ANALYTICAL SOLUTION OF STAGNATION FLOW OF A MICROPOLAR FLUID TOWARDS A VERTICAL PERMEABLE SURFACE

by

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In this paper, stagnation flow of a micropolar fluid towards a vertical permeable surface with two cases, Newtonian fluid (K = 0) and non-Newtonian fluid (K = 1) are studied in presence of suction and injection. The transformed non-linear equations are solved analytically by homotopy analysis method and some results are compared with numerical solutions for validity. Analytical results for the velocity profiles, the temperature profiles, the skin friction coefficient and the local Nusselt number are presented for various values of the flow parameters and also these results demonstrate obvious effect of suction and injection on temperature profiles on investigation of such flows, particularly for non-Newtonian fluid.

Key words: *micropolar fluid, non-Newtonian fluid, stagnation flow, homotopy analysis method*

Introduction

The theory of micropolar fluids which was originally formulated by Eringen [1] can be used to explain the flow of crystals, animal blood, paints, polymers, *etc*. The theory introduces new material parameters, an additional independent vector field – the microrotation – and new constitutive equations which must be solved simultaneously with the usual equations for Newtonian flow. Ramachandran *et al*. [2] studied laminar mixed convection in two-dimensional stagnation flows around surfaces. He considered both cases of an arbitrary wall temperature and arbitrary surface heat flux variations and found that a reversed flow developed in the buoyancy opposing flow region, and dual solutions are found to certain range of the buoyancy parameter. Hassanien *et al*. [3] extended Ramachandran's work to micropolar fluid. They considered both assisting and opposing flows, but the existence of dual solutions was not reported [4]. Devi *et al.* extended the problem posed by Ramachandran *et al*. [2] to the unsteady case, and they found that dual solution exist for a certain range of the buoyancy parameter when the flow is opposing. Similar problem for steady and unsteady cases, for a vertical surface immersed in a micropolar

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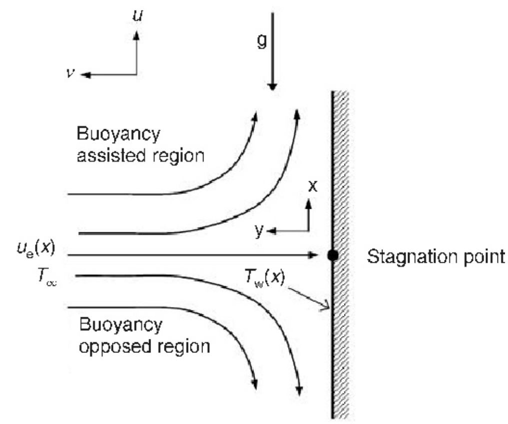

Figure 1. Physical model and co-ordinate system

fluid was investigated by Lok *et al.* [5, 6]. Existence of dual solutions was reported in [5] only for the opposing flow regime. The present study will show that dual solutions exist in the opposing flow regime and they continue into that of the assisting flow regime, *i. e.* when the buoyancy force acts in the same direction as the inertia force. Sketch of the problem is depicted in fig. 1. Recently, much effort put on constructing an analytic solution of these equations. One of these techniques is homotopy analysis method (HAM), which was introduced by Liao [13-18]. This method has been successfully applied to solve many types of non-linear problems [19-21].

Problem formulation

Consider a laminar two-dimensional stagnation flow of an incompressible micropolar impinges normal to a vertical heated plate. It is assumed that the free stream velocity U and the temperature of the plate $T_w(x)$ vary linearly with the distance x from the stagnation point, *i. e.* $U = ax$ and $T_w(x) = T_\infty + bx$, where a and b are positive constants. Under these assumptions along with the Boussinesq approximation, the steady laminar boundary layer equations governing the flow are as follows:

$$\frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} \quad v\frac{\partial u}{\partial y} \quad U\frac{\mathrm{d}U}{\mathrm{d}x} \quad \frac{\mu \quad k}{\rho}\frac{\partial^2 u}{\partial y^2} \quad \frac{k}{\rho} \quad \mathrm{g}\beta(T \quad T_\infty) \tag{2}$$

$$\rho j \quad u\frac{\partial N}{\partial x} \quad v\frac{\partial N}{\partial y} \quad \gamma\frac{\partial^2 N}{\partial y} \quad k \;\, 2N \quad \frac{\partial u}{\partial y} \tag{3}$$

$$u\frac{\partial T}{\partial x} \quad v\frac{\partial T}{\partial y} \quad \alpha\frac{\partial^2 T}{\partial y^2} \tag{4}$$

subject to the boundary conditions:

$$u \quad 0, \quad v \quad V_w, \quad N \quad \frac{1}{2}\frac{\partial u}{\partial y}, \quad T \quad T_w(x) \quad \text{at} \quad y \quad 0 \tag{5}$$

$$u \quad U(x), \quad N \quad 0, \quad T \quad T_\infty \quad \text{as} \quad y \quad \infty \tag{6}$$

where u and v are the velocity components along the x- and y-axes, respectively, T is the fluid temperature, N – the component of the microrotation vector normal to the x-y plane, ρ – the density, j – the micro-inertia density, m – the dynamic viscosity, k – the gyro-viscosity (or vortex viscosity), γ – the spin-gradient viscosity, and V_w – the uniform surface mass flux. The last term on the right-hand side of eq. (2) represents the influence of the thermal buoyancy force on the flow field, with "+" and "–" signs pertaining, respectively, to the buoyancy assisting and the buoyancy opposing flow regions. For the assisting flow, the x-axis points vertically upwards, while it points vertically downwards for the opposing flow. We assume that, $\gamma = (\mu + k/2)j = \mu(1 + K/2)j$, where $K = k/\mu$ is the material parameter. This assumption is invoked to allow the field of equations to predict the correct behavior in the limiting case when the microstructure effects become negligible and the total spin reduces to the angular velocity (see Ahmadi [7], Kline [8], or Gorla [9]). This assumption has also been used by the present authors to study different problems in micropolar fluids [10-12].

To seek similarity solutions for eqs. (1)-(4) subject to the boundary conditions (5), we introduce the following dimensionless similarity variables:

$$\eta \quad y\sqrt{\frac{U}{\nu x}}, \quad f(\eta) \quad \frac{\psi}{\sqrt{\nu x U}} \tag{7}$$

$$C(\eta) \quad \sqrt{\frac{\nu x}{U^3}}N, \quad \theta(\eta) \quad \frac{T \quad T_\infty}{T_\mathrm{w} \quad T_\infty} \tag{8}$$

where η is the independent similarity variable, $f(\eta)$ – the dimensionless stream function, $C(\eta)$ – the dimensionless microrotation, $\theta(\eta)$ – the dimensionless temperature, and ν – the kinematic viscosity of the fluid. Further, ψ is the stream function which is defined in the usual way as $u = \quad \psi/ \quad y$ and $v = - \quad \psi/ \quad x$ so as to identically satisfy eq. (1), using eq. (6), we get:

$$u \quad Uf\,(\eta), \quad v \quad \sqrt{\nu a}\,f(\eta) \tag{9}$$

where prime denotes differentiation with respect to η. Using eqs. (6) and (7), eqs. (2)-(4) reduce to the following ordinary differential equations or similarity equations:

$$(1 \quad K)f \quad ff \quad 1 \quad f^{\,2} \quad KC \quad \lambda\theta \quad 0 \tag{10}$$

$$1 \quad \frac{K}{2}\ C \quad fC \quad f\,C \quad K(2C \quad f\ ) \quad 0 \tag{11}$$

$$\frac{1}{\mathrm{Pr}}\theta \quad f\theta \quad f\theta \quad 0 \tag{12}$$

The boundary conditions (5) now become:

$$f(0) \quad f_\mathrm{w}, \quad f\,(0) \quad 0, \quad C(0) \quad \frac{1}{2}f\,(0), \quad \theta(0) \quad 1 \tag{13}$$

$$f\,(\eta) \quad 1, \quad C(\eta) \quad 0, \quad \theta(\eta) \quad 0, \quad \text{as} \quad \eta \quad \infty \tag{14}$$

where Pr is the Prandtl number and $f_\mathrm{w} = f(0) = -V_\mathrm{w}/(\nu a)^{1/2}$ is a constant (suction/injection parameter) with $f_\mathrm{w} > 0$ corresponds to mass suction, $f_\mathrm{w} < 0$ corresponds to mass injection, and $f_\mathrm{w} = 0$ is for an impermeable plate. Further, λ (= constant) is the mixed convection or buoyancy parameter which is defined as $\lambda = \quad \mathrm{Gr_x/Re_x^2}$, where $\mathrm{Gr_x} = g\beta(T_\mathrm{w} - T_\infty)x^3/\nu^2$ is the local Grashof number, $\mathrm{Re_x}$ – the local Reynolds number, and the " " sign has the same meaning as in eq. (2). We notice that when $\lambda = 0$, eqs. (8) and (10) are decoupled and a purely forced convection situation results. In this case, the flow field is not affected by the thermal field. The sign of λ characterizes the nature of the departure from this situation. For $\lambda = \mathrm{Gr_x/Re_x^2} > 0$, buoyancy forces act in the direction of the mainstream and fluid is accelerated in the manner of a favorable pressure gradient (assisting flow). When $\lambda = -\mathrm{Gr}_x/\mathrm{Re}_x^2 < 0$, buoyancy forces oppose the motion, retarding the fluid in the boundary layer, acting as an adverse pressure gradient (opposing low).We also notice that when $K = 0$ (Newtonian fluid), eqs. (8) and (10) reduce to those of Ramachandran *et al.* [2], when $f_w = 0$ (impermeable plate), this problem reduces to those considered by Hassanien *et al*. [3] or Lok *et al*. [5].

Homotopy analysis solution

In this section, HAM is employed to solve eqs. (10)-(12) subject to boundary conditions (13) and (14). We choose the initial guesses and auxiliary linear operators in the following form:

$$f_0(\eta) = f_w - 1 + x - e^{-\eta}$$

$$\theta_0(\eta) = e^{-\eta} \tag{15}$$

$$C_0(\eta) \quad \frac{1}{2} e \quad \eta$$

As the initial guess approximation for $f(\eta)$, $\theta(\eta)$, and $C(\eta)$:

$$L_1(f) \quad f \quad f, \quad L_2(\theta) \quad \theta \quad \theta, \quad L_3(C) \quad C \quad C \tag{16}$$

As the auxiliary linear operator which has the property:

$$L(c_1 + c_2\eta + c_3 e^{-\eta}) = 0, \quad L(c_4 + c_5 e^{-\eta}) = 0, \quad L(c_6 + c_7 e^{-\eta}) = 0 \tag{17}$$

and $c_i(i = 1\text{-}7)$ are constants. Let p [0, 1] denotes the embedding parameter and $\hbar$ indicates non-zero auxiliary parameters. Then, we construct the following equations.

Zero-order deformation problems

$$(1 \quad p)L_1[f(\eta, p) \quad f_0(\eta)] \quad p\hbar_1 N_1[f(\eta, p)]; \tag{18}$$

$$(1 \quad p)L_2[\theta(\eta, p) \quad \theta_0(\eta)] \quad p\hbar_2 N_2[\theta(\eta, p)]; \tag{19}$$

$$(1 \quad p)L_3[C(\eta, p) \quad C_0(\eta)] \quad p\hbar_3 N_3[C(\eta, p)]; \tag{20}$$

$$f(0, p) \quad f_w; \quad f(0, p) \quad 0; \quad f(\infty, p) \quad 1; \tag{21}$$

$$\theta(0, p) \quad 1; \quad \theta(\infty, p) \quad 0; \tag{22}$$

$$C(0, p) \quad \frac{1}{2} f(0, p); \quad C(\infty, p) \quad 0 \tag{23}$$

$$N_1[f(\eta, p)] \quad (1 \quad K)\frac{d^3 f(\eta, p)}{d\eta^3} \quad f(\eta, p)\frac{d^2 f(\eta, p)}{d\eta^2}$$

$$1 \quad \frac{df(\eta, p)}{d\eta}^{3} \quad K\frac{dC(\eta, p)}{d\eta} \quad \lambda\theta(\eta, p) \quad 0 \tag{24}$$

$$N_2[C(\eta,p)] = \left(1+\frac{K}{2}\right)\frac{\mathrm{d}^2C(\eta,p)}{\mathrm{d}\eta^2} - C(\eta,p)\frac{\mathrm{d}f(\eta,p)}{\mathrm{d}\eta} + f(\eta,p)\frac{\mathrm{d}C(\eta,p)}{\mathrm{d}\eta} - K\left[2C(\eta,p)+\frac{\mathrm{d}^2f(\eta,p)}{\mathrm{d}\eta^2}\right] = 0 \tag{25}$$

$$N_3[\theta(\eta,p)] = \frac{1}{\mathrm{Pr}}\frac{\mathrm{d}^2\theta(\eta,p)}{\mathrm{d}\eta^2} - \theta(\eta,p)\frac{\mathrm{d}f(\eta,p)}{\mathrm{d}\eta} + f(\eta,p)\frac{\mathrm{d}\theta(\eta,p)}{\mathrm{d}\eta} = 0 \tag{26}$$

For $p = 0$ and $p = 1$, we have:

$$\begin{aligned} &f(\eta,0) = f_0(\eta);\quad f(\eta,p) = f(\eta);\\ &\theta(\eta,0) = \theta_0(\eta);\quad \theta(\eta,1) = \theta(\eta);\\ &C(\eta,0) = C_0(\eta);\quad C(\eta,1) = C(\eta); \end{aligned} \tag{27}$$

When p increases from 0 to 1 then $f(\eta,p)$ vary from $f_0(\eta)$ to $f(\eta)$, $\theta(\eta,p)$ vary from $\theta_0(\eta)$ to $\theta(\eta)$ and $C(\eta,p)$ vary from $C_0(\eta)$ to $C(\eta)$. By Taylor's theorem and using eqs. (27), we can write:

$$f(\eta,p) = f_0(\eta) + \sum_{j=1}^{\infty} f_j(\eta)p^j,\quad f_j(\eta) = \frac{1}{j!}\frac{\partial^j[f(\eta,p)]}{\partial p^j} \tag{28}$$

$$\theta(\eta,p) = \theta_0(\eta) + \sum_{j=1}^{\infty} \theta_j(\eta)p^j,\quad \theta_j(\eta) = \frac{1}{j!}\frac{\partial^j[\theta(\eta,p)]}{\partial p^j} \tag{29}$$

$$C(\eta,p) = C_0(\eta) + \sum_{j=1}^{\infty} C_j(\eta)p^j,\quad C_j(\eta) = \frac{1}{j!}\frac{\partial^j[C(\eta,p)]}{\partial p^j} \tag{30}$$

For simplicity, we suppose $\hbar_1 = \hbar_2 = \hbar_3 = \hbar$, which $\hbar$ is chosen in such a way that these three series are convergent at $p = 1$. Therefore we have through eqs. (28)-(30):

$$f(\eta) = f_0(\eta) + \sum_{j=1}^{\infty} f_j(\eta) \tag{31}$$

$$\theta(\eta) = \theta_0(\eta) + \sum_{j=1}^{\infty} \theta_j(\eta) \tag{32}$$

$$C(\eta) = C_0(\eta) + \sum_{j=1}^{\infty} C_j(\eta) \tag{33}$$

M^th-order deformation problems

$$L[f_j(\eta) - \chi_j f_{j-1}(\eta)] = \hbar R_j^f(\eta) \tag{34}$$

$$f_j(0) = f_j'(0) = f_j'(\infty) = 0 \tag{35}$$

$$R_j^f(\eta) = (1+K)f_{j-1}''' + \sum_{n=0}^{j-1}(f_{j-1-n}f_n'' - f_{j-1-n}'f_n') + KC_{j-1}' + \lambda\theta_{j-1} \tag{36}$$

$$L[\theta_j(\eta) - \chi_j\theta_{j-1}(\eta)] = \hbar R_j^\theta(\eta) \tag{37}$$

$$\theta_j(0) \quad \theta_j(\infty) \quad 0 \tag{38}$$

$$R_j^{\theta}(\eta) \quad \frac{1}{\Pr}\theta_{j\ 1} \quad \sum_{n\ 0}^{j\ 1}(\ \theta_{j\ 1} \quad f_n \quad f_{j\ 1\ n}\theta_n)$$

$$L[C_j(\eta) \quad \chi_j C_{j\ 1}(\eta)] \quad \hbar R_j^C(\eta) \tag{39}$$

$$C_j(0) = C_j(\infty) = 0 \tag{40}$$

$$R_j^C(0) \quad 1 \quad \frac{K}{2}\ C_{j\ 1} \quad \sum_{n\ 0}^{j\ 1}(\ C_{j\ 1}f_n \quad f_{j\ 1\ n}C_n) \quad K(2C_{j\ 1} \quad f_{j\ 1}) \tag{41}$$

$$\chi_j \quad \begin{matrix} 0, & j \quad 1 \\ 1, & j \quad 1 \end{matrix} \tag{42}$$

Convergence of the HAM solutions

As pointed by Liao, the convergence of the solution depends upon the value of the auxiliary parameter $\hbar$. Figures 2-4 show the admissible value of $\hbar$, $-1 < \hbar < -0.3$ for θ' and C , and $-0.8 < \hbar < -0.2$ for f .

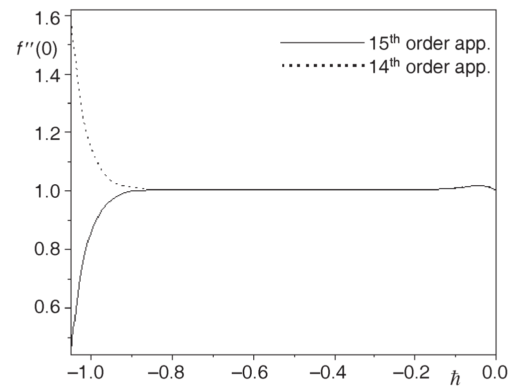

Figure 2. $\hbar$ – **curve of** f **(0) at** K **= 1, Pr = 1,** λ **= –1, and** f_w **= 0**

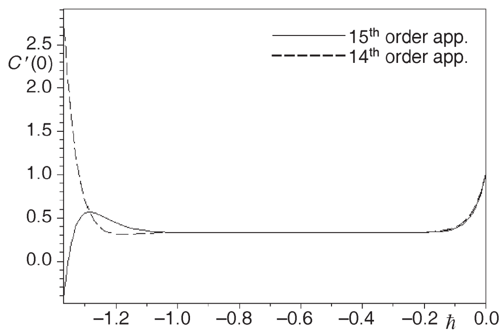

Figure 3. $\hbar$ – **curve of** $C'(0)$ **at** K **= 1, Pr = 1,** λ **= –1, and** f_w **= 0**

Results and discussion

Figure 5 illustrates the effects of suction and injection on velocity profile, as can be seen the value of velocity increases with raise of f_w and microrotation profile also has a similar trend as shown in fig. 6

Figure 7 depicts that the value of temperature decreases by increasing f_w and the temperature profile moves closer to the wall.

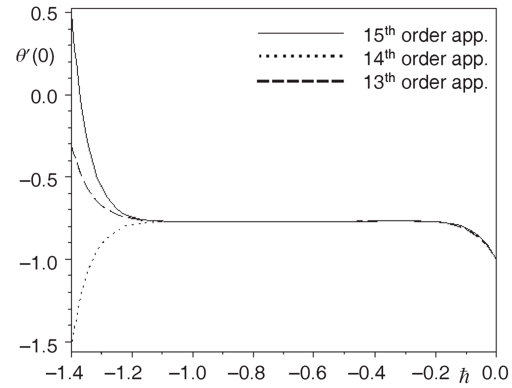

Figure 4. $\hbar-$ **curve of** $\theta'(0)$ **at** $K = 1$, **Pr = 1,** $\lambda = -1$, **and** $f_w = 0$

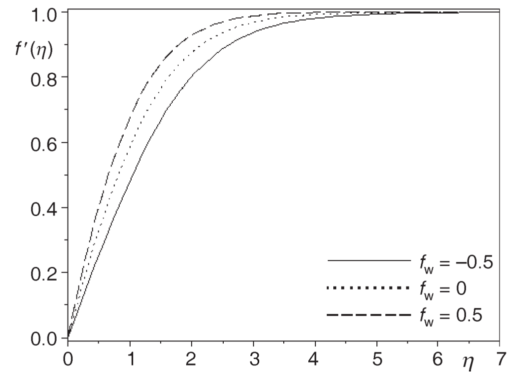

Figure 5. Velocity profile f' **for various** f_w **when** $K = 1$, **Pr = 1, and** $\lambda = -1$

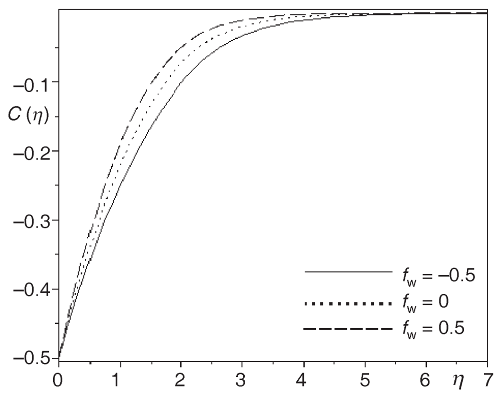

Figure 6. Microrotation profiles f' **for various** f_w **when** $K = 1$, **Pr = 1, and** $\lambda = -1$

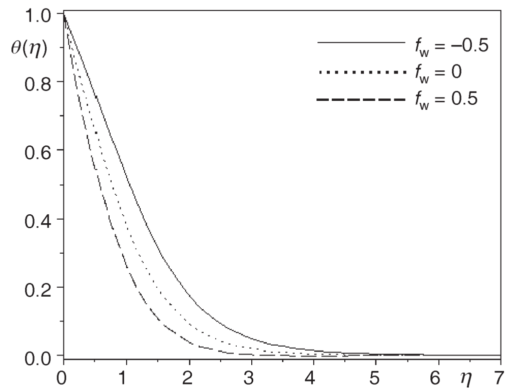

Figure 7. Temperature profiles θ **for various** f_w **when** $K = 1$, **Pr = 1, and** $\lambda = -1$

Figure 8 displays the behavior of velocity profile for two values of K, from the figure the value of velocity for Newtonian fluid is more than non-Newtonian case.

Figures 9 and 10 show the effect of f_w on the local Nusselt number and the skin friction coefficient, respectively; by increasing the value of λ the values of f (0) and $\theta'(0)$ increase and this is more noticeable for Nusselt number.

The comparison of HAM results with numerical ones has been made in figs.11 and 12 that are in excellent agreement which suggest that the HAM could be a useful and effective tool in solving systems of non-linear differential equations of engineering problems.

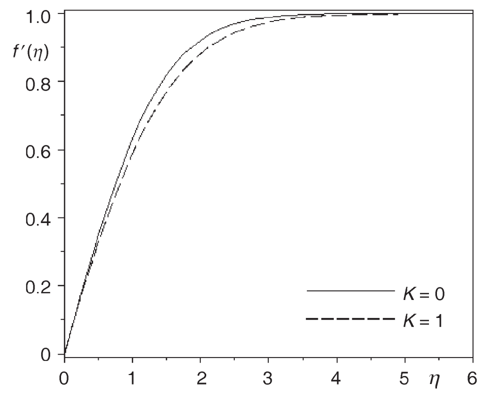

Figure 8. Velocity profiles f' **for various** K **when Pr = 1, and** $\lambda = -1$

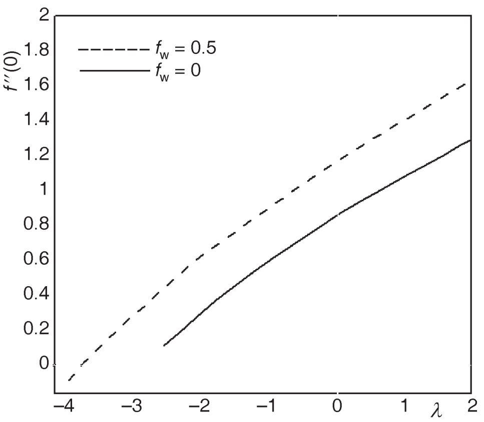

Figure 9. Variation with λ of the skin friction coefficient f (0) for f_w = 0 and 0.5 when Pr = 1 and K = 1

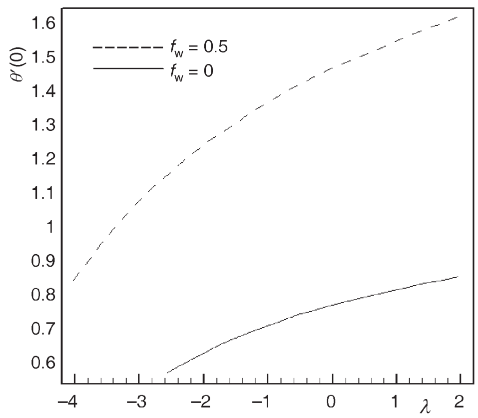

Figure 10. Variation with l of the local Nusselt number $-\theta'(0)$ for f_w = 0 and 0.5 when Pr = 1 and K = 1

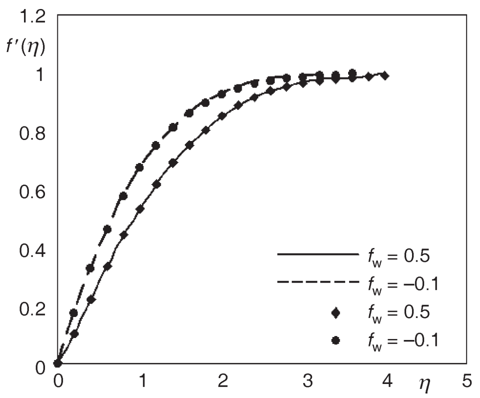

Figure 11. Comparison between HAM (◆, ●) and numerical solution (,) for f_w = –0.1 and 0.5 when Pr = 1, λ = –1, and K = 1

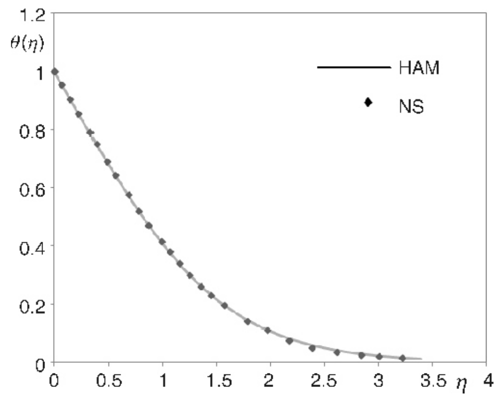

Figure 12. Comparison between HAM and numcerical solution for f_w = 0 when Pr = 1, λ = –1, and K = 1

Conclusions

This communication deals with the stagnation flow of a micropolar fluid towards a vertical permeable surface. HAM solution has been obtained for the problem. The results are sketched and discussed for the fluid and flow parameters variations. It is found that HAM results agree well with the numerical results. It is concluded that HAM provides a simple and easy way to control and adjust the convergence region for strong nonlinearity and is applicable to highly non-linear problems.

Nomenclature

a, b – positive constants, [–]
f – dimensionless stream function, [–]
f_0 – suction/injection parameter, [–]
g – acceleration due to gravity, [Ls^{-2}]

Gr_x – local Grashof number ($= g\beta(T_w - T_\infty)x^3/\nu^2$, [–]
h – dimensionless angular velocity, [–]
j – microinertia density, [kgL^3]
K – material parameter, [–]
N – component of the microrotation vector normal to the x-y plane, [–]
Pr – Prandtl number ($= -V_w/\nu a)^{1/2}$), [–]
Re_x – local Reynolds number ($= UD/\nu$), [–]
T – fluid temperature, [K]
T_w – plate temperature, [K]
T_∞ – ambient temperature, [K]
u, v – velocity components along the x- and y-directions, respectively, [Ls^{-1}]
U – free stream velocity, [Ls^{-1}]
V_w – uniform surface mass flux, [kgm^{-1}]
x, y – Cartesian co-ordinates along the surface and normal to it, respectively, [–]

Greek letters

α – thermal diffusivity, [m^{2}s^{-1}]
β – thermal expansion coefficient, [K^{-1}]
γ – spin gradient viscosity, [m^{2}s^{-1}]
η – similarity variable, [m]
θ – dimensionless temperature, [–]
κ – vortex viscosity, [m^{2}s^{-1}]
λ – buoyancy or mixed convection parameter, [–]
μ – dynamic viscosity, [m^{2}s^{-1}]
ν – kinematic viscosity, [m^{2}s^{-1}]
ρ – fluid density, [kgm^{-3}]
ψ – stream function, [–]

Subscripts

w – condition at the wall
∞ – ambient condition

Superscript

$'$ – differentiation with respect to η

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