

## EFFECT OF VARIABLE VISCOSITY AND SUCTION/INJECTION ON THERMAL BOUNDARY LAYER OF A NON-NEWTONIAN POWER-LAW FLUIDS PAST A POWER-LAW STRETCHED SURFACE

by

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*The analysis of laminar boundary layer flow and heat transfer of non-Newtonian fluids over a continuous stretched surface with suction or injection has been presented. The velocity and temperature of the sheet were assumed to vary in a power-law form, that is  $u = U_0 x^m$ , and  $T_w(x) = T_\infty + Cx^b$ . The viscosity of the fluid is assumed to be inverse linear function of temperature. The resulting governing boundary-layer equations are highly non-linear and coupled form of partial differential equations and they have been solved numerically by using the Runge-Kutta method and Shooting technique. Velocity and temperature distributions as well as the Nusselt number were studied for two thermal boundary conditions: uniform surface temperature ( $b = 0$ ) and cooled surface temperature ( $b = -1$ ), for different parameters: variable viscosity parameter  $\theta_r$ , temperature exponent  $b$ , blowing parameter  $d$  and Prandtl number. The obtained results show that the flow and heat transfer characteristics are significantly influenced by these parameters.*

*Key words: variable viscosity, heat transfer, non-Newtonian power-law fluids, stretching sheet*

### Introduction

In recent time, the study of two-dimensional boundary layer flow over a stretching sheet has been studied with increasing interests for its industrial and engineering applications. For example, drawing of a polymer sheet or filaments extruded continuously from a die through quiescent fluids, the cooling of a metallic plate in a cooling path, the aerodynamic extrusion of plastic sheets, crystal growing, the boundary layer along a liquid film in condensation process and the continuous casting are the practical applications of moving surfaces and also the materials manufactured by extrusion processes and heat treated materials traveling between a feed roll and wind up roll or on a conveyer belt possesses the characteristics of a moving continuous surfaces.

There are various applications in which significant temperature differences between the body surface and the ambient fluid exist. It is usually assumed that the sheet is inextensible, but in some different situations like in polymer industry in which it is necessary to deal with a

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stretching plastic sheet as mentioned by Crane [1]. He assumed the velocity of the sheet to vary linearly as the distance from the slit and the finally got the analytical solution. Many authors have extended the work of Crane [1] to study heat and mass transfer under different physical situation (see for instance [2-5]). Seddeek *et al.* [6] studied the effects of radiation and thermal diffusivity on heat transfer over a stretching surface with a variable heat flux.

Thermal transport from a heated moving surface to a quiescent non-Newtonian fluid is of interest in many practical industrially important processes such as multiphase mixtures, food products, biological fluids, natural products, and agricultural and dairy wastes. The interest in studying flow and heat transfer characteristics of non-Newtonian fluids has increased in the last four decades because of their important usage and wide range of applications. Considerable efforts have been directed at these characteristics to control the quality of the final product of these processes because of the growing use of these fluids in various manufacturing and processing industries such as hot rolling, wire drawing, continuous casting, glass fiber production, and paper production (Chabra [7], Altan *et al.* [8], Fisherr [9]). Its worth mentioning here that many of the inelastic non-Newtonian fluids encountered in chemical engineering processes, are known to follow the empirical Ostwald-de Waele model or the so-called "power-law model" in which the shear stress varies according to a power function of the strain rate. Hassanien *et al.* [10] studied the flow and heat transfer in a power-law fluid over a non-isothermal stretching sheet. Abo-Eldahab [11] studied the MHD free-convection flow of a non-Newtonian power-law fluid at stretching surface with uniform free-stream. Seddeek [12] examined the problem of power-law non-Newtonian fluid over a continuous stretched surface with thermal radiation.

In all of the above-mentioned studies the viscosity of the fluid was assumed to be constant. However, it is known that this physical property may change significantly with temperature. When the effect of viscosity is included in the analysis, Gary *et al.* [13] and Mehat *et al.* [14] have found that the flow characteristics substantially are changed compared with constant viscosity cases. Ling *et al.* [15] presented a very interesting theoretical investigation of temperature-dependent fluid viscosity which effect on the forced convection through a semi-infinite porous medium bounded by an isothermal flat plate. Hydrodynamic flows of a viscous and incompressible fluid have been studied under different physical conditions with variable fluid properties by Seddeek *et al.* [16] and Gahely *et al.* [17]. Also, Salem [18] studied the influence of thermal conductivity and variable viscosity on the flow of a micropolar fluid past a continuously semi-infinite moving plate with suction or injection. Ali [19] studied the similarity solutions of the laminar boundary layer equations describing heat and flow in a quiescent fluid driven by a stretched surface subject to suction or injection. Recently, Eldabe *et al.* [20] investigated the effect of variable viscosity on mixed convective heat and mass transfer in a non-Newtonian fluid at a peristaltic surface.

No attempt has been made to investigate non-linear boundary layer flow and heat transfer of non-Newtonian power-law fluid with variable viscosity over a continuous stretched surface with suction or injection at the wall in the presence of two thermal boundary conditions of uniform surface temperature ( $b = 0$ ) and cooled surface temperature ( $b = -1$ ). So we are going to investigate it in this article. In this work the two-dimensional continuity, momentum and energy equations have been reduced to a system of non-linear ordinary differential equations, which are solved numerically by using the fourth order Runge-Kutta method and shooting technique. The influence of the temperature-dependent fluid viscosity parameter and suction/injection parameter on the velocity and temperature distributions as well as the heat transfer is investigated and analyzed with the help of their graphical representations.

### Mathematical formulation

Let us consider a steady, two-dimensional flow on a power-law stretched surface with suction or injection moving through a quiescent ambient non-Newtonian fluid of ambient temperature  $T_\infty$ . The x-axis runs along the continuous surface in the direction of motion and y-axis is perpendicular to it (fig.1), where the temperature of the moving surface is given by  $T_w(x) = T_\infty + Cx^b$ .

The non-Newtonian fluid model used in this study is power-law model (Ostwald-de Waele rheological model) with the parameters defined by Bird *et al.* [21]:

$$\tau = \mu \left| \frac{\Delta \Delta}{2} \right|^{n-1} \Delta \quad (1)$$

where  $\tau$  is the stress tensor,  $\Delta$  – the symmetrical rate of deformation tensor,  $\mu$  – the consistency coefficient, and  $n$  – the power-law index. The two-parameter rheological eq. (1) is known as Ostwald-de Waele model, or more commonly, the power-law model. When  $n = 1$ , eq. (1) represents a Newtonian fluid with dynamic coefficient of viscosity. If  $n < 1$ , the fluid is said to be “pseudo-plastic” and if  $n > 1$ , the fluid is called “dilatant”. In order to develop the expression of  $\tau_{xy}$ , the shear stress component of stress tensor reduce to (see [21]) the form:

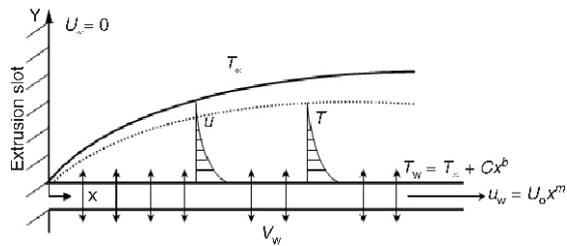


Figure 1. Co-ordinate and flow model

$$\tau_{xy} = \mu \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \quad (2)$$

Using this relationship, the steady two-dimensional boundary-layer equations for non-Newtonian fluid have the form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right) = 0 \quad (4)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} - \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial y^2} = 0 \quad (5)$$

where  $u$  is the velocity component in the x-direction,  $v$  – the velocity component in the y-direction,  $\mu$  – the dynamic viscosity,  $\rho$  – the density,  $c_p$  – the specific heat at constant pressure,  $T$  – the temperature inside the boundary layer, and  $k$  – the thermal conductivity.

The boundary condition associated with eqs. (3), (4), and (5) are:

$$\begin{aligned} u &= U_0 x^m, \quad v = v_w(x), \quad T = T_\infty + Cx^b \quad \text{at } y = 0 \\ u &= 0, \quad T = T_\infty \quad \text{at } y = \infty \end{aligned} \quad (6)$$

where  $U_0$  is the dimensional constant,  $v_w(x)$  – the suction velocity,  $C$  – the constant, and  $b$  – the temperature exponent parameter. Also, two thermal boundary conditions of uniform temperature ( $b = 0$ ) and cooled surface temperature ( $b = -1$ ) are considered. It should be noted that posi-

tive and negative  $m$  indicates that the surface is accelerated or decelerated from the extruded slit, respectively.

Here, it is assumed that the fluid properties are isotropic and constant, except for fluid viscosity  $\mu$ , which is assumed to vary as inverse linear function of temperature given by [22]:

$$\frac{1}{\mu} = \frac{1}{\mu_\infty} [1 - \gamma(T - T_\infty)] \quad (7)$$

or

$$\frac{1}{\mu} = a(T - T_r) \quad (8)$$

where

$$a = \frac{\gamma}{\mu_\infty} \text{ and } T_r = T_\infty - \frac{1}{\gamma} \quad (9)$$

Both  $a$  and  $T_r$  are constants, and their values depend on the reference state and the thermal property of the fluid, *i. e.*  $\gamma$ . In general,  $a > 0$  for liquids and  $a < 0$  for gases.

Now we define the following dimensionless variables:

$$\eta(x, y) = \frac{y}{x}, \quad \psi = U_0 x^{m+1} f(\eta) \quad (10)$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} = \frac{T - T_\infty}{C x^b}$$

For similarity solution  $u = \frac{\partial \psi}{\partial x}$ , and  $v = \frac{\partial \psi}{\partial y}$  (11)

we get

$$u = U_0 x^m f'(\eta), \quad v = U_0 x^m [\eta f'(\eta) - (m+1)f(\eta)] \quad (12)$$

With the help of eq. (12), the eqs. (4)-(6) reduces to:

$$f'''' + [(f')^{n-1} - (n-1)f'(f')^{n-2}] \text{Re} \frac{\theta_r}{\theta} [(q-r)f''^2 - rff'] \quad (13)$$

$$+ (f')^{n-1} f' \frac{\theta}{\theta - \theta_r}$$

$$- \theta \text{Pr} (b\theta f' - r\theta) \quad (14)$$

with the boundary conditions

$$f(0) = 1, \quad f'(0) = \frac{d}{r}, \quad f(\eta_\infty) = 0 \quad (15)$$

$$\theta(\eta_\infty) = 0, \quad \theta(0) = 1$$

where  $q = -1$ ,  $r = m + 1$ , and  $m = n/(n-2)$ .

In the equations, the primes denote the differentiation with respect to the similarity variable  $\eta$ ,  $\theta_r = (T_r - T_\infty)/(T_w - T_\infty) = -1/\gamma(T_w - T_\infty)$  is the dimensionless variable viscosity parameter,  $\text{Re} = \rho(U_0 x^m)^{2-n} x^n / \mu_\infty$  – the generalized Reynolds number,  $d = V_w x^{-m} / U_0$  – the suction blowing parameter such that  $d > 0$  indicates wall injection and  $d < 0$  indicates wall suction, and  $\text{Pr}$  is the Prandtl number ( $\rho C_p / k$ ).

From the definition of  $\theta_r$  in eq. (7), it is noted that as  $\theta_r \rightarrow \infty$  means  $\mu = \mu_\infty = \text{constant}$  and  $\theta_r \rightarrow 0$ . For the case of fluid-heating,  $T_w > T_\infty$ , Elbashbeshy *et al.* [23] showed that  $\theta_r$  cannot take the value between zero and one and suggested that  $\theta_r > 1$  for gases and  $\theta_r < 0$  for liquids.

The important physical quantity our interest is the Nusselt number Nu, which take the form:

$$\text{Nu} = \frac{x \left. \frac{\partial T}{\partial y} \right|_{y=0}}{T_w - T_\infty} = \theta_r(0) \quad (16)$$

### Numerical procedure

The eqs. (13) and (14) along with the boundary conditions (15) constitute a one-parameter two-point boundary value problem. The ordinary differential equations were written as a system of five coupled first-order equations in terms of five dependent variables  $z_n$  ( $n = 1, 2, \dots, 5$ ) where  $z_1 = f$ ,  $z_2 = f'$ ,  $z_3 = f''$ ,  $z_4 = \theta$ , and  $z_5 = \theta'$ . Thus

$$\begin{aligned} z_1' &= z_2, & z_1(0) &= \frac{d}{r} \\ z_2' &= z_3, & z_2(0) &= 1 \\ z_3' &= \text{Re}[|z_3|^{n-1} (n-1)z_3|z_3|^{n-2}] + \frac{\theta_r}{\theta_r} \frac{z_4}{\theta_r} [(q-r)z_2^2 - z_3z_1] \\ & & & - |z_3|^{n-1} z_3 \frac{z_5}{z_4 \theta_r}, & z_3(0) &= \alpha_1 \\ z_4' &= z_5, & z_4(0) &= 1 \\ z_5' &= p_r [bz_4z_2 - z_1z_5r], & z_5(0) &= \alpha_2 \end{aligned}$$

where  $\alpha_1$  and  $\alpha_2$  are determined such that  $z_2(\infty) = 0$  and  $z_4(\infty) = 0$ . The essence of this method is reduce the boundary value problem to an initial value problem and then use a shooting numerical technique [24, 25] to guess  $\alpha_1$  and  $\alpha_2$  until the boundary condition  $z_2(\infty) = 0$  and  $z_4(\infty) = 0$  are satisfied. The resulting differential equations are then easily integrated using fourth-order classical Runge-Kutta method.

### Results and discussion

To study the behavior of the dimensionless velocity function  $f$ ,  $\eta$  viscosity function  $\mu/\mu_\infty$ , and temperature profiles  $\theta$ ,  $\eta$ , the curves are drawn for various values of the parameters that describe the flow. Figures 2 and 3 represent the dimensionless velocity  $f$  for several values of  $\theta$  with  $n = 0.2$  (pseudo-plastic fluid) in two cases: suction ( $d = -1$ ) and injection ( $d = 1$ ) when ambient Prandtl number  $\text{Pr} = 10$ . In each case, increasing the temperature-dependent viscosity parameter  $\theta_r$  will decrease the velocity near the plate surface and increase it as  $\eta \rightarrow \eta_\infty$ . This can be explained physically when the parameter  $\theta_r$  increases, the fluid viscosity decreases. Thus the boundary layer thickness is incremented. Also, these figures shows that the hydrodynamic boundary layer thickness increases for injection and decrease for suction. Further, it is found from these figures that the effects of variable viscosity parameter are more pronounced for cooled surface temperature ( $b = -1$ ) than the case of uniform surface temperature ( $b = 0$ ). This is

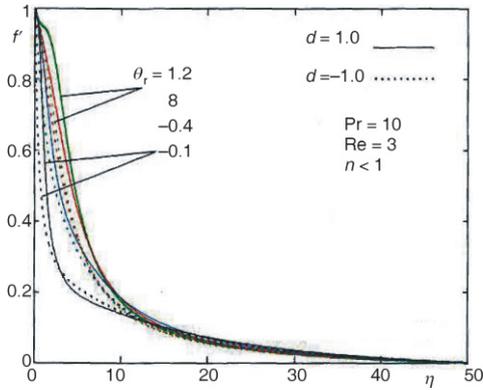


Figure 2. Dimensionless velocity profiles for different values of  $\theta_r$  for  $b = -1$

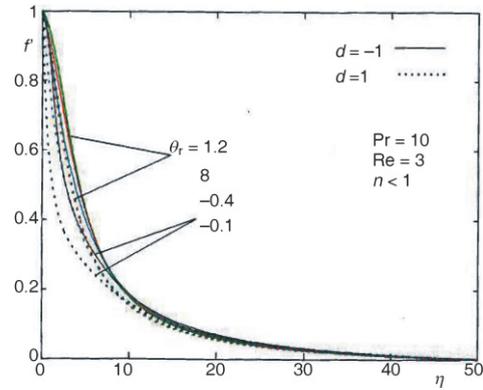


Figure 3. Dimensionless velocity profiles for different values of  $\theta_r$  for  $b = 0.0$

because heat is transferred to the moving surface for  $b < 0$ . On the other hand, from figs. 4 and 5 we found that the increase in the viscosity-variation parameter  $\theta_r$  leads to decrease in the viscosity near the surface of the plate and this approaches to unity value at the outer-edge of the boundary layer for every values of the viscosity-variation parameter considered here. This outcome is consistent with the result for velocity profiles presented in figs. 2 and 3.

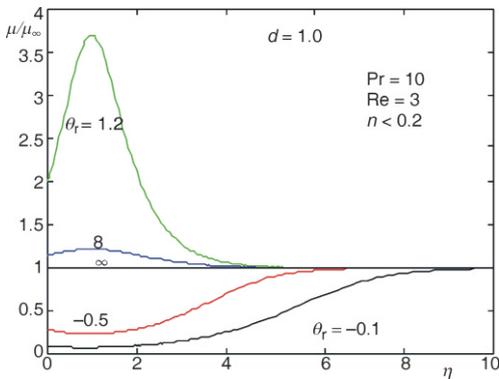


Figure 4. The viscosity profiles for different values of  $\theta_r$  for  $b = -1$

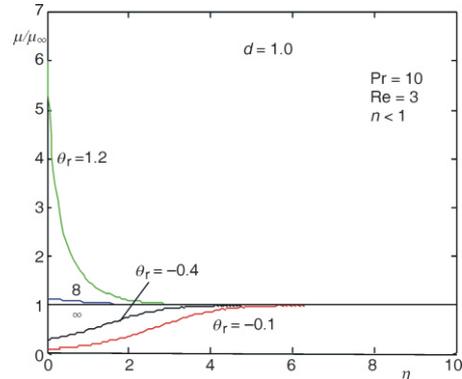


Figure 5. The viscosity profiles for different values of  $\theta_r$  for  $b = 0.0$

The effect of the increase of viscosity parameter on the temperature profiles with  $n = 0.2$  for uniform surface temperature ( $b = 0$ ) when ambient prandtl number  $Pr = 10$  is shown in fig. 6. The corresponding temperature profiles for cooled surface temperature ( $b = -1$ ) are depicted in fig. 7. From fig. 6 it is observed that the increase of the temperature dependent fluid viscosity parameter increases the thermal boundary layer thickness, which leads to increase of temperature profile for both cases of constant fluid suction or injection. It is observed from fig. 7 that for  $b = -1$ , the temperature profile exhibits overshoot within the boundary layer before reaching to zero value at the edge of the boundary layer. This implies that the temperature of the fluid near the wall is greater than that at the wall. Also, fig. 7 shows that the temperature profiles is found to decrease as  $\theta_r$  increases closed to the wall. But far away from the wall these profiles increase with the increase

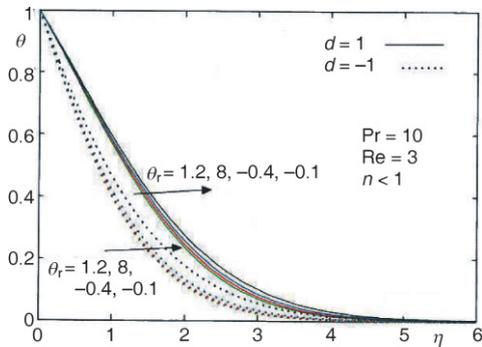


Figure 6. Dimensionless temperature profiles for different values of  $\theta_r$  for  $b = 0.0$

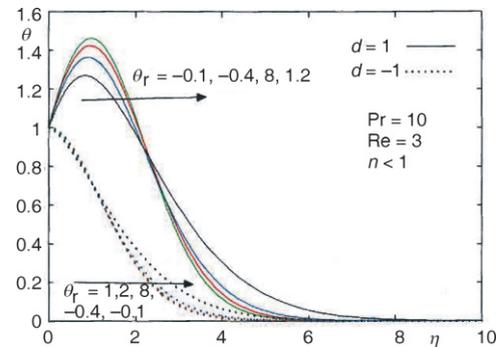


Figure 7. Dimensionless temperature profiles for different values of  $\theta_r$  for  $b = -1$

of  $\theta_r$ . A comparison between two cases ( $d = 1$  and  $d = -1$ ) reveals that for the same values of  $\theta_r$ , the thermal boundary-layer thickness is larger for injection than for suction.

Figure 8 represents the results for the heat transfer rate plotted as the Nusselt number  $Nu = hx/k$  vs. the blowing parameter  $d$  for cooled surface temperature  $b = -1$  and for several values of  $\theta_r$  and  $Pr$ . For given value of  $n$  the heat transfer rate is decreased with increase in value of blowing parameter  $d$ . Also, increasing the Prandtl number will increase adverse heat transfer for the injection case  $d > 0$  and increase heat transfer for the suction case  $d < 0$ . On other hand, negative value of  $Nu$  indicates that heat is transferred to the surface rather than from it as for positive value of  $Nu$  (which can also be understood by the corresponding overshoot in temperature profiles exhibited in fig. 7). Also, it is clear from this figure that heat transfer coefficient increases when the variable viscosity parameter increases for the case  $d > 0$ , however this increase for  $Pr = 7.0$  is greater than for  $Pr = 0.7$  as the boundary layer is thinner for  $Pr = 7.0$  than for  $Pr = 0.7$ . Thus, fluids which have a larger Prandtl number are more sensitive to the variation of viscosity parameter than fluids with a smaller Prandtl number.

Figure 9 demonstrates the local Nusselt number  $Nu$  as a function of blowing parameter  $d$  for uniform surface temperature  $b = 0$  and for various values of variable viscosity parameter  $\theta_r$ . For given values of  $n$  and  $Pr$ , the heat transfer rate is decreased with the increase of the value

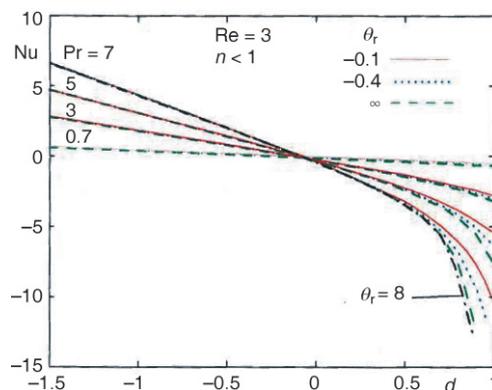


Figure 8. Values of Nusselt number for different values of  $Pr$  and  $\theta_r$  for  $b = -1$

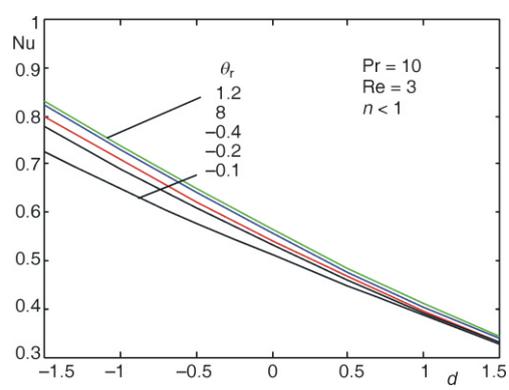


Figure 9. Values of Nusselt number for different values of  $\theta_r$  for  $b = 0.0$

of  $d$ . Also, increasing the viscosity parameter leads to an decrease in Nusselt number. Also, we found that the results for Nusselt number depicted in fig. 8 are in agreement with the results for the temperature profiles shown in fig. 5, as the increase of  $\theta_r$  gives rise to an increase in surface temperature, and thus reduces the rate of heat transfer. Furthermore, it is clear that suction  $d < 0$  enhances heat transfer coefficient much better than blowing  $d > 0$ , and the thickness of the thermal boundary layer is reduced. Thus suction can be used for cooling the surface much faster than blowing.

## Conclusions

The present work deals with the hydrodynamic flow of heat transfer of a non-Newtonian power-law fluid past a stretching surface, taking into consideration the effects suction or injection at the surface. The fluid viscosity is assumed to vary as an inverse linear function of temperature. Numerical solution was obtained for the velocity, temperature, and heat transfer coefficients for two thermal boundary conditions, uniform and cooled surface temperature. It was found that the velocity decreases and the corresponding dynamic viscosity of the fluid decreases near the surface owing to an increase of viscosity variation parameter  $\theta_r$ . In addition, the effect of the viscosity variation parameter  $\theta_r$  on the dimensionless dynamic viscosity in the case of cooled surface temperature ( $b = -1$ ) is stronger than its effect in the case of uniform surface temperature ( $b = 0$ ) and the dynamic viscosity approaches unity at the outer edge of the boundary layer for every values of considered here. Also, for cooled surface temperature ( $b = -1$ ) heat flows to or from the surface depending on  $d$  and  $Pr$ , and for all  $d > 0$ , effects of variable viscosity parameter  $\theta_r$  on the heat transfer coefficient are found to be more significant for fluids which have larger Prandtl number than fluids with smaller Prandtl number. Furthermore, in the imposition of suction, the Nusselt number is increased with the increasing of Prandtl number whereas injection shows the opposite effect. Finally, it is clear that the suction  $d < 0$  enhances the heat transfer coefficient much better than blowing  $d > 0$ , and the thickness of the thermal boundary layer is reduced. Thus suction with variable viscosity can be used for cooling the surface faster than blowing. Therefore, the results obtained here to explore the basic thermal behavior of a continuous stretching surface might be useful in achieving the design for relevant manufacturing processes.

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## Nomenclature

$b$	– temperature exponent parameter	$n$	– fluid power-law index
$C$	– dimensional constant, [ $\text{Km}^{-b}$ ]	$T$	– temperature, [K]
$c_p$	– specific heat, [ $\text{Jkg}^{-1}\text{K}^{-1}$ ]	$U_0$	– dimensional constant [ $\text{m}^{1-p}\text{s}^{-1}$ ]
$d$	– dimensionless injection parameter, [ $v_w/U_0^{1-p}$ ]	$V_w$	– uniform surface mass flux, [ $\text{kgm}^{-1}$ ]
$f$	– dimensionless stream function	$u, v$	– velocity components along x- and y-axes, respectively, [ $\text{ms}^{-1}$ ]
$h$	– heat transfer coefficient, [ $\text{Wm}^{-2}\text{K}^{-1}$ ]	$x, y$	– Cartesian co-ordinates along x- and y-axes, respectively, [m]
$k$	– thermal conductivity, [ $\text{Wm}^{-1}\text{K}^{-1}$ ]		

### Greek symbols

- $\alpha$  – thermal diffusivity, [ $\text{m}^2\text{s}^{-1}$ ]  
 $\eta$  – dimensionless similarity variable ( $= y/x$ )  
 $\theta$  – dimensionless temperature,  $(T - T_\infty)/(T_w - T_\infty)$ ;  
 $(T - T_\infty)/(Cx^b)$   
 $\mu$  – dynamic viscosity, [ $\text{kgm}^{-1}\text{s}^{-1}$ ]  
 $\rho$  – density, [ $\text{kgm}^{-3}$ ]

### Subscripts

- w – condition at the surface  
 $\infty$  – condition at ambient medium

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