ESTIMATION OF THE LENGTH CONSTANT OF A LONG COOLING FIN BY AN ANCIENT CHINESE ALGORITHM

by

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In this paper, an ancient Chinese algorithm is used to estimate the length constant of a long cooling fin, and an approximate solution formulation is obtained. The obtained results show that this method is a simple but promising method without any requirement for advanced calculus.

Key words: *ancient Chinese mathematics, long cooling fin, approximate solution*

Introduction

The Nine Chapters is the oldest and most influential work in the history of Chinese mathematics [1]. The Chapter 7 of the Nine Chapters is the Ying Buzu Shu, which is the oldest method for approximating real roots of an equation. Consider an algebraic equation:

$$f(x) = 0 \tag{1}$$

The basic idea of the Ying Buzu Shu is to give two trial-roots, x_1 and x_2, which lead to the remainders $f(x_1)$ and $f(x_2)$, respectively, and the approximate solution can be written in the following form:

$$x_3 = \frac{x_2 f(x_1) - x_1 f(x_2)}{f(x_1) - f(x_2)}. \tag{2}$$

The ancient Chinese mathematical algorithm was furthered extended to solve nonlinear differential equations by the famous Chinese mathematician, Dr. Ji-Huan He, see refs. [1-6], and many followers found Ji-Huan He's idea extremely simple for engineering applications, see refs. [7-15]. In this paper, we will also follow Ji-Huan He's idea to estimate the length constant of a long cooling fin.

Mathematical model for a long cooling fin

Figure 1 shows a cooling fin of thin rectangular section projecting from a hot plate held at a fixed temperature. The fin loses heat by convection to the surrounding air, and the governing equation, found by expressing the heat balance in a small length element, is [16]:

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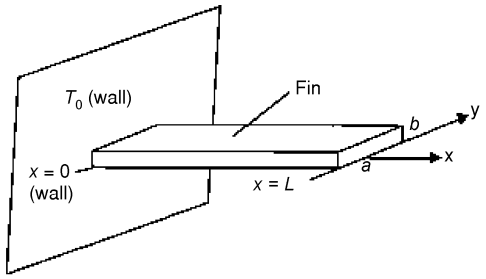

Figure 1. Geometry of a cooling fin

$$Kab\frac{d^2T}{dx^2} - 2ahT^n = 0 \qquad (3)$$

with boundary conditions

$$T(0) = T_0, \qquad T(l) = 0 \qquad (4)$$

Here T is the excess temperature above that of the surrounding air at a distance x along the fin, K is the thermal conductivity, a is the height of the fin, b is its thickness ($b \ll a$) and h is a convective heat transfer coefficient. The index n may vary from 1 to 5/4 according to the conditions in the surrounding air.

The equation may be simplified by dividing throughout by Kab. It may then be written as follows:

$$\frac{d^2T}{dx^2} - cT^n = 0 \qquad (3)$$

with boundary conditions

$$T(0) = T_0, \quad T(l) = 0 \qquad (4)$$

where $c = 2h/Kb$.

Estimation of the length constant of a long cooling fin

Similar to He's frequency-amplitude formulation [2, 6], we choose two trial functions:

$$T_1(x) = T_0 e^{-x} \qquad (5)$$

and

$$T_2(x) = T_0 e^{-\lambda x} \qquad (6)$$

Substituting eq. (5) and eq. (6) into, respectively, eq. (4), we obtain the following residuals:

$$R_1(x) = T_0 e^{-x} - c(T_0 e^{-x})^n \qquad (7)$$

and

$$R_2(x) = \lambda^2 T_0 e^{-\lambda x} - c(T_0 e^{-\lambda x})^n \qquad (8)$$

According to He's frequency-amplitude formulation (2, 6), we can approximately determine λ^2 in the following form:

$$\lambda^2 = \frac{\lambda_1^2 R_2(0) - \lambda_2^2 R_1(0)}{R_2(0) - R_1(0)} = \frac{\lambda^2 T_0 - cT_0^n - \lambda^2 (T_0 - cT_0^n)}{\lambda^2 T_0 - cT_0^n - (u_0 - cT_0^n)} = cT_0^{n-1} \qquad (9)$$

So the mathematical form of the temperature along a long cooling fin is:

$$T(x) = T_0 e^{-c^{1/2} u_0^{(n-1)/2} x} \qquad (10)$$

In order to verify the correctness of the obtained frequency, we consider two special cases.

Case 1

If $c = 1$, $n = 5/4$, $T_0 = 5$, eqs. (3) and (4) reduce to:

$$\frac{d^2T}{dx^2} - T^{5/4} = 0 \tag{11}$$

with boundary conditions

$$T(0) = 5, \quad T(l) = 0 \tag{12}$$

Then we can obtain the length constant of a long cooling fin as follows:

$$\lambda = 5^{1/8} \tag{13}$$

Therefore, we can get the mathematical form of the temperature along a long cooling fin:

$$T(x) = T_0 e^{-1.22284454499385x} \tag{14}$$

Comparison of the approximate solution, eq. (14), with that in ref. [2] is illustrated in fig. 2 showing a good agreement.

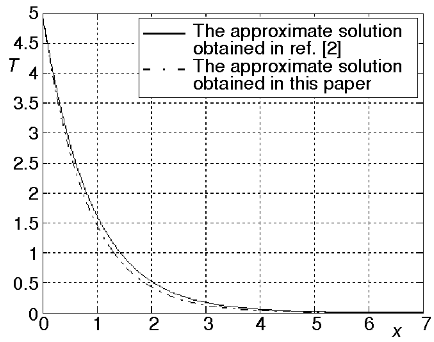

Figure 2 Comparison of the approximate solution obtained in this paper with the approximate solution obtained in ref. [16]

Case 2

If $c = 2$, $n = 5/4$, $T_0 = 1.5$, according to eqs. (3) and (4), we can obtain the following model:

$$\frac{d^2T}{dx^2} - 2T^{5/4} = 0 \tag{15}$$

with boundary conditions

$$T(0) = 1.5, \quad T(l) = 0 \tag{16}$$

Its approximate length constant and its approximate solution are, respectively, as follows:

$$\lambda = \sqrt{2 \cdot 1.5^{1/4}} \tag{17}$$

and

$$T(x) = T_0 e^{-1.48773782616449x} \tag{18}$$

which agrees well with the approximate solution obtained in ref. [16] as shown in fig. 3.

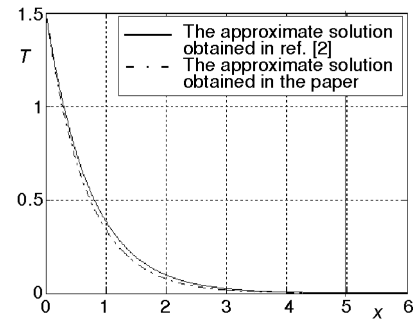

Figure 3. Comparison of the approximate solution obtained in this paper with the approximate solution obtained in ref. [16]

Conclusion

In this paper, we introduce the solution procedure using the basic concept of the ancient Chinese algorithm, and apply the method to obtain the length constant of a long cooling fin. The obtained solutions are in good agreement with ones in ref. [16]. The results show that the solution procedure of the ancient Chinese algorithm is of deceptive simplicity and the method might find wide applications.

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