## EFFECT OF RADIATION AND POROSITY PARAMETER ON MAGNETOHYDRODYNAMICS FLOW DUE TO STRETCHING SHEET IN POROUS MEDIA

by

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An analysis is made for the steady two-dimensional flow of a viscous incompressible electrically conducting fluid in the vicinity of a stagnation point on a stretching sheet. Fluid is considered in a porous medium under the influence of (1) transverse magnetic field and (2) volumetric rate of heat generation/absorption in the presence of radiation effect. Rosseland approximation is used to model the radiative heat transfer. The governing boundary layer equations are transformed to ordinary differential equations by taking suitable similarity variables. In the present reported work the effect of porosity parameter, radiation parameter, magnetic field parameter and the Prandtl number on flow and heat transfer characteristics have been discussed. Variation of above discussed parameters with the ratio of free stream velocity parameter to stretching sheet parameter have been graphically represented.

Key words: magnetohydrodynamics, heat transfer, stretching sheet, Rosseland approximation

## 1. Introduction

The flow of an incompressible viscous fluid over a stretching surface is important in various processes. It is used to create polymers of fixed cross-sectional profiles, cooling of metallic or glass plates. Aerodynamics shaping of plastic sheet by forcing through a die and boundary layer along a liquid film in condensation processes are among the other areas of application. The production of sheeting material, which includes both metal and polymer sheets arises in a number of industrial manufacturing processes. In technical processes concerning polymer involves the drawing of strips. Strips which are extruded from a die through a stagnant fluid with controlled cooling system may become sometime stretched. The stretching surfaces undergo cooling/heating that causes surface velocity and temperature variations. The study of heat transfer and flow field is necessary for determining the quality of the final product. In technological processes at high temperature, thermal radiation effect cannot be neglected.

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Present research field has attracted many researchers in recent years due to its astounding applications. Gupta *et al.* [1] analyzed stagnation-point flow towards a stretching surface. They reported in their research work that a boundary layer is formed when stretching velocity is less than the free stream velocity. As the stretching velocity exceeds the free stream velocity than an inverted boundary layer is formed. Viscous incompressible fluid striking on a permeable stretching surface with heat generation or absorption has been studied by Attia [2].

Many researchers have analyzed magnetic field parameter on two-dimensional stagnation point flow of a viscous incompressible stagnation point flow of a viscous incompressible fluid. Attia [3] investigated effect of increasing magnetic field on velocity boundary layer thickness. AL-Harbi [4] studied continuously stretching surface in presence of suction/blowing with variable velocity and thermal conductivity. Stretching sheet with velocity and temperature proportional to the distance from stagnation point has been investigated by Jat *et al.* [5]. Timol *et al.* [6] analyzed unsteady boundary layer flow of non-Newtonian fluid past a semi-infinite plate. Effects of Ohmic heating and viscous dissipation on steady flow with variable free stream has been investigated by Singh *et al.* [7]. Many other phenomenon for effects of heat transfer flows have been discussed in detail by Bejan *et al.* [8], and White [9].

The radiation effect takes place at high temperature. Free convection heat transfer with radiation effect near the isothermal stretching sheet and over a flat sheet near the stagnation point have been investigated, respectively, by Ghaly *et al.* [10] and Pop *et al.* [11]. They found that a boundary layer thickness increases with radiation. The radiative effect on the heat transfer from an arbitrarily stretching surface with non-uniform surface temperature in a porous medium has been studied by Rashad [12]. An important contribution by Prandtl was to show that the Navier-Stokes equations can be simplified to yield an approximate set of boundary layer equations, see Schlichting [13]. A steady and unsteady boundary layer flow and heat transfer past a vertical stretching sheet in a viscous fluid-saturated porous medium has been analyzed by Ishak *et al.* [14]. They explained the effects of the porosity of the medium.

The authors in the present manuscript studied the steady two-dimensional flow of a viscous incompressible electrically conducting fluid in the vicinity of a stagnation point on a stretching sheet. Fluid is considered in a porous medium under the influence of transverse magnetic field, volumetric rate of heat generation/absorption in the presence of radiation effect. Rosseland approximation is used to model the radiative heat transfer. The linear stretching of the sheet is assumed because of its simplicity in the modeling of the flow and



Figure 1. Physical model of the problem

heat transfer over stretching surface. It also permits the similarity solution, which are useful in understanding the interaction of flow field with temperature field.

### Formulation of the problem

Consider steady two-dimensional flow of a viscous incompressible electrically conducting fluid in the vicinity of a stagnation point on a stretching sheet. Fluid is in a porous medium in the presence of transverse magnetic field and volumetric rate of heat generation/absorption

with radiation effect. The stretching sheet has uniform temperature  $T_w$  and linear velocity  $u_w(x)$ . It is assumed that external field is zero, the electric field owing to polarization of charges and Hall effect are neglected. Stretching sheet is placed in the plane y = 0 and x-axis is taken along the sheet. The fluid occupies the upper half plane *i. e.* y > 0 as shown in fig. 1. The governing equations of continuity, momentum and energy under the influence of externally imposed transverse magnetic field with radiation in the boundary layer are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u - \frac{vu}{k'}$$
(2)

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = K \frac{\partial^2 T}{\partial y^2} + Q(T - T_{\infty}) - \frac{\partial q_r}{\partial y}$$
(3)

Here  $q_r$  is approximated by Rosseland approximation, which gives:

$$q_{\rm r} = -\frac{4\sigma_{\rm s}}{3k} \frac{\partial T^4}{\partial y} \tag{4}$$

It is assumed that the temperature difference within the flow is so small that  $T^4$  can be expressed as a linear function of  $T_{\infty}$ . This can be obtained by expanding  $T^4$  in a Taylor series about  $T_{\infty}$  and neglecting the higher order terms. Thus we get:

$$T^{4} \cong T_{\infty}^{4} + 4(T - T_{\infty})T_{\infty}^{3}$$
$$T^{4} \cong 4T_{\infty}^{3}T - 3T_{\infty}^{4}$$

Here, in free stream u = U(x) = bx, eq. (2) reduces to:

$$U\frac{\mathrm{d}U}{\mathrm{d}x} = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \frac{\sigma B_0^2}{\rho}U - \frac{\nu U}{k'} \tag{5}$$

Eliminating  $(-1/\rho)(\partial p/\partial x)$  from eq. (2) using eq. (5), we get:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\mathrm{d}U}{\mathrm{d}x} + v\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}(u-U) - \frac{v(u-U)}{k'} \tag{6}$$

Boundary conditions are:

$$y = 0: \quad u = u_w(x) = cx, \quad v = 0, \quad T = T_w$$
$$y = \infty: \quad u = U(x) = bx, \quad T = T_\infty$$

## Method of solution

Introducing the stream function  $\psi(x,y)$  as defined by:

$$u = \frac{\partial \psi}{\partial y}$$
 and  $v = \frac{\partial \psi}{\partial x}$  (7)

and the similarity variable:

$$\eta = \sqrt{\frac{c}{v}} y \text{ and } \psi(c, y) = \sqrt{cv} x f(\eta)$$
 (8)

in the eqs. (3) and (6), we get:

$$f''' + ff'' - (f')^2 - (M+N)(f'-\lambda) + \lambda^2 = 0$$
(9)

and

$$\left(\frac{3+4R}{3}\right)\theta'' + \Pr\theta'f + \Pr S\theta = 0 \tag{10}$$

where the corresponding boundary conditions are reduce to:

$$f(0) = 0$$
,  $f'(0) = 1$ ,  $\theta(0) = 1$ , and  $f'(\infty) = \lambda$ ,  $\theta(\infty) = 0$ 

The governing boundary layer and thermal boundary layer eqs. (9) and (10) with these boundary conditions are solved using Runge-Kutta fourth order technique along with shooting method (Jain *et al.* [15]).

The wall shear stress  $\tau_w$  can be related to the skin fraction coefficient  $C_f$  according to:

$$C_{\rm f} = \frac{\tau_w}{\rho(u_w)^2}$$
  
$$\tau_w = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)_{y=0}$$
  
$$C_{\rm f} = x f''(\eta)$$
(11)

The rate of heat transfer in terms of the Nusselt number at the sheet is given by:

$$\mathrm{Nu}_{\mathrm{x}} = \frac{xq_{\mathrm{w}}}{K(T_{\mathrm{w}} - T_{\mathrm{w}})}$$

where

$$q_{\rm w} = -\left(K\frac{\partial T}{\partial y} + \frac{16\sigma_{\rm s}T_{\rm s}^3}{3k}\frac{\partial T}{\partial y}\right)_{y=0}$$
$$Nu_{\rm x} = -x\left(\frac{3+4R}{3}\right)\theta' \tag{12}$$

#### **Result and discussion**

Runge-Kutta fourth order method with the help of shooting method is used to solve eqs. (9) and (10) for different values of M,  $\lambda$ , N, S, R, and Pr taking step size 0.001. While numerical simulation, step size 0.002 and 0.003 were all checked and values of f'(0) and

2	f''(0)		
λ	Pop <i>et al.</i> [11]	Present paper	
0.1	-0.9694	-0.973710	
0.2	-0.9181	-0.921592	
0.5	-0.6673	-0.667685	
2	2.0174	2.0174765	
3	4.7290	4.7290671	

Table 1. Values of f''(0) for different values of  $\lambda$  are compared with the results obtained by Pop *et al.* [11]

 $\theta'(0)$  were found in each case correct up to five decimal places. Hence the scheme used in this is paper stable and accurate.

It is observed from tab. 1 that the numerical values of f''(0) in the present paper for different value of  $\lambda$ , when M = 0, and N = 0are in good agreement with results obtained by Pop *et al.* [11]. The skin-friction coefficient and Nusselt number presented by eqs. (11) and (12), are directly proportional f''(0)and  $-\theta'(0)$ , respectively. The effects of M,  $\lambda$ , N, S, R, and Pr on  $\theta'(0)$  have been presented through tabs. 2 and 3. We notice that for a

fixed value of M,  $\lambda$ , N, S, R, and Pr,  $-\theta'(0)$  decrease with the increase of the radiation parameter. However, the Nusselt number increases with increase of the Prandtl number. The physical reason for this trend is that at higher Prandtl number, fluid has thinner thermal boundary layer, which increases the gradient of the temperature. Consequently, the local Nusselt number increases as Pr increases.

Table 2. Values of rate of heat transfer for different values of Prandtl number  $\lambda$ , heat source/sink parameter S, radiation parameter R in absence of porosity parameter N and Hartmann number M

		- heta'(0)			
S	R	$M=0, N=0, \lambda=0.1$		$M=0, N=0, \lambda=2$	
		Pr = 0.01	Pr = 0.71	Pr = 0.01	Pr = 0.71
-0.1	1	0.33499	0.44968	0.33736	0.60180
-0.1	5	0.33380	0.36863	0.33456	0.41968
-0.1	10	0.33360	0.35219	0.33399	0.37978
0.1	1	0.33410	0.39364	0.33650	0.55893
0.1	5	0.33357	0.35053	0.33430	0.40304
0.1	10	0.33346	0.34239	0.33385	0.37038

Table 3. Values of rate of heat transfer for different values of Prandtl number  $\lambda$ , heat source/sink parameter *S*, radiation parameter *R* in presence of porosity parameter *N* and Hartmann number *M* 

		- heta'(0)			
S	R	$M=1, N=2, \lambda=0.1$		$M = 1, N = 2, \lambda = 2$	
		Pr = 0.01	Pr = 0.71	Pr = 0.01	Pr = 0.71
-0.1	1	0.33466	0.42784	0.337470	0.60934
-0.1	5	0.33374	0.36204	0.334590	0.42208
-0.1	10	0.33255	0.34868	0.334005	0.38101
0.1	1	0.33380	0.37010	0.336610	0.56684
0.1	5	0.33348	0.34379	0.334330	0.40548
0.1	10	0.33341	0.33884	0.333890	0.37166

λ	<i>f''</i> (0)			
	M = 0, N = 0	M = 1, N = 1	M = 1, N = 2	M = 1, N = 5
0.1	-0.973710	-1.59830100	-1.8339830	-2.4067698
0.2	-0.921592	-1.45470750	-1.6598185	-2.1618680
0.5	-0.667685	-0.97012145	-1.0910265	-1.3925573
2	2.0174765	2.459622	2.6539304	3.167219
3	4.7290671	5.503061	5.8528081	6.796776
5	8.5383344	9.045596	9.5265590	10.844605

Table 4. Values of f'(0) for different values of ratio of free stream velocity parameter to stretching sheet parameter  $\lambda$ , Hartmann number *M* and porosity parameter *N* 

Table 5. Values of f''(0) for different values of Hartmann number M taking ratio of free stream velocity parameter to stretching sheet parameter  $\lambda = 0.1, 2$ , and porosity parameter N = 2

м	<i>f'</i> (0)		
11/1	$\lambda = 0.1, N = 2$	$\lambda = 2, N = 2$	
0.5	-1.7201470	2.558575	
1	-1.8339830	2.659304	
2	-2.0479470	2.835211	
5	-2.5695147	3.320945	

Figure 2(a) and (b) shows that the boundary layer thickness decreases considerably as  $\lambda$  increases at the points where  $f'(\eta)$  reaches the boundary condition. The increase in the value of  $\lambda$  implies that free stream velocity increases in comparison to stretching velocity, which results in the increase in pressure and straining motion near stagnation point and hence thinning of boundary layer takes place. The phenomenon of thinning of boundary layer thus implies increased shear stress at the sheet, which is seen in tab. 4 when M = 0. It is important to note that for  $\lambda =$ 

= 1, there is no formation of boundary layer because the sheet velocity is equal to free stream velocity.



Figure 2. Velocity profile  $f(\eta)$  vs. similarity variable  $\eta$  at different values of (a)  $\lambda = 0.1, 0.2$ , and 0.5, and (b)  $\lambda = 2, 3$ , and 5 taking Hartmann number M = 0 and porosity parameter N = 0

The Hartmann number represents the importance of magnetic field on the flow. The presence of transverse magnetic field sets in Lorentz force, which results in retarding force on

the velocity field and therefore as Hartmann number increases, so does the retarding force and hence the velocity profiles decrease. This is shown in fig. 3(a) when  $\lambda < 1$ . In case when  $\lambda > 1$ , which is just opposite to  $\lambda < 1$ , as expected that the velocity profiles increase with the increase in the Hartmann number as shown in fig. 3(b). This explains the results presented in tab. 5, is that the shear stress at the sheet decreases due to increase in the Hartmann number when  $\lambda < 1$ , while it increases with the increase in the Hartmann number when  $\lambda > 1$ . Figure 4(a) and (b) show that the effect of porosity parameter *N* depends on  $\lambda$ . When  $\lambda < 1$ , velocity profile decreases as porosity parameter increases. The velocity profiles increase with the increase in the porosity parameter when  $\lambda > 1$ .



Figure 3. Velocity profile  $f(\eta)$  vs. similarity variable  $\eta$  at different values of Hartmann number M = 0.5, 1, 2, 5 taking (a)  $\lambda = 0.1$  and (b)  $\lambda = 2$ , and porosity parameter N = 2



Figure 4. Velocity profile  $f(\eta)$  vs. similarity variable  $\eta$  at different values of porosity parameter N = 0, 1, 2, 5 taking (a)  $\lambda = 0.1$  and (b)  $\lambda = 2$ , and Hartmann number M = 1

It is also observed from figs. 5(a) and (b) that the temperature profile decreases with an increase in the Prandtl number. This is in agreement with the physical fact that at higher Prandtl number, fluid has a thinner thermal boundary layer and this increases the gradient of temperature. On the other hand, figs. 6(a) and (b) show, as expected, that an increase of radiation parameter leads to an increase of the temperature profiles. Therefore higher value of radiation parameter implies higher surface heat flux.



Figure 5. Temperature profile  $\theta(\eta)$  vs. similarity variable  $\eta$  at different values of Prandtl number Pr = 0.01, 1, 2, 5 taking porosity parameter N = 2, (a)  $\lambda = 0.1$  and (b)  $\lambda = 2$ , heat source/sink parameter S = 0, radiation parameter R = 1, and Hartmann number M = 1



Figure 6. Temperature profile  $\theta(\eta)$  vs. similarity variable  $\eta$  at different values of radiation parameter R = 1, 2, 3, 5 taking Prandtl number Pr = 2, porosity parameter N = 2, (a)  $\lambda = 0.1$  and (b)  $\lambda = 2$ , heat source/sink parameter S = 0, and Hartmann number M = 1

#### Conclusions

In this paper, steady two dimensional flow of a viscous incompressible electrically conducting fluid in the vicinity of a stagnation point on a stretching sheet with free stream velocity is studied. Fluid is considered in a porous medium under the influence of transverse magnetic field, volumetric rate of heat generation/absorption in the presence of radiation effect. Rosseland approximation is use to model the radiative heat transfer. Numerical solution for the governing equations has been obtained using Runge-Kutta fourth order technique along with shooting method. The results indicate that:

- increasing stretching parameter increase the velocity components but decreases the velocity boundary layer thickness,
- fluid velocity decreases due to increase in the Hartmann number for  $\lambda < 1$ , while reverse effect is observed, when  $\lambda > 1$ ,
- effect of porosity parameter depends on  $\lambda$ , which is same as Hartmann number,

- Nusselt number increases with increase of the Prandtl number, whereas it decreases with increase of radiation parameter, and
- temperature decreases with increase in Prandtl number, while increases with increase in radiation parameter.

#### Nomenclature

- $B_0$ magnetic field intensity, [T]
- skin-friction coefficient, [-]  $C_{\rm f}$
- $C_p$ specific heat at constant pressure,  $[Jkg^{-1}K^{-1}]$ - stretching sheet parameter, [-] с
- thermal conductivity,  $[Wm^{-1}K^{-1}]$ K
- k mean absorption coefficient, [-] \_
- k' permeability of the medium, [m<sup>2</sup>]
- Hartmann number, (=  $\sigma B_0^2 / c\rho$ ), [-] М
- Ν porosity parameter, (= v/ck), [-]\_
- Nu<sub>x</sub> Nusselt number, [-]
- Pr Prandtl number,  $(= \mu C_p/K)$ , [-]
- pressure, [Nm<sup>2</sup>]
- 0 \_ volumetric rate of heat
- generation/absorption, [W]
- radiative heat flux, [Wm<sup>-2</sup>]  $q_{\rm r}$
- rate of heat transfer,  $[Wm^{-2}]$  $q_{\rm w}$
- R - radiation parameter,  $(4\sigma_s T_{\infty}^3/Kk)$ , [-]
- S heat source/sink parameter, \_
- $(= Q/c\rho C_{\rm p}), [-]$
- Т fluid temperature, [K]
- $T_{\rm w}$
- temperature of stretching sheet, [K]  $T_{\infty}$
- free stream temperature, [K]

U(x)- free stream velocity (= bx), [ms<sup>-1</sup>]

- velocity components along x- and u. v
  - y-axes, respectively, [ms<sup>-1</sup>]
- velocity of stretching sheet, [ms<sup>-1</sup>]  $u_{\rm w}(x)$
- Cartesian co-ordinates along x*x*, *y* and y-axes, respectively, [m]

Greek symbols

- similarity variable,  $[=(c/v)^{1/2}y]$ θ dimensionless temperature,  $[(T - T_{\infty})/(T_{w} - T_{\infty})], [-]$ λ ratio of free stream velocity parameter to stretching sheet parameter, (=b/c), [-]coefficient of viscosity, [kgms<sup>-1</sup>] μ v kinematic viscosity, [ms<sup>-2</sup>] - density, [kgm<sup>-3</sup>] ρ σ electrical conductivity, [Wm<sup>-2</sup>K<sup>-4</sup>]  $\sigma_{
  m s}$ - Stefan-Boltzmann constant
  - shear stress, [Pa]
- $\tau_{w}$  stream function Ψ

Superscript

- differentiation with respect to  $\eta$ 

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