radiative heat transfer is analyzed with inhomogeneous and anisotropic scattering media. Three burners' flames of the furnace are modeled as uniform heat generation zones. The effects of various influencing parameters such as conduction-radiation parameter, scattering albedo and anisotropic scattering coefficient on the temperature and wall heat flux distributions are investigated. It is evidenced from the results that the magnitude of the ratio of

the radiant heat transfer to the conduction heat transfer should be at least unity to require a radiation heat transfer calculation ( $N_{cr} \leq 1$ ). Also, when the radiation heat transfer becomes dominant disturbance is clearly seen in the heat flux contours.

According to the results, this method is strongly recommended and its formulation with the blocked-off-region procedure develops an accurate and simple numerical tool to deal with 3-D radiative transfer in complex geometries with inhomogeneous media.

## Nomenclature

$A_1$	– scattering phase function coefficient, [–]	$\varepsilon$	– emissivity of a surface, [–]
$I$	– radiation intensity, [ $\text{Wm}^{-2}\text{sr}^{-1}$ ]	$\theta$	– dimensionless temperature, [–]
$\bar{I}$	– dimensionless radiation intensity, [–]	$\kappa$	– absorption coefficient, [ $\text{m}^{-1}$ ]
$k$	– thermal conductivity, [ $\text{Wm}^{-1}\text{K}^{-1}$ ]	$\rho$	– reflectivity of the surface, [–]
$L$	– length, [m]	$\sigma$	– Stefan-Boltzmann constant (= $5.670 \cdot 10^{-8}$ ), [ $\text{Wm}^{-2}\text{K}^{-4}$ ]
$M$	– number of discrete directions, [–]	$\sigma_s$	– scattering coefficient, [ $\text{m}^{-1}$ ]
$N_{cr}$	– conduction-radiation parameter (= $k\beta/4\sigma T^3$ ), [–]	$\tau$	– optical thickness (= $\beta L$ ), [–]
$n$	– unit vector normal to the surface, [–]	$\Phi$	– scattering phase function, [–]
$Q$	– dimensionless heat flux, [–]	$\omega$	– scattering albedo, [–]
$\tilde{Q}'''$	– dimensionless heat source per unit volume, [–]	$\Omega$	– direction vector, [–]
$q$	– heat flux, [ $\text{Wm}^{-2}$ ]	<i>Subscripts</i>	
$\tilde{q}'''$	– heat source per unit volume, [ $\text{Wm}^{-3}$ ]	$b$	– black body
$r$	– position vector, [m]	$m$	– discrete direction
$S$	– source term, [ $\text{Wm}^{-3}$ ]	$p$	– nodal point
$T$	– temperature, [K]	$r$	– radiation
$w$	– quadrature weights, [–]	*	– reference quantity
$x, y, z$	– co-ordinate, [m]	<i>Superscripts</i>	
$X, Y, Z$	– dimensionless co-ordinate, [–]	'	– incoming direction
<i>Greek symbols</i>		$n$	– iteration step
$\beta$	– extinction coefficient, [ $\text{m}^{-1}$ ]		

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