DETERMINATION OF TEMPERATURE DISTRIBUTION FOR ANNULAR FINS WITH TEMPERATURE DEPENDENT THERMAL CONDUCTIVITY BY HPM

by

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In this paper, homotopy perturbation method has been used to evaluate the temperature distribution of annular fin with temperature-dependent thermal conductivity and to determine the temperature distribution within the fin. This method is useful and practical for solving the nonlinear heat transfer equation, which is associated with variable thermal conductivity condition. The homotopy perturbation method provides an approximate analytical solution in the form of an infinite power series. The annular fin heat transfer rate with temperature-dependent thermal conductivity has been obtained as a function of thermo-geometric fin parameter and the thermal conductivity parameter describing the variation of the thermal conductivity.

Key words: *homotopy perturbation method, numerical method, annular fin, thermal conductivity in heat transfer*

Introduction

Most scientific problems and phenomena require high performance heat transfer components with progressively smaller weights, volumes, costs. So, one of the most significant importance is the optimization of the design of fins for high performance, light weight and compact heat transfer components. Extended surfaces are widely utilized in various industrial applications. Kern *et al*. [1] have presented an extensive review on this topic. In addition, large amount of papers exist on the problem of convective fins. It has provided the optimum dimensions of straight fins, circular fins and spines of different profiles with several numerical examples by Aziz [2]. Constant thermal conductivity was considered in previous study.

Additionally, a considerable amount of research has been conducted into the variable thermal parameters. It has utilized the regular perturbation method and a numerical solution method to calculate a closed form solution for a straight convecting fin with temperature dependent thermal conductivity by Aziz *et al*. [3].

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There are few phenomena in different fields of science occurring linearly. Most scientific problems such as heat transfer are inherently nonlinear. We know that except a limited number of these problems, most of them do not have analytical solution. Therefore, these nonlinear equations should be solved using other methods. Some of them are solved using numerical techniques. In the numerical method, stability and convergence should be considered so as to avoid divergence or inappropriate results. In the analytical perturbation method, we should exert the small parameter in the equation. Therefore, finding the small parameter and exerting it into the equation are difficulties of this method. Since there are some limitations with the common perturbation method, and also because the basis of the common perturbation method is upon the existence of a small parameter, developing the method for different applications is very difficult.

Homotopy perturbation method (HPM) which was recently developed by Ji-Huan He [4, 5] is one of the most successful and efficient methods in solving nonlinear equations. In contrast to previously introduced analytic methods, HPM is independent of any small or large parameter. In the works of previous authors Ganji *et al.* [6, 7] and others [8, 9] have successfully applied HPM in solving different types of nonlinear problems i.e. coupled, decoupled, homogeneous and non-homogeneous equations arising in different physical problems such as heat transfer, fluid flow, oscillatory systems and *etc*. The aim of this paper is to give the analytic solution of the nonlinear equation of the annular fins with temperature dependent thermal conductivity and compare the HPM results with numerical results given by Arslanturk [10].

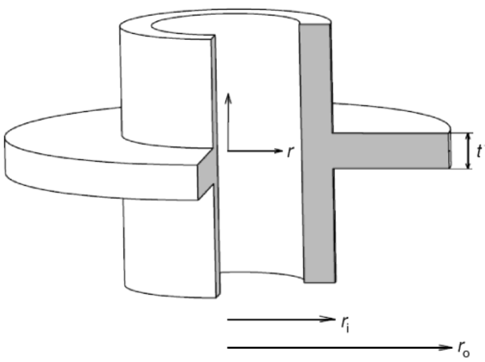

Figure 1. Geometry of a straight fin

Problem description

An annular fin with temperature-dependent thermal conductivity as shown in fig. 1 is considered in predicting the fin geometry. The fin of thickness t, base radius r_i, and tip radius r_o is exposed to a convective environment at the constant ambient temperature T_∞ and heat transfer coefficient h. The base temperature T_b of the fin is constant, and the fin tip insulated. Since the fin is assumed to be thin, the temperature distribution within the fin does not depend on axial direction. The one-dimensional energy balance equation is given:

$$t\frac{d}{\mathrm{d}r}\left[k(T)r\frac{\mathrm{d}T}{\mathrm{d}r}\right] = 2hr(T - T_\infty) \tag{1}$$

$$T = T_\mathrm{b} \quad \text{at} \quad r = r_\mathrm{i}, \quad \frac{\mathrm{d}T}{\mathrm{d}r} = 0 \;\; \text{at} \;\; r = r_o \tag{2}$$

The thermal conductivity of the fin material is assumed to be a linear function of temperature according to $k(T) = \mathrm{k}_\infty[1 + \kappa(T - T_\mathrm{b})]$ where k_b is the thermal conductivity at the ambient base temperature of the fin and λ is the parameter describing the variation of the thermal conductivity. Employing the following dimensionless parameters:

$$\theta = \frac{T - T_\infty}{T_\mathrm{b} - T_\infty},\ \mathrm{Bi} = \frac{hr_\mathrm{i}}{k_\infty},\ \beta = \kappa\ T_\mathrm{b} - T_\infty\ ,\ \xi = \frac{r - r_\mathrm{i}}{r_\mathrm{i}},\ \lambda = \frac{r_\mathrm{o}}{r_\mathrm{i}},\ \delta = \frac{t}{r_\mathrm{i}} \tag{3}$$

the formulation of the problem reduces to:

$$\theta'' + \beta\ \theta'\ ^2 + \beta\theta\theta'' + \frac{\beta}{(1+\xi)}\theta\theta' + \frac{1}{(1+\xi)}\theta' - \frac{2\mathrm{Bi}}{\delta}\theta = 0, \quad 0 < \xi < \lambda - 1 \tag{4}$$

$$\theta = 1 \quad \text{at} \quad \xi = 0 \tag{5}$$

$$\theta' = 0 \quad \text{at} \quad \xi = \lambda - 1. \tag{6}$$

Application of homotopy perturbation method to fin temperature distribution

We set up the homotopy perturbation method formulation [9] by rewriting eq. (4) as:

$$(1-p)L(\theta - \theta_0) + \\ + p\left(\theta'' + \beta\ \theta'\ ^2 + \beta\theta\theta'' + \frac{\beta}{(1+\xi)}\theta\theta' + \frac{1}{(1+\xi)}\theta' - \frac{2Bi}{\delta}\theta\right) = 0 \tag{7}$$

where p is the homotopy parameter, and L is the linear operator as $L \equiv \mathrm{d}^2/\mathrm{d}\xi^2$.We seek a perturbation solution for θ in the form of a power series in p as under:

$$\theta(\xi) = \theta_0(\xi) + p\theta_1(\xi) + p^2\theta_2(\xi) + \cdots = \sum_{i=0}^{\infty} p^i\theta_i(\xi) \tag{8}$$

Assuming that the series (20) converge for $p = 1$, the final solution for θ is given by:

$$\theta(\xi) = \sum_{i=0}^{\infty} \theta_i(\xi) \tag{9}$$

The HPM is an analytical method that has been used to solve effectively, easily, and accurately a large class of linear and nonlinear, ordinary or partial, deterministic or stochastic differential equations with approximate which converge rapidly to accurate solutions. This technique will be used to solve the evolution equations derived here as follows. We begin with initial approximations:

$$\theta_0(\xi) = 1 \tag{10}$$

Substituting for θ from eq. (8) into eq. (7), and equating like powers of p on both sides, we obtain, for $n \geq 1$:

$$L(\theta_n) = -\beta\sum_{m=0}^{n-1}\left(\theta'_m\theta'_{n-m-1} + \theta_m\theta''_{n-m-1} + \frac{1}{1+\xi}\theta_m\theta'_{m-n-1}\right) - \frac{1}{1+\xi}\theta'_{n-1} + \frac{2\mathrm{Bi}}{\delta}\theta_{n-1} \tag{11}$$

with boundary conditions:

$$\theta_n\ \ 0\ = 0, \quad \theta_n\ \ \lambda - 1\ = 0 \tag{12}$$

As with the initial approximation, eq. (12) must be first solved for θ_n. Below we present first few terms of the expansions (13):

$$\theta_1(\xi) = \frac{\mathrm{Bi}}{\delta}\xi^2 - \frac{2\mathrm{Bi}(\lambda - 1)}{\delta}\xi, \tag{13}$$

$$\theta_2(\eta) = \frac{\mathrm{Bi}^2}{6\delta^2}\xi^4 + \frac{2\mathrm{Bi}^2(1-\lambda)}{3\delta^2}\xi^3 - \frac{\mathrm{Bi}\delta(1+2\beta)}{\delta^2}\xi^2 + \cdots \tag{14}$$

According to eq. (9) the assumption $p \to 1$, we get:

$$\theta(\xi) = \theta_0(\xi) + \theta_1(\xi) + \theta_2(\xi) + \cdots \tag{15}$$

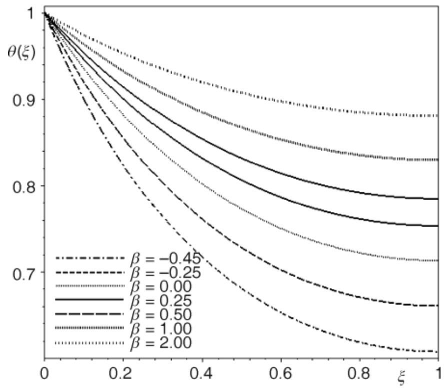

Figure 2. Temperature distribution by HAM for various β when λ = 2, δ = 1/3, and Bi -= 0.1

Conclusion

In this study, temperature distribution of annular fin with temperature-dependent thermal conductivity was analyzed using HPM. When compared with other numerical methods, it is clear that HPM provides highly accurate analytic solutions for nonlinear problems. Figures 2 and 3 show $\theta(\xi)$ that are obtained by using homotopy perturbation method for various values of β and δ when λ = 2. Finally, as shown in fig. 4, it has been attempted to show the accuracy, capabilities and wide-range applications of the homotopy perturbation method in comparison with the numerical solution of nonlinear temperature distribution of annular fin with temperature-dependent thermal conductivity.

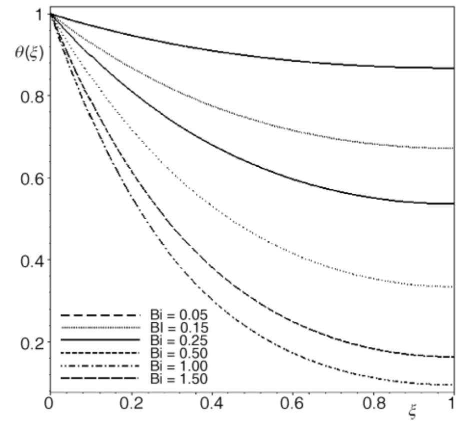

Figure 3. Temperature distribution by HAM for various Bi when λ = 2, δ = 1.3 and β = 0.3

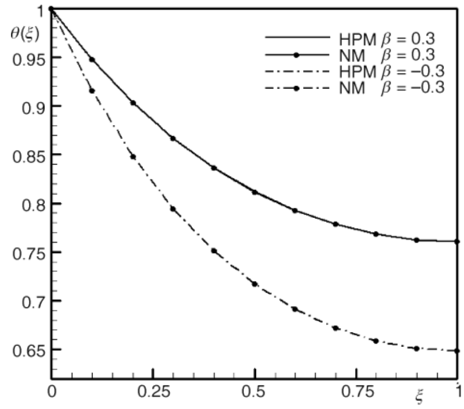

Figure 4. The comparison between HPM and numerical solution for $\theta(\xi)$ for λ = 2 δ = 1/3 and Bi = 0.1

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