DOUBLE TRIALS METHOD FOR NONLINEAR PROBLEMS ARISING IN HEAT TRANSFER

by

Chun-Hui HE [a] ***and Ji-Huan HE*** [b*]

[a] Department of Material Science and Engineering, Northeastern University, Qinhuangdao Branch, Qinhuangdao, China

[b] National Engineering Laboratory of Modern Silk, Soochow University, Suzhou, China

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According to an ancient Chinese algorithm, the Ying Buzu Shu, in about second century BC, known as the rule of double false position in West after 1202 AD, two trial roots are assumed to solve algebraic equations. The solution procedure can be extended to solve nonlinear differential equations by constructing an approximate solution with an unknown parameter, and the unknown parameter can be easily determined using the Ying Buzu Shu. An example in heat transfer is given to elucidate the solution procedure.

Key words: *ancient Chinese mathematics, approximate solution*

Introduction

The *Ying Buzu Shu* is the oldest method to solve algebraic equations, it traces back as far as in about second century BC, and it is known as the rule of double false position in West after 1202 AD, a detailed description is given in ref. [1].

Consider an algebraic equation:

$$f(x) = 0 \tag{1}$$

The basic idea of the *Ying Buzu Shu* is to choose two trials (approximate solutions) x_1 and x_2, and we have two remainders $f_1 = f(x_1)$ and $f_2 = f(x_2)$ respectively, and the approximate solution can be updated:

$$x_3 = \frac{x_2 f_1 - x_1 f_2}{f_1 - f_2} \tag{2}$$

The ancient Chinese mathematical algorithm was further extended to solve nonlinear differential equations, especially nonlinear oscillators, see refs. [1-3]. In this paper, we will suggest a new method called the double trials method to solve nonlinear differential equations.

* Corresponding author; e-mail: hejihuan@suda.edu.cn

The double trials method

The double trials method is proposed in this paper to solve nonlinear differential equations, the solution procedure is as follows: (1) to construct an approximate solution with an unknown parameter; (2) to substitute the approximate solution to the governing equation to have a residual equation; (3) to determine the unknown parameter using the basic concept of the *Ying Buzu Shu;* (4) to repeat the solution process by updating the approximate solution with a new unknown parameter.

In order to elucidate the solution procedure, we consider the following dimensionless nonlinear differential equation describing combined convection and radiation cooling of a lumped system [4]:

$$\frac{\mathrm{d}u}{\mathrm{d}t}+u+\varepsilon u^4=0 \quad u(0)=1\,, \tag{3}$$

where ε is the dimensionless radiation parameter.

Assume the solution can be expressed in the form:

$$u=\frac{1}{1+at} \tag{4}$$

where a is an unknown constant.

Substituting eq. (4) into eq. (3), we have the following residual:

$$R(t)=-\frac{a}{(1+at)^2}+\frac{1}{1+at}+\varepsilon\frac{1}{(1+at)^4} \tag{5}$$

Now we should use the *Ying Buzu Shu* to determine the unknown parameter. Assume that a_1 = 0 and a_2 = 1, we have $R_1(0)$ = 1 + ε and $R_2(0) = \varepsilon$, respectively. The unknown parameter, *a,* in eq. (4), can be determined using the following formula:

$$a=\frac{a_1R_2(0)-a_2R_1(0)}{R_2(0)-R_1(0)} \tag{6}$$

that is

$$a=\frac{a_1R_2(0)-a_2R_1(0)}{R_2(0)-R_1(0)}=\frac{-(1+\varepsilon)}{\varepsilon-(1+\varepsilon)}=1+\varepsilon \tag{7}$$

We, therefore, obtain the following approximate solution:

$$u=\frac{1}{1+(1+\varepsilon)t} \tag{8}$$

We can also assume that the solution can be expressed in the form:

$$u = \mathrm{e}^{-bt} \tag{9}$$

The residual is:

$$R(t) = (1-b)\mathrm{e}^{-bt}+\varepsilon\mathrm{e}^{-4bt} \tag{10}$$

Choose two arbitrary values of *b, i. e.,* b_1 = 0 and b_2 = 1, we have $R_1(0)$ = 1 + ε and $R_2(0) = \varepsilon$, respectively. The unknown parameter, *b*, can be determined as:

$$b = \frac{b_1 R_2(0) - b_2 R_1(0)}{R_2(0) - R_1(0)} = 1 + \varepsilon \tag{11}$$

The approximate solution reads:

$$u = \mathrm{e}^{-(1+\varepsilon)t} \tag{12}$$

The solution process can be continued by constructing a new approximate solution with a new unknown parameter, and a similar formula can be constructed to determine the parameter, which will be discussed in details in a forthcoming paper.

Conclusion

The suggested solution procedure is extremely simple and effective, and it is extremely accessible to engineers and non-mathematicians.

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