FLOW AND HEAT TRANSFER CHARACTERISTICS DOWNSTREAM OF A POROUS SUDDEN EXPANSION: A NUMERICAL STUDY

by

Khalid N. ALAMMAR
Department of Mechanical Engineering, King Saud University, Riyadh, Saudi Arabia

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Incompressible, axisymmetric laminar flow downstream of a porous expansion is simulated. Effect of the Darcy number and inertia coefficient on flow and heat transfer characteristics downstream of the expansion is investigated. The simulation revealed circulation downstream of the expansion. Decreasing the Darcy number is shown to decrease the circulation region. The Nusselt number, friction coefficient, and pressure drop are shown to increase, while reattachment and location of maximum heat transfer move upstream with decreasing Darcy number. Similar effects are observed with increasing inertia coefficient.

Key words**:** *friction, numerical, laminar, porous, expansion, heat transfer*

Introduction

The axisymmetric sudden expansion flow is encountered in numerous applications, including orifices, burners, and industrial duct and pipe systems. From a fundamental point of view, it is one of the most popular flows involving separation and reattachment. Many studies have been conducted to examine physics of the flow, which have revealed many characteristics of the flow. For example, it has been shown that with increasing expansion ratio, turbulence intensity downstream of the expansion increases and reattachment moves downstream of the expansion, Eaton *et al.* [1]. So [2] has reported that inlet Reynolds number has little or no effect on reattachment length. Shahnam *et al.* [3] have reported that a decrease in the expansion ratio increases the circulation region. Laser-Doppler velocimetry (LDV) measurements of Durrett *et al.* [4] have clearly identified a weak secondary recirculation zone (corner eddy). Stieglmeier *et al.* [5] investigated the effect of expansion angle on flow characteristics downstream of an axisymmetric expansion. They reported that the diffuser geometry influences the separated shear layer appreciably over the entire length of the diffuser section, and that production of turbulence after separation was higher in the case of 14 and 18-degree expansions compared to the 90-degree expansion.

As pertaining to the heat transfer aspect of the flow, experimental work of Baughn *et al.* [6] has shown that the local Nusselt number in the separated, reattached, and redevelopment regions were up to nine times higher than those for fully-developed flows in pipes having the same diameter as the pipe downstream. Maximum and average heat transfer

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* Author's e-mail: alammar@ksu.edu.sa

enhancements were shown to increase strongly with decreasing expansion ratio. Location of the maximum Nusselt number moved downstream as the expansion ratio increased, and unlike maximum normalized turbulence intensity, which was insensitive to Reynolds number, the peak values of Nusselt number correlated fairly well with the Reynolds number based on the upstream pipe diameter irrespective of the expansion ratio. For a given expansion ratio, the ratio of peak to fully-developed Nusselt number was reported to decrease as the Reynolds number increases. The work has also shown that a minimum Nusselt number takes place around one step height (*H*) downstream of the expansion for expansion ratios (*d/D*) below 0.5, suggesting the presence of a secondary circulation.

Said *et al.* [7] have investigated flow and heat transfer characteristics of pulsating flow downstream of an abrupt expansion for different frequencies (5 to 35 Hz), Reynolds numbers (5 and $10 \cdot 10^5$), expansion ratios (0.2 and 0.6), and Prandtl numbers (0.7 and 7.0). They reported that the influence of pulsation on the mean time-averaged Nusselt number is insignificant for fluids having a Prandtl number less than unity, while the increase is around 30% for fluids having a Prandtl number greater than unity. For all pulsating frequencies, the variation in the mean time-averaged Nusselt number, maximum Nusselt number, and its location with Reynolds number and diameter ratio exhibit similar characteristics to steady flows.

Evidently, the problem of axisymmetric sudden expansion has received much attention. However, effect of porous expansion on flow and heat transfer characteristics has not received much attention. Flow in porous media has many important applications, including industrial furnaces, heat exchangers, thrusters, *etc*. Accordingly, they have been widely investigated experimentally and numerically for many years, *e. g*., Jiang *et al.* [8] and Jiang *et al.* [9, 10] and the references therein. Porous media can intensify fluid mixing and can increase the surface area in contact with fluids, making them effective in heat transfer augmentation. Due to reduced thermal contact resistance and porosity, heat transfer enhancement is greater in sintered porous media than non-sintered, Jiang *et al.* [8].

Alkam *et al.* [11] investigated the thermal performance of a conventional concentric tube. Numerical results obtained showed that porous substrates of optimum thicknesses yield the maximum improvement in the heat exchanger performance with moderate increase in the pumping power. Al-Nimr *et al.* [12] numerically investigated forced convection in a concentric annulus partially filled with a porous medium. They reported a considerable increase in heat transfer as compared with the annulus without a porous medium. Mohamad [13] numerically investigated the heat transfer augmentation for flow in a pipe or a channel partially or fully filled with porous material. It was shown that partially filling the channel with porous substrates can reduce the thermal entrance length and enhance heat transfer.

Paval *et al.* [14] investigated the effects of porosity, porous material diameter, thermal conductivity, and Reynolds number on heat transfer rate and pressure drop in pipes. They reported that higher heat transfer rates can be achieved using porous inserts. They also show that for an accurate simulation of heat transfer through porous media, an accurate effective thermal conductivity is needed. Tak *et al.* [15] investigated the effect of thermal radiation and magnetic field on heat and mass transfer characteristics of natural convection on a vertical surface embedded in a saturated Darcian porous medium. They found that the Nusselt number increases and the Sherwood number decreases with the increase in radiation, while both the Nusselt number and Sherwood number decrease with the magnetic field.

In this work, incompressible, axisymmetric laminar flow downstream of a porous sudden expansion is simulated. The expansion ratio is 0.4 and the Reynolds and Prandtl numbers were 400 and 0.74, respectively. The flow is fully-developed prior to expansion. Schematic of the axisymmetric porous pipe with sudden expansion is shown in fig. 1.

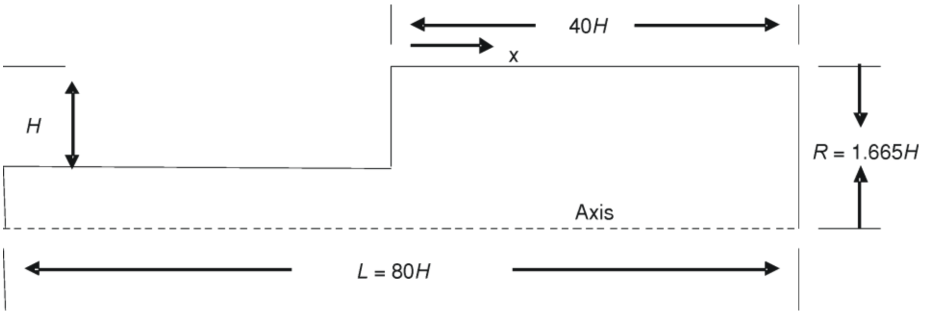

Figure 1. Schematic of the axisymmetric porous pipe with sudden expansion

The mathematical model

The mathematical model consisted of the following:

– *Continuity*

$$\frac{\partial(\rho u)}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\ r\rho v\ =0 \tag{1}$$

– *X-momentum*

$$\frac{\partial\ \rho uu}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\ r\rho vu\ =-\frac{\partial p}{\partial x}+\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)-\frac{\mu u}{K}-\frac{\rho F}{\sqrt{K}}|u|u \tag{2}$$

– *R-momentum*

$$\frac{\partial\ \rho uv}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\ r\rho vv\ =-\frac{\partial p}{\partial r}+\frac{\partial}{\partial x}\left(\mu\frac{\partial v}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial v}{\partial r}\right)-\frac{\mu v}{K}-\frac{\rho F}{\sqrt{K}}|u|v-\frac{\mu v}{r^2} \tag{3}$$

– *Energy*

$$\frac{\partial\ \rho cuT}{\partial x}+\frac{1}{r}\frac{\partial}{\partial r}\ r\rho cvT\ =\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{1}{r}\frac{\partial}{\partial r}\left(rk\frac{\partial T}{\partial r}\right) \tag{5}$$

The numerical procedure

Fluent 6.2 [16] solver was used. The structured grid was built using Gambit 2.0. The mesh consisted of approximately 50,000 structured cells. The simulation was carried out using the semi-implicit method with pressure-linked equations (SIMPLE) [17] and second-order schemes. The linearized equations were solved using Gauss-Seidel method, in conjunction with an algebraic Multigrid scheme [18]. No-slip condition was applied at the walls, and a heat flux of 3 W/m^2 was applied at the wall downstream of the expansion. Velocity inlet and pressure outlet were prescribed. Due to small overall temperature changes in the fluid, properties were assumed constant. In addition, transport properties of the fluid and porous medium were assumed identical. For the energy equation, this assumption fixed the Prandtl number for the flow field.

Uncertainty analysis

There are mainly two sources of uncertainty in computational fluid dynamics (CFD), namely modeling and numerical [19]. Modeling uncertainty is approximated through analytical or experimental validation while numerical uncertainty can be approximated through grid independence. Numerical uncertainty has two main sources, namely truncation and round-off errors. Higher order schemes have less truncation error, and as was outlined earlier, the discretization schemes invoked were second-order. In explicit schemes, round-off error increases with increasing iterations, and is reduced by increasing significant digits (machine precision). However, having used Gauss-Seidel iterative procedure in a steady-state simulation renders the calculation insensitive to round-off error.

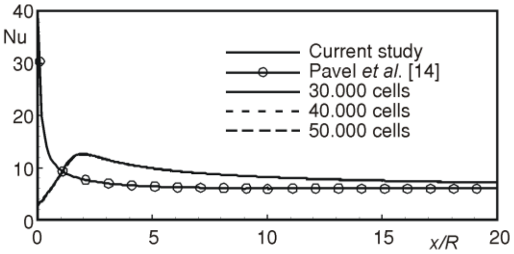

Figure 2. Distribution of the Nusselt number

A comparison between the current model and numerical work of Paval *et al.* [14] for a developing flow in a porous straight pipe is depicted in fig. 2 (lower profiles). The Nusselt number prediction of the two simulations are for Darcy number of 0.01, Re = 100, and the inertia coefficient $F = 0$. The two profiles are in excellent agreement. Therefore, we assume the modeling uncertainty to be negligible. The numerical error in the current simulation is approximated by the discrepancy with different cell count (upper profiles). Again, the uncertainty is negligible. Since the profiles are practically overlapping, we assume the numerical uncertainty to be negligible.

Results and discussion

Effect of the Darcy number on the friction coefficient downstream of the expansion is depicted in fig. 3(a) for F of 0.1. Overall, the friction coefficient is shown to increase with decreasing Darcy number. Decreasing Darcy number increases mixing and hence wall friction. For the same reason, reattachment length is shown to decrease with decreasing Darcy number. As expected, the friction coefficient downstream of the expansion approaches the value for a non-porous pipe flow with increasing Darcy number.

Effect of the Darcy number on the friction coefficient downstream of the expansion is depicted in figs. 3(b) and 3(c) for F of 0.5 and 1.0. Increasing the inertia coefficient is shown to increase the friction coefficient and decrease the reattachment length. Increasing the inertial coefficient has the same effect as decreasing the Darcy number. Hence, the similar influence on the friction coefficient.

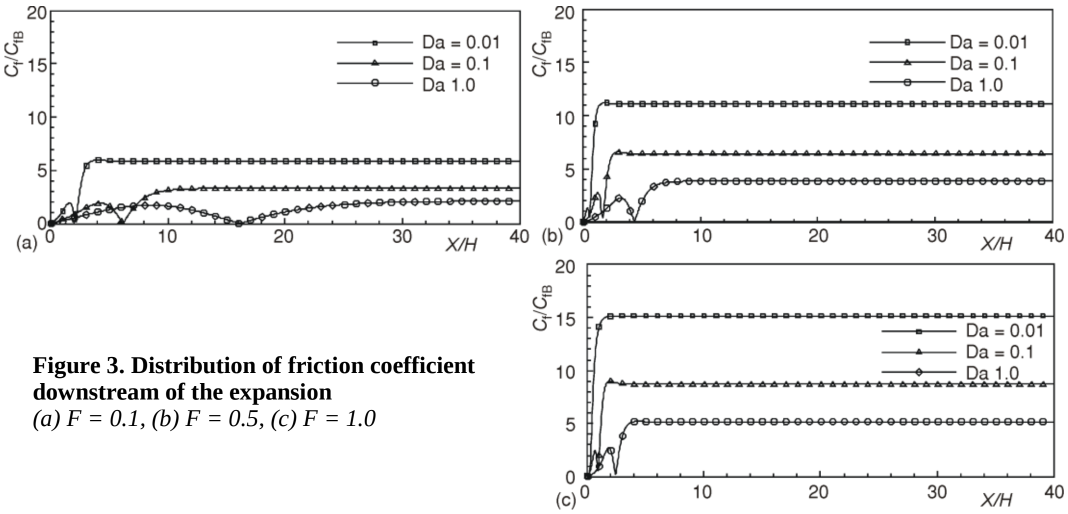

Figure 3. Distribution of friction coefficient downstream of the expansion
(a) F = 0.1, (b) F = 0.5, (c) F = 1.0

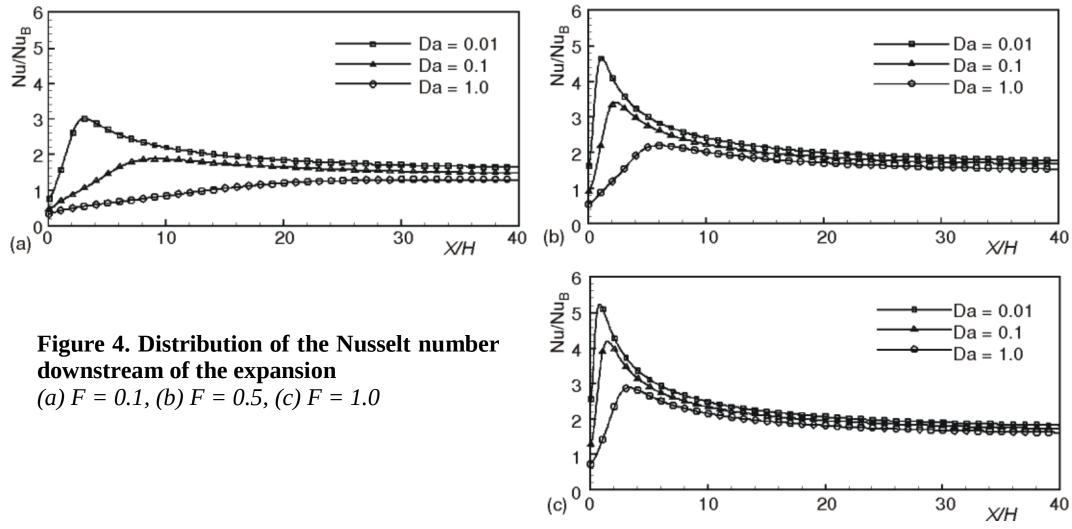

Figure 4. Distribution of the Nusselt number downstream of the expansion
(a) F = 0.1, (b) F = 0.5, (c) F = 1.0

Effect of the Darcy number on the Nusselt number downstream of the expansion is depicted in fig. 4(a) for *F* of 0.1. Due to enhanced mixing, decreasing Darcy number is shown to increase heat transfer. Following the reattachment length, maximum heat transfer is shown to shift upstream with decreasing Darcy number.

Effect of the Darcy number on the Nusselt number downstream of the expansion is depicted in figs. 4(b) and 4(c) for *F* of 0.5 and 1.0. Increasing the inertia coefficient is shown to increase the Nusselt number and shift location of maximum heat transfer upstream. Increasing the inertial coefficient has the same effect as decreasing the Darcy number. Hence, the similar influence on the Nusselt number

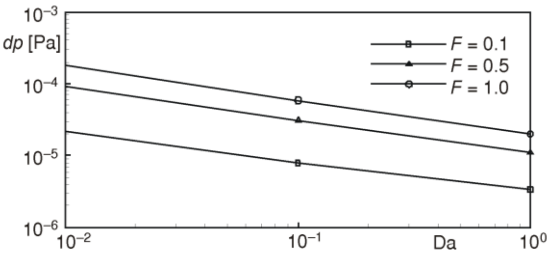

Figure 5. Pressure drop as a function of Darcy number

Effect of the Darcy number and the inertia coefficient on the pressure drop is depicted in fig. 5. As expected, the pressure drop increases with decreasing Darcy number. However, the pressure drop is greater with increasing inertia coefficient for a given Darcy number, suggesting minimizing the inertia coefficient in porous media design.

Effect of the Darcy number on the stream function downstream of the expansion is depicted in figs. 6(a) through 6(c) for *F* of 0.1. Due to increasing diffusion effect of porosity, decreasing the Darcy number is shown to decrease the circulation region. Similar effect is seen with increasing *F* as shown in figs. 6(c) through 6(e) for Darcy number of 1.0.

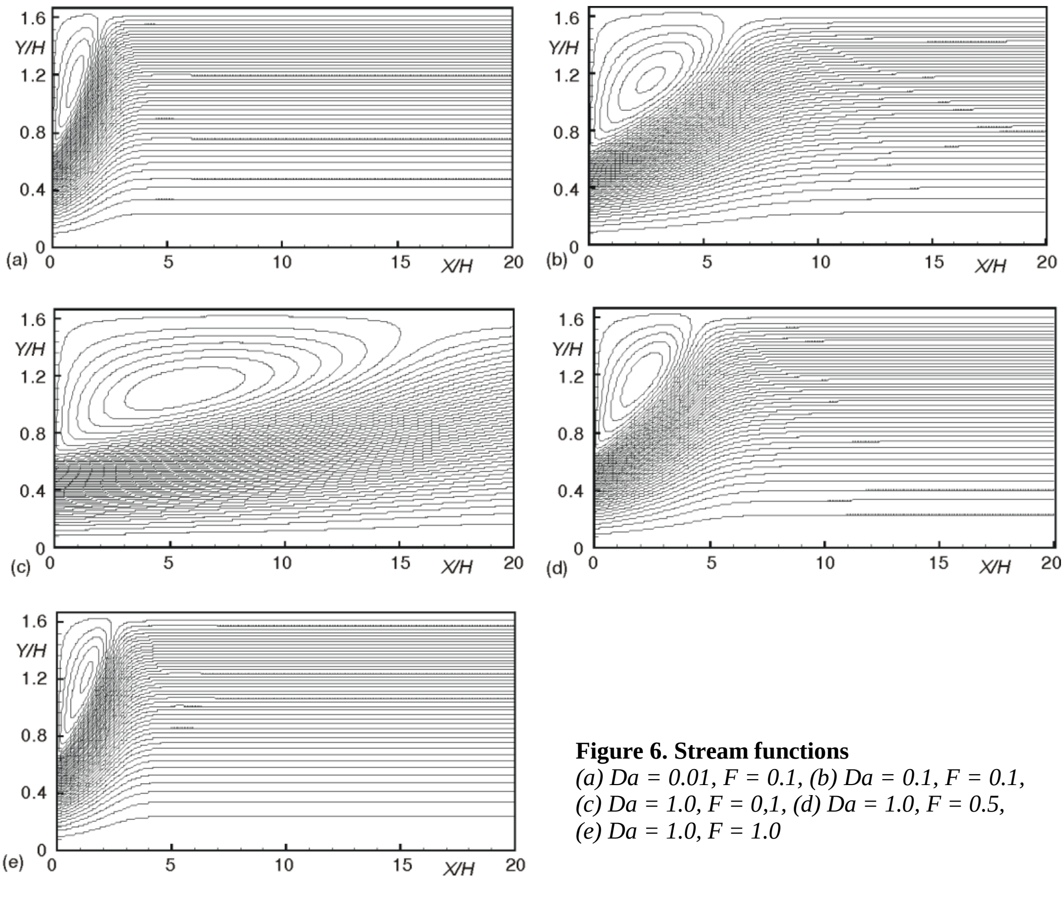

Figure 6. Stream functions
(a) Da = 0.01, F = 0.1, (b) Da = 0.1, F = 0.1, (c) Da = 1.0, F = 0,1, (d) Da = 1.0, F = 0.5, (e) Da = 1.0, F = 1.0

Conclusions

Incompressible, axisymmetric laminar flow downstream of a porous expansion was simulated. Effect of the Darcy number and inertia coefficient on flow and heat transfer characteristics downstream of the expansion was investigated. The simulation revealed circulation downstream of the expansion. Decreasing the Darcy number was shown to decrease the circulation region. Nusselt number, friction coefficient, and pressure drop were shown to increase with decreasing Darcy number. Reattachment and location of maximum heat transfer moved upstream with decreasing Darcy number. As expected, similar effects were observed with increasing inertia coefficient. However, the pressure drop was significantly greater with increasing inertia coefficient than with decreasing Darcy number, suggesting minimizing the inertia coefficient in porous media design.

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Nomenclature

C_f – friction coefficient (= $\tau_w/0.5\rho U_m^2$)
C_{fB} – baseline friction coefficient (= 0.04)
c – specific heat, [kJkg^{-1}K^{-1}]
D – pipe diameter downstream of expansion, [m]
Da – Darcy number (= K/R^2)
dp – pressure of the fluid, [Pa]
F – inertia coefficient
H – step height, [m]
h – heat transfer coefficient [= $q/(T_w - T_m)$], [Wm^{-2}K^{-1}]
K – permeability
k – fluid thermal conductivity, [Wm^{-1}K^{-1}]
L – pipe length, [m]
Nu – Nusselt number (= hD/k)
Nu$_B$ – clear pipe Nusselt number (= 4.36)
p – pressure of the fluid, [Pa]
q – heat flux through the wall, [Wm^{-2}]
R – pipe radius downstream of expansion, [m]
r – tube radius (= 10 mm)
Re – Reynolds number (= $\rho U_a D/\mu$), (= 400)
T_w – wall temperature, [K]
T_{in} – inlet temperature, [K]
T_m – ($4xq/\mathrm{Re}\mu C_p$) + T_{in}, [K]
U_a – area-averaged velocity downstream of expansion, [ms^{-1}]
u – axial velocity of the fluid, [ms^{-1}]
$|u|$ – velocity magnitude [= $(u^2 + v^2)]^{1/2}$, [ms^{-1}]
v – radial velocity of the fluid, [ms^{-1}]

Greek symbols

μ – dynamic viscosity of the fluid, [kgm^{-1}s^{-1}]
ρ – density of the fluid, [kgm^{-3}]
τ_w – wall shear stress, [Pa]

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