A THERMO-ELECTRO-HYDRODYNAMIC MODEL FOR VIBRATION-ELECTROSPINNING PROCESS

by

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Original scientific paper
UDC: 678.7:537.632/.636:536.24:517.91/.95
DOI: 10.2298/TSCI11S1131X

In this paper, a thermo-electro-hydrodynamic model of the vibration-electrospinning process is first established. The model can offer in-depth insight into physical understanding of many complex phenomena which can not be fully explained experimentally. It is a powerful tool to controlling over physical characters.

Key words: *thermo-electro-hydrodynamic model, vibration, electrospinning*

Introduction

Electrospinning is one of the most popular methods to produce polymer nanofibers. The relatively high production rate and simplicity of the setup make electrospinning highly attractive to both academia and industry, and much work on experimental investigation, mathematical modeling, and numerical analysis is appeared in open literature [1-12]. However, some polymer with high viscosity can not be electrospun in a satisfactory way. Vibration-electrospinning was first suggested theoretically in ref. [13], the novel strategy produces finer nanofibers than those obtained without vibration [14], and can produce nanofibers which can not be done by electrospinning without vibration. In this paper, a thermo-electro-hydrodynamic model of the vibration-electrospinning process which can be applied to numerical study is established. The vibration-electrospinning setup is proposed, as illustrated in fig. 1.

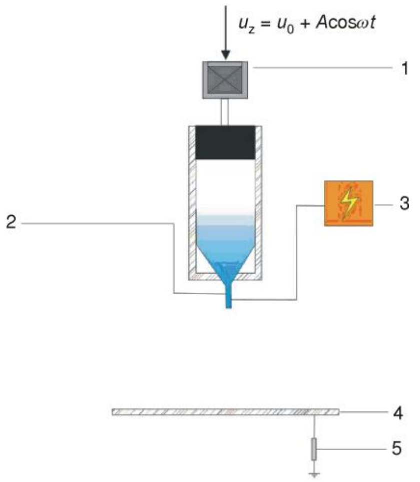

Figure 1. Vibration-electrospinning setup
(1) vibration piston: $u_r = 0$, $u_\theta = 0$, $u_z = u_0 + A\cos\omega t$, (2) nozzle, (3) high voltage supply, (4) grounded collecting plate, (5) resistance

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Thermo-electro-hydrodynamic model

This model consists of modified Maxwell's equations governing the electrical field in the jet, the modified Navier-Stokes equations governing heat and the jet under the influence of electric field, vibration force and air drag, and constitutive equations describing behavior of the jet. The governing equations are:

(1) Maxwell's equation

$$\frac{\partial q_e}{\partial t} + \nabla \cdot \vec{J} = 0 \tag{1}$$

where q_e is the electric charge, and $\vec{J}$ is the current.

(2) Continue equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{u}) = 0 \tag{2}$$

where ρ is the density of the jet and $\vec{u}$ is the velocity of the jet.

(3) Momentum equation

$$\rho \frac{D\vec{u}}{Dt} = \nabla \cdot \boldsymbol{t} + \rho \vec{f} + q_e \vec{E} + (\nabla \vec{E}) \cdot \boldsymbol{P} + \vec{F}_{\boldsymbol{v}} + \vec{F}_{\boldsymbol{f}} \tag{3}$$

where $\boldsymbol{t}$ is the stress tensor, $\vec{E}$ – the electric field, $\boldsymbol{P}$ – the polarization, $\vec{F}_v$ – the vibration force, and $\vec{F}_f$ – the air drag.

$$\vec{F}_{\boldsymbol{v}} = k_r \overline{\Delta p}[1 + \varepsilon_r \cos(\omega_q t + \varphi)] - k_q \overline{Q} \varepsilon_{\boldsymbol{q}} \omega_q \sin \omega_q t \tag{4}$$

$$Q(t) = \overline{Q}(1 + \varepsilon_{\boldsymbol{q}} \cos \omega_q t) \tag{5}$$

$$\Delta p(t) = \overline{\Delta p}[1 + \varepsilon_r \cos(\omega_q t + \varphi)] \tag{6}$$

where k_r and k_q are modified coefficients, $\overline{\Delta p}$ is the average pressure drop in the needle, $\overline{Q}$ – the average fluid flux in the needle, ε_r – a pulsant amplitude coefficient of the pressure drop, ε_q – a pulsant amplitude coefficient of the fluid flux, and ω_q is the pulsant frequency of the fluid flux.

$$\vec{F}_{\boldsymbol{f}} = C_f |\vec{u}| \vec{u} \tag{7}$$

where C_f is the frictional resistance coefficient.

(4) Energy equation

$$\rho c_p \frac{DT}{Dt} = Q_h + \nabla \cdot \vec{q} + \vec{J} \cdot \vec{E} + \vec{E} \cdot \frac{D\boldsymbol{P}}{Dt} + Q_v + Q_f \tag{8}$$

where $\vec{q}$ is the heat, Q_h – the source term, Q_v – the energy loss caused by the vibration force, and Q_f – the energy loss caused by the air drag.

This set of conservation laws can constitute a closed system when it is supplemented by appropriate constitutive equations for the field variables such as polarization. The most general theory of constitutive equations determining the polarization, electric conduction current, heat flux, and Cauchy stress tensor has been developed by Eringen *et al.* [15, 16].

$$\boldsymbol{P} = \varepsilon_p \vec{E} \tag{9}$$

$$\vec{J} = k\vec{E} + q_e \vec{u} + \sigma_T \nabla T \tag{10}$$

$$\vec{q} = \kappa \nabla T + \kappa_E \vec{E} \tag{11}$$

$$\boldsymbol{t} = -\tilde{p}\underline{\underline{\boldsymbol{I}}} + \eta[\nabla \underline{\boldsymbol{v}} + (\nabla \underline{\boldsymbol{v}})^t] \tag{12}$$

$$\eta = -\frac{\partial \psi_0}{\partial T} + \frac{1}{2\rho}\frac{\partial \varepsilon_p}{\partial T}\vec{E}\cdot\vec{E} \tag{13}$$

where ψ_0 and ε_p are the material moduli, κ is the electric conduction, and κ_E and σ_T are known as *Peltier* and *Seebeck* coefficients.

One dimensional thermo-electro-hydrodynamic model

A complete thermo-electro-hydrodynamic model which considers the couple effects of thermal field, electric field, vibration force and air drag is established. But it is too complex for numerical analysis. Therefore, a one-dimensional thermo-electro-hydrodynamic model which can be applied to numerical study is derived. As illustrated in fig. 2 the governing equations are:

(1) Maxwell's equations

The current $\boldsymbol{J}$ is composed of four parts: (1) the Ohmic bulk conduction current, $J_c = \pi r^2 kE$; (2) the surface convection current, $J_s = 2\pi r\sigma u$; (3) the current caused by polarization, $J_p = 2\pi r\varepsilon_p Eu$; and (4) the current caused by temperature gradient, $J_T = \pi r^2 \sigma T \partial T/\partial z$.

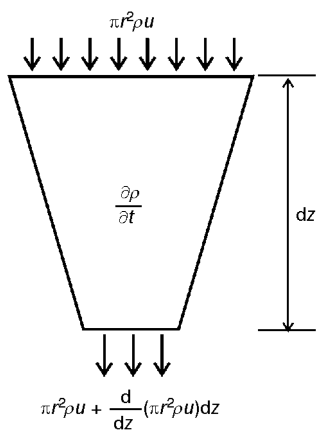

Figure 2. Schematic of a control volume

$$\frac{\partial}{\partial t}[2\pi r(\sigma + \varepsilon_p E)] + \frac{\partial}{\partial z}\left[2\pi r(\sigma + \varepsilon_p E)u + \right. \\ \left. + \pi r^2 kE + \pi r^2 \sigma T \frac{\partial T}{\partial z}\right] = 0 \tag{14}$$

where σ is the surface electric charge density, and E is the axial electric field.

(2) Continue equation

$$r^2 \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}(r^2 \rho u) = 0 \tag{15}$$

where r is the radius of the jet and u is the axial velocity of the jet.

(4) Momentum equation

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + g + \frac{2\sigma E}{\rho r} + \frac{1}{r^2}\frac{\partial \tau}{\partial z} + \frac{1}{r^2}\varepsilon_p E\frac{\partial E}{\partial z} + F_v + F_f \tag{16}$$

where p is the fluid pressure, F_v is the axial vibration force, and F_f is the axial air drag.

$$p = \kappa\gamma - \frac{\varepsilon - \bar{\varepsilon}}{8\pi}E^2 - \frac{2\pi}{\bar{\varepsilon}}\sigma^2 \tag{17}$$

$$\kappa = \frac{1}{R_1} + \frac{1}{R_2} \tag{18}$$

where R_1, and R_2 are principal radius of curvature, ε is the dielectric constant of fluid, and $\bar{\varepsilon}$ the dielectric constant of air.

(5) Energy equation

$$\rho c_\rho \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial z} \right) = Q_h + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z} + k_E E \right) + \left[2\pi r(\sigma + \varepsilon_p E)u + \pi r^2 kE + \pi r^2 \sigma T \frac{\partial T}{\partial z} \right] E + \\ + \varepsilon_p E \left(\frac{\partial E}{\partial t} + u \frac{\partial E}{\partial z} \right) + Q_v + Q_f \tag{19}$$

Conclusion

A complete thermo-electro-hydrodynamic model which considers the couple effects of thermal field, electric field, vibration force and air drag is first established. The disadvantage of this model is that it is too complex for numerical analysis. Therefore, a one-dimensional thermo-electro-hydrodynamic model which can be applied to numerical study is derived. The model can offer in-depth insight into physical understanding of many complex phenomena which can not be fully explained experimentally. It is a powerful tool to controlling over physical characters. Many basic properties are tunable by adjusting electrospinning parameters such as voltage, flow rate, and others. According to the established model, a lot of experiments will be carried with the vibration-electrospinning setup to testify the technology can produce nanofibers which can not be done by electrospinning without vibration.

Acknowledgements

The present work is supported financially by National Natural Science Foundation of China under Grant No. 10802021, National Natural Science Foundation of Shanghai under Grant No.08ZR1400300 and Shanghai Rising-Star Program under Grant No.09QA1400100.

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Paper submitted: July 1, 2010
Paper revised: September 1, 2010
Paper accepted: November 11, 2010