EFFECT OF TEMPERATURE-DEPENDENCY OF SURFACE EMISSIVITY ON HEAT TRANSFER USING THE PARAMETERIZED PERTURBATION METHOD

by

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Knowledge of the temperature dependence of the physical properties such surface emissivity, which controls the radiative problem, is fundamental for determining the thermal balance of many scientific and industrial processes. The current work studies the ability of a strong analytical method called parameterized perturbation method (PPM), which unlike classic perturbation method do not need small parameter, for nonlinear heat transfer equations. The results are compared with the numerical Runge-Kutta method showed good agreement.

Key words: *radiative heat transfer, thermal dependency, surface emissivity, PPM*

Introduction

Thermal radiation occurs in many industrial applications such as heating, cooling and drying, as well as in the energy conversion phenomenon which include fuel combustion and solar radiation. It also occurs in advanced technologies, for example during rapid solidification by means of thermal sprays and in radiators of drop liquid for airships [1]. Also variable-emissivity materials have received considerable attention due to their potential applications in thermal control devices of modern spacecraft, optical switching element and smart windows.

Knowledge of the temperature dependence of the surface emissivity, which controls the radiative problem, is essential for determining the thermal balance of these phenomena.

The surface emissivity in generally depends on surface temperature, wavelength, surface material geometry (curvature, roughness), direction of observation, and often changes with oxidation, melting, coating and even surface pollution. Also, emissivity can be influenced by the method of fabrication, thermal cycling, and chemical reaction with the environment. What is clear is that total emissivity depends mainly on temperature, which may

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be significant. Unfortunately, pervious works assumed emissivity as a constant coefficient. Using temperature dependent emissivity inserts some additional nonlinear terms and increases the power of the equation.

This scientific problem is modeled by an ordinary or partial differential equation. In most cases, analytical solutions cannot be applied to this problem, so this equation should be solved using special techniques. In recent years, much attention has been devoted to the newly developed methods to construct an analytic solution of this equation; such as homotopy perturbation method (HPM) and parameterized perturbation method (PPM) [2-9].

Therefore in the present work we examine the nonlinear heat equation with a variable physical property and attempt to obtain its solution using PPM.

Analysis of the parametrized perturbation method

The parameterized perturbation method was first proposed in 1999 in [10]. According to [10], an expanding parameter is introduced by a linear trans-formation:

$$\theta = \xi v + b, \tag{1}$$

where ξ is the perturbation parameter, by substituting eq. (1) into an original equation in order to have no secular term in the equation; we can obtain the un-known constant parameter b, then, the solution is expanded in the form:

$$v = \sum_{i=0}^{n} \xi^i v_i = v_0 + \xi v_1 + \xi^2 v_2 + \xi^3 v_3 + \ldots, \tag{2}$$

Here ξ is an artificial bookkeeping parameter. Unlike traditional perturbation methods, we keep $v_0(0) = v(0)$ and $\Sigma_{i=1} v_i(0) = 0$.

Application of PPM

By considering three different type of variation for emissivity versus temperature (no-variation, linear variation and second power variation), three unlike ODEs are solved as:

$$\frac{d(\xi v + b)}{d\tau} + (\xi v) + \varepsilon(\xi v)^4 = 0, \tag{3}$$

$$\frac{d(\xi v)}{d\tau} + \xi v + \varepsilon_a (\xi v)^4 + \varepsilon_a \beta (\xi v)^5 = 0 \tag{4}$$

$$\frac{d(\xi v)}{d\tau} + \xi v + \varepsilon_a (\xi v)^4 + \varepsilon_a \beta (\xi v)^5 + \varepsilon_a \alpha (\xi v)^6 = 0 \tag{5}$$

By substituting transformation equation into these equations, Inserting v terms, rearranging ξ-terms, summation of solutions and subsequently using inverse transformation, the temperature distribution for different conditions will be obtained.

All analytical expressions gained by PPM are in very good agreement with numerical results and can be used in many calculations related to industries. To show the effect of emissivity dependency on temperature, analytical results for variations of temperature deviations in correspond to the time are illustrated in fig. 1.

It is obvious that in reality when the surface emissivity is dependent on temperature variation, the value of temperature gradient, which is stronger in early seconds of the heat transfer, is dramatically less than the situation in which the emissivity is assumed to be constant. This difference is bigger for large amounts of initial emissivity. This issue may be

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seems negligible in industry while it is an important parameter for some specific applications like micro or nano scale devices or optical switching elements. Using current analytical expressions assist scientists in the field of radiative heat transfer to calculate an exact value of temperature even the emissivity is variable with temperature.

Conclusion

In this paper, the parameterized perturbation method (PPM) is applied to obtain the solution of heat transfer equation involved convection and radiation terms. Results are compared with solutions of numerical Runge–Kutta method and good agreements are obtained. PPM is very simple while you just employ a transformation equation and solve the original equation without the separation of linear and nonlinear terms.
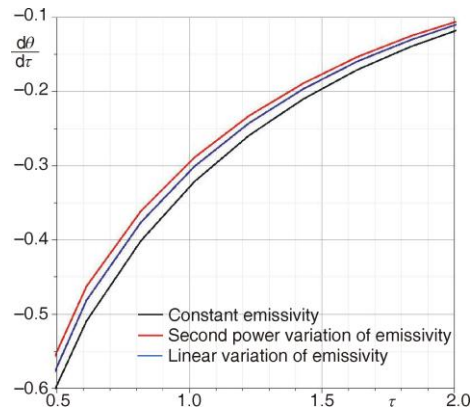

Figure 1. Variation of temperature deviation (temperature gradient) for
$\varepsilon = \varepsilon_a = 0.5, \alpha = \beta = 1$

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