

RAYLEIGH-TAYLOR INSTABILITY OF TWO SUPERPOSED MAGNETIZED VISCOUS FLUIDS WITH SUSPENDED DUST PARTICLES

by

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The linear Rayleigh-Taylor instability of two superposed incompressible magnetized fluids is investigated incorporating the effects of suspended dust particles and viscosity. The basic magnetohydrodynamic set of equations have been constructed and linearized. The dispersion relation for 2-D and 3-D perturbations is obtained by applying the appropriate boundary conditions. The condition of Rayleigh-Taylor instability is investigated for potentially stable and unstable modes, which depends upon magnetic field, viscosity and suspended dust particles. The stability of the system is discussed by applying the Routh-Hurwitz criterion. It is found that the Alfvén mode comes into the dispersion relation for perturbations in x, y-directions and in only x-direction, while it does not come into y-directional perturbation. The stable configuration is found to remain stable even in the presence of suspended dust particles. Numerical calculations have been performed to see the effects of various parameters on the growth rate of Rayleigh-Taylor instability. It is found that magnetic field and relaxation frequency of suspended dust particles both have destabilizing influence on the growth rate of Rayleigh-Taylor instability. The effects of kinematic viscosity and mass concentration of dust particles are found to have stabilized the growth rate of linear Rayleigh-Taylor instability.

Key words: *hydromagnetic instabilities, magnetohydrodynamics, Rayleigh-Taylor instability, suspended dust particles, viscosity*

Introduction

The problem of Rayleigh-Taylor (R-T) instability has importance in space plasma physics and astrophysics providing explanation for a large number of observations between streaming fluids. In laser fusion process, the high temperature with thermal energy exchange is important and the R-T instability plays a crucial role in laser plasma experiments. It is encountered frequently in the laboratory plasma with the magnetic fusion problem. It occurs when a lighter fluid supports a heavier against gravity whereupon they tend to interchange their posi-

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tions. The problem of R-T instability is studied widely by many authors in different types of medium. Chandrasekhar [1] has given the detailed contributions to this classical problem of R-T instability under different assumptions of hydrodynamics and hydromagnetics. Roberts [2] has investigated the effect of finite kinematic viscosity and magnetic resistivity on the R-T instability of two superposed incompressible fluids. Ogbonna *et al.* [3] have analyzed the stability of a plane interface separating two viscous superposed fluids and concluded that the stability criterion is independent of viscosity but the magnetic field has stabilizing influence. Mikaelian [4] has discussed the effect of viscosity on the R-T instability. Lange *et al.* [5] have investigated the fingering instability in a water sand mixture experimentally in the context of R-T instability of two stratified fluids of different densities and dynamic viscosities. El-Ansary *et al.* [6] have investigated the R-T instability for three fluid systems taking the effects of surface tension and rotation. Thus the problem of linear R-T instability is widely discussed in different types of medium for different cases.

In addition to this, the problem of R-T instability of two superposed visco-elastic fluids is widely discussed by many authors due to its industrial and chemical importance. In this view, Sharma *et al.* [7] have investigated the R-T instability of Rivlin-Ericksen elastico-viscous fluid through porous medium in presence of horizontal magnetic field and uniform rotation. Recently, Kumar *et al.* [8] have discussed R-T instability of two superposed viscous-viscoelastic fluids and found the stabilization effect of both kinematic viscosity and kinematic viscoelasticity on the growth rate of R-T instability.

In the case of dusty plasma or plasma with dust, there are problems in which dust grain will be in motion relative to the surrounding gas or plasma. In this case dust is subjected to a viscous drag force, which is given by Stokes law [9]. Owing to the relevance of suspended dust particles in number of astrophysical situations and laboratory problems, various workers have incorporated the effect of suspended dust particles in the R-T instability analysis. Scanlon *et al.* [10] have given a novel account of the theoretical studies focused on the effect of suspended dust particles on the onset of Bernard convection. Sharma *et al.* [11] have studied the effect of suspended particles in an infinitely conducting gas layer in hydromagnetics where destabilizing influence of these particles is pointed out. Sharma *et al.* [12] have also analyzed the R-T instability for a medium consisting of two superposed fluids including suspended particles and shown the criteria determining stability and instability of the system. Sanghvi *et al.* [13] have discussed the hydromagnetic Kelvin-Helmholtz (K-H) instability in the presence of suspended particles and finite Larmor radius (FLR) effect and found that suspended dust particles have stabilizing influence on the system. Also, Gupta *et al.* [14] have discussed the R-T instability of two superposed magnetized viscous fluids with neutral particles. The effects of rotation and FLR corrections on the R-T instability of two superposed magnetized conducting fluids with suspended dust particles are analyzed by Sharma *et al.* [15,16]. Kumar *et al.* [17] have investigated the R-T instability of two superposed Rivlin-Ericksen visco-elastic fluids in presence of suspended dust particles.

In this light, Sunil *et al.* [18] have discussed the R-T instability of two superposed hydromagnetic porous medium in the presence of suspended dust particles. In this work the condition of R-T instability is obtained but effect of viscosity of the medium was not considered. In the present work, we wish to discuss R-T instability with viscosity of the medium, which has definite role in connection with suspended dust particles.

Recently, Khan *et al.* [19] have discussed the K-H discontinuity in two superposed viscous conducting fluids in the presence of horizontal magnetic field. They found that viscosity,

porosity, and surface tension have stabilizing influence on the growth rate of the unstable mode. El-Sayed [20] has investigated the transverse hydromagnetic K-H instability including effects of suspended particles, viscosity and FLR corrections. He has discussed perturbation only in y-direction and solved the R-T and K-H problem. In his static result there is no effect of magnetic field on the dispersion relation and condition of R-T instability. We find that the terms due to magnetic field do not come into dispersion relation due to consideration of 2-D perturbations. Thus it is of interest to discuss the problem of R-T instability taking 3-D perturbations. In the present problem we have considered the different configuration of perturbations and tried to explain the effect of magnetic field along with suspended dust particles and viscosity on the condition of R-T instability and growth rate of R-T instability.

In the light of above discussion, we find that investigation of the problem of R-T instability is of current interest in thermal physics. It is important to note that in various works [13, 15, 18], the R-T instability is discussed with and without suspended particles and magnetic field but the effect of viscosity is not included in the analysis. In the present paper, we analyze the R-T instability at the plane interface of two superposed incompressible conducting viscous fluids of different densities in the presence of uniform horizontal magnetic field including the effects of suspended dust particles and viscosity of the medium. We have also discussed stable and unstable arrangements as separate cases.

Basic equations of the problem

We consider two superposed incompressible fluids of variable density and constant viscosity with horizontal magnetic field $\vec{H}(H, 0, 0)$ and gravity field $\vec{g}(0, 0, -g)$ in the two regions (see fig. 1). The fluids are taken to be conducting in which non-conducting suspended dust particles have been incorporated. The relevant basic magnetohydrodynamic (MHD) equations of the problem are:

$$\rho \frac{d\vec{u}}{dt} = \mu_e \left[(\vec{H} \cdot \nabla) \vec{H} - \nabla \left(\frac{H^2}{2} \right) \right] - \nabla p + KN(\vec{v} - \vec{u}) - \rho \vec{g} \quad (1)$$

$$\frac{\partial \rho}{\partial t} + (\vec{u} \cdot \nabla) \rho = 0 \quad (2)$$

$$\frac{\partial \vec{H}}{\partial t} + (\vec{u} \cdot \nabla) \vec{H} - (\vec{H} \cdot \nabla) \vec{u} = 0 \quad (3)$$

$$\vec{u} = 0, \text{ and } \vec{H} = 0, \quad (4)$$

where $d/dt = \partial/\partial t + (\vec{u} \cdot \nabla)$ is the convective derivative, $\vec{u}(u, v, w)$, ρ , μ , μ_e denote the velocity, density, viscosity, and the magnetic permeability of the fluid, respectively. $\vec{v}(\vec{x}, t)$ and N describe the velocity and the number density of the suspended dust particles. If we assume identical dust particles in shape and size then the net effective drag force of the suspended dust particles on the fluid per unit volume is $KN(\vec{v} - \vec{u})$ where the Stokes drag coefficient $K = 6\pi\mu a$, and a being the suspended dust particle radius.

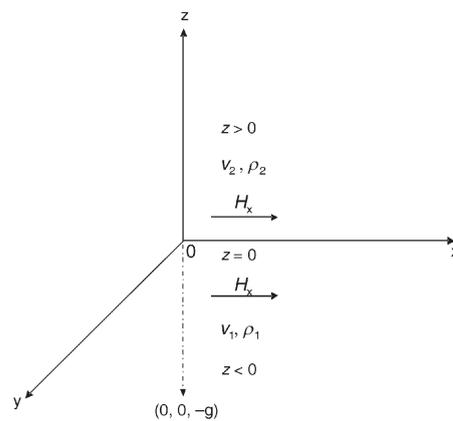


Figure 1. Schematic diagram of the considered R-T configuration

We know that the force exerted by the fluid on the dust particles is equal and opposite to that of the force exerted by the dust particles on the fluid, thus there is an extra force term in equation of motion of the dust particles equal in magnitude but opposite in sign. The equations of motion and continuity including dust particles can be written as:

$$mN \frac{\partial \bar{v}}{\partial t} - (\bar{v} \cdot \nabla) \bar{v} = KN(\bar{u} - \bar{v}) \quad (5)$$

and

$$\frac{\partial N}{\partial t} + (N \bar{v}) = 0 \quad (6)$$

where mN is the mass of the suspended dust particles per unit volume.

Linearized perturbation equations of the problem

Let $\delta\rho$, δp and $\bar{h}(h_x, h_y, h_z)$ denote the perturbations in the density, pressure and magnetic field, respectively. Then by neglecting the equilibrium and most perturbed terms, the linearized perturbation equations of the configuration are:

$$\rho \frac{\partial \bar{u}}{\partial t} - \delta p - \frac{\mu_e}{4\pi} [(\bar{h} \cdot \nabla) \bar{H} - KN(\bar{v} - \bar{u}) - \bar{g}\delta\rho - \mu^2 \bar{u}] = \frac{\partial w}{\partial x} - \frac{\partial \bar{u}}{\partial z} - \frac{\partial \mu}{\partial z} \quad (7)$$

$$\frac{\partial \delta\rho}{\partial t} - wD\rho \quad (8)$$

$$\frac{\partial \bar{h}}{\partial t} - (\bar{u} \cdot \nabla) \bar{H} \quad (9)$$

$$\frac{m}{K} \frac{\partial}{\partial t} (1 - \bar{v} \cdot \nabla) \bar{u} \quad (10)$$

$$\bar{u} = 0, \quad \text{and} \quad \bar{h} = 0 \quad (11)$$

where $D = d/dz$.

Dispersion relation and discussions

Equations (7)-(11) are the linearized perturbation equations of the present problem. To get better insight into the effect of magnetic field on the R-T instability problem depending upon 2-D and 3-D configuration, we discuss three different cases of the R-T instability: (1) general case having perturbations of the variables in both x, y-directions (2) the perturbation only in x-direction, and (3) the perturbation only in y-direction.

The R-T instability with perturbations in both x, y-directions

Assuming all the perturbations of the form:

$$\exp(ik_x x + ik_y y + nt) \quad (12)$$

where k_x and k_y are the wavenumbers of perturbations along x, y directions providing $(k^2 = k_x^2 + k_y^2)$ and n is the growth rate of harmonic disturbance.

Equations (7)-(11) can be written in the component form using eq. (12), as follows:

$$[\rho(1 - \tau n) - mN]nu - (1 - \tau n)ik_x \delta p - \mu(1 - \tau n)(D^2 - k^2)u - (ik_x w - Dw)(1 - \tau n)D\mu \quad (13)$$

$$[\rho(1 - \tau n) - mN]nv - (1 - \tau n)ik_y \delta p - \mu(1 - \tau n)(D^2 - k^2)v - (ik_y w - Dw)(1 - \tau n)D\mu - \frac{\mu_e H}{4\pi}(ik_x h_y - ik_y h_x)(1 - \tau n) \quad (14)$$

$$[\rho(1 - \tau n) - mN]nw - (1 - \tau n)D\delta p - \mu(1 - \tau n)(D^2 - k^2)w - \frac{g}{n}(1 - \tau n)(D\rho)w - \frac{\mu_e H}{4\pi}(ik_x h_z - Dh_x)(1 - \tau n) - 2(1 - \tau n)(D\mu)(Dw) \quad (15)$$

$$n\bar{h} - ik_x H\bar{u} \quad (16)$$

$$ik_x u - ik_y v - Dw = 0 \quad (17)$$

$$ik_x h_x - ik_y h_y - Dh_z = 0 \quad (18)$$

where $\tau = m/K$ is the relaxation time of the suspended dust particles and $\nu = \mu/\rho$, is the kinematic viscosity of the fluid.

On eliminating δp between eqs. (13)-(15) and using eqs. (16)-(18), we obtain following differential equation in w :

$$n(\tau n - 1)[D(\rho Dw) - k^2 \rho w] - n\{D[mN(Dw)] - k^2(mN)w\} - \mu(\tau n - 1)(D^2 - k^2)^2 w - \frac{gk^2}{n}[(D\rho)(\tau n - 1)w] - \frac{\mu_e H^2 k_x^2}{4\pi n}(\tau n - 1)(D^2 - k^2)w - (\tau n - 1)\{D[(D\mu)(D^2 - k^2)w] - 2k^2(D\mu)(Dw)\} = 0 \quad (19)$$

Now we consider the case where two superposed fluids have uniform densities ρ_1 and ρ_2 and uniform viscosities μ_1 and μ_2 . The fluids are separated by a horizontal boundary $z = 0$. Then, in each region of constant density and viscosity, eq. (19) reduces to:

$$(D^2 - k^2)(D^2 - K^2)w = 0 \quad (20)$$

where

$$K^2 = k^2 - \frac{n}{\nu} \left[1 - \frac{mN}{\rho(1 - \tau n)} - \frac{\mu_e k_x^2 H^2}{4\pi n^2 \rho} \right] \quad (21)$$

Since w must vanish both when $z \rightarrow -\infty$ (in the lower fluid) and $z \rightarrow \infty$ (in the upper fluid), the general solutions of eq. (20) appropriate to the regions are:

$$w_1 = A_1 \exp(kz) + B_1 \exp(K_1 z), \quad (z < 0) \quad (22)$$

$$w_2 = A_2 \exp(-kz) + B_2 \exp(-K_2 z), \quad (z > 0) \quad (23)$$

where $A_1, B_1, A_2,$ and B_2 are arbitrary constants and K_1 and K_2 are the positive square roots of eq. (21) for the two regions, respectively.

The above solutions must satisfy certain boundary conditions. Following Chandrashekhar [1] these conditions require at an interface ($z = 0$):

$$w = 0$$

$$\text{and } Dw = 0 \quad (24)$$

$$\mu(D^2 - k^2)w = 0$$

Integrating eq. (19) across the interface $z = 0$, we get:

$$\rho_2 \frac{\mu_2}{n} (D^2 - k^2) Dw_2 \Big|_{z=0} - \rho_1 \frac{\mu_1}{n} (D^2 - k^2) Dw_1 \Big|_{z=0} - \frac{mN}{\tau n - 1} (Dw_2 - Dw_1) \Big|_{z=0} = 0 \quad (25)$$

$$\frac{\mu_e k_x^2 H^2}{4\pi n^2} (Dw_2 - Dw_1) \Big|_{z=0} - \frac{gk^2}{n^2} (\rho_2 - \rho_1) w_0 - \frac{2k^2}{n} (\mu_2 - \mu_1) (Dw)_0 = 0$$

where w_0 and $(Dw)_0$ are the common values of w_1 and w_2 and similarly, Dw_1 and Dw_2 at $z = 0$.
Applying the boundary conditions as given by eq. (24) into eqs. (22) and (23), we obtain:

$$A_1 + B_1 = A_2 + B_2 \quad (26)$$

$$kA_1 + K_1 B_1 = -kA_2 - K_2 B_2 \quad (27)$$

$$\mu_1 [2k^2 A_1 - (K_1^2 - k^2) B_1] - \mu_2 [2k^2 A_2 - (K_2^2 - k^2) B_2] = 0 \quad (28)$$

$$\frac{gk^2}{2n^2} (\rho_2 - \rho_1) - \frac{k^2}{n} (\mu_2 - \mu_1) - \rho_1 \frac{mN}{\tau n - 1} - \frac{\mu_e k_x^2 H^2}{4\pi n^2} A_1$$

$$- \frac{gk^2}{2n^2} (\rho_2 - \rho_1) - \frac{kK_1}{n} (\mu_2 - \mu_1) B_1$$

$$\frac{gk^2}{2n^2} (\rho_2 - \rho_1) - \frac{k^2}{n} (\mu_2 - \mu_1) - \rho_2 \frac{mN}{\tau n - 1} - \frac{\mu_e k_x^2 H^2}{4\pi n^2} A_2$$

$$- \frac{gk^2}{2n^2} (\rho_2 - \rho_1) - \frac{kK_2}{n} (\mu_2 - \mu_1) B_2 = 0 \quad (29)$$

Eliminating the constants $A_1, B_1, A_2,$ and B_2 from the set of eqs. (26)-(29), we obtain the following determinant:

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ k & K_1 & k & K_2 \\ 2k^2\mu_1 - \mu_1(K_1^2 - k^2) & 2k^2\mu_2 - \mu_2(K_2^2 - k^2) & & \\ \alpha & \beta & \gamma & \delta \end{vmatrix} \quad (30)$$

where

$$\alpha = \frac{gk^2}{2n^2} (\rho_2 - \rho_1) - \frac{k^2}{n} (\mu_2 - \mu_1) - \rho_1 \frac{mN}{\tau n - 1} - \frac{\mu_e k_x^2 H^2}{4\pi n^2}$$

$$\beta = \frac{gk^2}{2n^2} (\rho_2 - \rho_1) - \frac{kK_1}{n} (\mu_2 - \mu_1)$$

$$\gamma = \frac{gk^2}{2n^2} (\rho_2 - \rho_1) - \frac{k^2}{n} (\mu_2 - \mu_1) - \rho_2 \frac{mN}{\tau n - 1} - \frac{\mu_e k_x^2 H^2}{4\pi n^2}$$

$$\delta = \frac{gk^2}{2n^2} (\rho_2 - \rho_1) - \frac{kK_2}{n} (\mu_2 - \mu_1)$$

On solving the determinant (30), we get the following characteristic equation:

$$(K_1 - k)[2k^2\nu(\rho_1 - \rho_2)a - b] - 2k(c - d) - (K_2 - k)[f - 2k^2\nu(\rho_1 - \rho_2)h] = 0 \quad (31)$$

where

$$\begin{aligned} a &= \rho_2 \frac{mNn}{\tau n - 1} \frac{k}{n} (K_2 - k)(\mu_2 - \mu_1) \frac{\mu_e k_x^2 H^2}{4\pi n^2} \\ b &= \rho_2 n \frac{mNn}{\tau n - 1} \frac{\mu_e k_x^2 H^2}{4\pi n} \frac{gk}{n^2} (\rho_2 - \rho_1) \frac{2mN}{\tau\nu - 1} (\rho_1 - \rho_2) \frac{2\mu_e k_x^2 H^2}{4\pi n^2} \\ c &= \rho_1 n \frac{mNn}{\tau n - 1} \frac{\mu_e k_x^2 H^2}{4\pi n} \rho_2 \frac{mN}{\tau n - 1} \frac{k}{n} (K_2 - k)(\mu_2 - \mu_1) \frac{\mu_e k_x^2 H^2}{4\pi n^2} \\ d &= \rho_2 n \frac{mNn}{\tau n - 1} \frac{\mu_e k_x^2 H^2}{4\pi n} \rho_1 \frac{mN}{\tau n - 1} \frac{k}{n} (K_1 - k)(\mu_2 - \mu_1) \frac{\mu_e k_x^2 H^2}{4\pi n^2} \\ f &= \rho_1 n \frac{mNn}{\tau n - 1} \frac{\mu_e k_x^2 H^2}{4\pi n} \frac{gk}{n^2} (\rho_2 - \rho_1) \frac{2mNn}{\tau n - 1} (\rho_1 - \rho_2) \frac{2\mu_e k_x^2 H^2}{4\pi n^2} \\ h &= \rho_1 \frac{mN}{\tau n - 1} \frac{k}{n} (K_1 - k)(\mu_2 - \mu_1) \frac{\mu_e k_x^2 H^2}{4\pi n^2} \end{aligned}$$

The characteristic eq. (31) shows the combined influence of magnetic field, permeability, kinematic viscosity, and suspended dust particles on the R-T instability of two superposed magnetized viscous fluids. Owing to character of the dispersion relation (31), K_1 and K_2 involve square roots; we restrict our treatment to the fluids of equal kinematic viscosities ($\nu_1 = \nu_2 = \nu$). Also for simplicity in the stability analysis, we further assume that the fluids to be highly viscous as given in Chandrasekhar [1]. Therefore, under the above restrictions we get:

$$K_{1,2} - k = \frac{n}{2k\nu_{1,2}} \frac{mNn}{2\rho_{1,2}\nu_{1,2}k(\tau n - 1)} \frac{\mu_e k_x^2 H^2}{8\pi n k \nu_{1,2} \rho_{1,2}} \quad (32)$$

Substituting the values of $K_1 - k$ and $K_2 - k$ from eq. (32) in the expression (31) and simplifying we obtain following dispersion relation:

$$\begin{aligned} \rho_1 n^2 (\tau n - 1) - n^2 mN \frac{\mu_e k_x^2 H^2}{4\pi} (\tau n - 1) - \rho_2 n^2 (\tau n - 1) - n^2 mN \frac{\mu_e k_x^2 H^2}{4\pi} (\tau n - 1) \\ - \tau n^3 - n^2 - \frac{2mN}{\rho_1 - \rho_2} - 2k^2 \nu \tau - n gk \tau \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} - \frac{2\mu_e k_x^2 H^2 \tau}{4\pi(\rho_1 - \rho_2)} - 2k^2 \nu \\ + gk \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} - \frac{2\mu_e k_x^2 H^2}{4\pi(\rho_1 - \rho_2)} = 0 \end{aligned} \quad (33)$$

Equation (33) represents the general dispersion relation for R-T instability of two superposed magnetized viscous fluids including suspended dust particles. In the absence of suspended dust particles ($\tau = 0$, and $N = 0$) this dispersion relation reduces to Ogbonna *et al.* [3] when vertical magnetic field is not considered in the dispersion relation.

The dispersion relation (33) is product of three factors and on equating the first and second factor equal to zero, we get:

$$\tau n^3 - n^2 - 1 - \frac{mN}{\rho_1} - \frac{\mu_e k_x^2 H^2 \tau n}{4\pi\rho_1} - \frac{\mu_e k_x^2 H^2}{4\pi\rho_1} = 0 \quad (34)$$

$$\tau n^3 - n^2 - 1 - \frac{mN}{\rho_2} - \frac{\mu_e k_x^2 H^2 \tau n}{4\pi\rho_2} - \frac{\mu_e k_x^2 H^2}{4\pi\rho_2} = 0 \quad (35)$$

Equations (34) and (35) give the dispersion relation for Alfvén mode modified by suspended dust particles and propagating in x-direction. The Alfvén velocity in both the dispersion relations is different and depends on the density of the upper and lower mediums. If we put $\tau = 0$, and $N = 0$ in both the equations, we get simple Alfvén wave propagation in x-direction. Thus we find that Alfvén modes are damped due to presence of suspended dust particles and this also gives a stable effect to the configuration of our system.

On equating third factor to zero it gives the following dispersion relation:

$$\tau n^3 - n^2 - 1 - \frac{2mN}{\rho_1 + \rho_2} - 2k^2 \nu \tau - n \frac{gk\tau}{\rho_1 + \rho_2} - \frac{2\mu_e k_x^2 H^2 \tau}{4\pi(\rho_1 + \rho_2)} - 2k^2 \nu - gk \frac{\rho_1 + \rho_2}{\rho_1 + \rho_2} - \frac{2\mu_e k_x^2 H^2}{4\pi(\rho_1 + \rho_2)} = 0 \quad (36)$$

The dispersion relation (36) represents the effect of suspended dust particles and viscosity and the condition of the R-T instability can be obtained. If we neglect effect of viscosity in eq. (36), we get the same dispersion relation that is obtained by Sunil *et al.* [18]. Thus eq. (36) is the dispersion relation modified due to viscosity as compared to Sunil *et al.* [18]. In the absence of viscosity and magnetic field, eq. (36) reduces to eq. (35) of Sanghvi *et al.* [13]. In present case we have taken 3-D perturbation and we are getting effect of magnetic field in the dispersion relation due to perturbation in x-direction. In Sanghvi *et al.* [13] the perturbation was taken only in y-direction and there was no effect of magnetic field in the dispersion relation. Thus our dispersion relation gives the complete information regarding the problem of R-T instability of two superposed magnetized fluids. If we ignore the effect of magnetic field then, eq. (36) gives the similar result that is obtained by Sharma *et al.* [16] excluding FLR corrections and rotation in that case. We find that in that case, the effect of magnetic field was taken in the basic equations of the problem but it does not appear in the dispersion relation due to the consideration of 2-D perturbations, but in the present problem we get the effect of magnetic field due to 3-D perturbations of the problem. Thus dispersion relation (36) is modified due to the inclusion of viscosity and magnetic field. We also note that in the absence of viscosity there is not any effect on the nature of equation. In both the cases with and without viscosity the dispersion relation is a cubic equation. Thus with viscosity no new mode is obtained in the dispersion relation.

In the case of Sunil *et al.* [18], they have taken the same density of the medium in the term mN/ρ whereas we have taken different densities of the medium which appears in dispersion relation as $(\rho_1 + \rho_2)/2$ instead of ρ in contribution of suspended dust particles.

We now discuss potentially stable and unstable configuration separately from our basic dispersion relation (36).

Stable configuration ($\rho_1 > \rho_2$)

If we apply this condition of $\rho_1 > \rho_2$ to eq. (36), we find that all the coefficients are positive, meaning thereby all the roots of the equation have negative real part satisfying the necessary condition of Routh-Hurwitz criterion. Therefore the stable configuration remains stable even in the presence of suspended dust particles. We have discussed the necessary condition of stability of the system using Routh-Hurwitz criterion but this is not sufficient condition for a system to be stable. Therefore for necessary and sufficient condition we must find Hurwitz minors. From eq. (36) we get:

$$\begin{aligned} \Delta_1 &= 1 - \frac{2mN}{\rho_1 - \rho_2} - 2k^2\nu\tau > 0 \\ \Delta_2 &= 2k^2\nu - 1 - \frac{2mN}{\rho_1 - \rho_2} - 2k^2\nu\tau - gk\tau \frac{\rho_1 - \rho_2}{\rho_1 - \rho_2} - \frac{2\mu_e k_x^2 H^2 \tau}{4\pi(\rho_1 - \rho_2)} > 2k^2\nu > 0 \quad (37) \\ \Delta_3 &= gk \frac{\rho_1 - \rho_2}{\rho_1 - \rho_2} - \frac{2\mu_e k_x^2 H^2}{4\pi(\rho_1 - \rho_2)} > \Delta_2 > 0 \end{aligned}$$

If we assume $\rho_2 > \rho_1$, then we find that all Δ are positive. Thus on applying necessary and sufficient condition to our system we find that for $\rho_2 > \rho_1$ the system is always stable. Thus we find that there is no change on the condition of stability of the system due to presence of suspended dust particles.

Unstable configuration ($\rho_2 > \rho_1$)

If we assume $\rho_2 > \rho_1$ and applying Routh-Hurwitz criterion to eq. (36), we find from the constant term that configuration shall be unstable according as:

$$\frac{\mu_e k_x^2 H^2}{2\pi} - gk(\rho_1 - \rho_2) < 0 \quad (38)$$

Thus the unstable configuration remains unaffected under the condition (38) in the presence of magnetic field. But the potentially unstable configuration can be stabilized for a certain wave number and by suitable choice of magnetic field, which is determined as

$$\frac{\mu_e H^2 k_x^2}{2\pi} > gk(\rho_2 - \rho_1) \quad (39)$$

On comparing our result with Sunil *et al.* [18] where viscosity is not included, we get the same condition of R-T instability. Thus, we find that viscosity of the medium has no effect on the condition of R-T instability but viscosity changes the growth rate of R-T instability of the medium. The result given by Sharma *et al.* [12] excluding magnetic perturbations, differs from us in the sense that we have shown that the potentially unstable configuration can be stabilized, for certain wavenumbers range by an appropriate value of magnetic field, while in their case it remains unstable for all wavenumbers.

The numerical calculations have been performed on the dispersion relation (36) to see the effect of magnetic field, relaxation frequency of suspended dust particles, kinematic viscosity, and mass concentration of dust particles on the growth rate of R-T instability. We write the dispersion relation (36) in dimensionless form as:

$$n^{*3} - n^{*2} [f_s^* (1 - 2\alpha) - 2k^{*2} v^*] + n^* [(\eta_2 - \eta_1) - 2V_A^{*2} - 2k^{*2} v^* f_s^*] + f_s^* [(\eta_2 - \eta_1) - 2V_A^{*2}] = 0 \quad (40)$$

where

$$n^* = \frac{n}{\sqrt{gk}}, \quad f_s^* = \frac{f_s}{\sqrt{gk}}, \quad v^* = \frac{\nu}{\sqrt{(gk)^3}}, \quad k^* = k\sqrt{gk}, \quad V_A^* = \sqrt{\frac{k}{g}} V_A \quad (41)$$

We have also substituted $\alpha' = mN/(\rho_1 + \rho_2)$ and $(\eta_2 - \eta_1) = [(\rho_2 - \rho_1)/(\rho_1 + \rho_2)]$ as mass concentration of dust particles and the Atwood number, respectively.

In figs. 2-5 we have plotted the growth rate of unstable R-T mode vs. dimensionless wavenumber for different values of magnetic field and no magnetic field, kinematic viscosity, relaxation frequency, and mass concentrations of suspended dust particles.

In fig. 2, we have depicted the dimensionless growth rate of R-T instability vs. dimensionless wavenumber of various values of magnetic field $V_A^* = 0.0, 1.0, \text{ and } 2.0$. The values of constant parameters are taken to be $\alpha = 0.5, f_s^* = 1.0, v^* = 0.5$ and $\eta_2 - \eta_1 = 1.5$. It is clear from the curves that the growth rate of R-T instability increases by increasing the value of magnetic field. The peak value of growth rate is minimum for the case of no magnetic field while it is max-

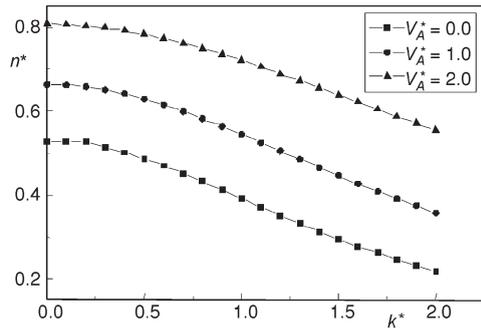


Figure 2. The growth rate of unstable R-T mode (positive real roots of n^*) against wavenumber for different values of magnetic field (V_A^*)

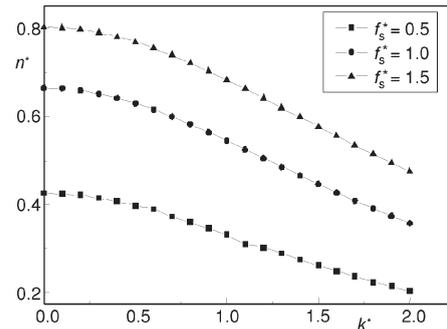


Figure 3. The growth rate of unstable R-T mode (positive real roots of n^*) against wavenumber for different values of relaxation frequency of suspended dust particles (f_s^*)

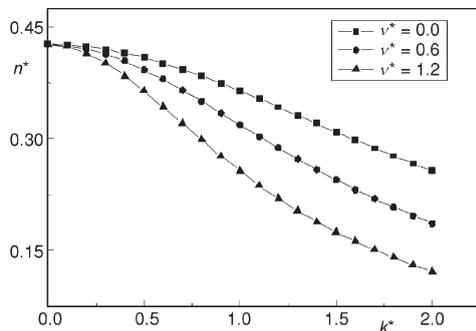


Figure 4. The growth rate of unstable R-T mode (positive real roots of n^*) against wavenumber for different values of kinematic viscosity parameter (v^*)

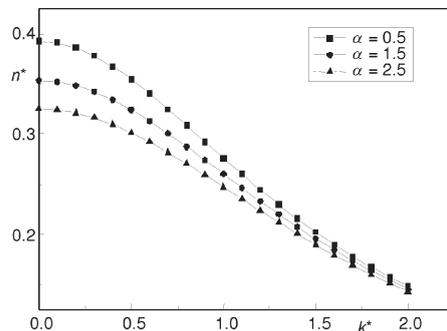


Figure 5. The growth rate of unstable R-T mode (positive real roots of n^*) against wavenumber for different values of mass concentration of suspended dust particles (α')

imum for the greater value of magnetic field (V_A^*). Thus magnetic field has destabilizing influence on the growth rate of the system. It is also seen from the curves that the growth rate decreases by increasing the value of dimensionless wavenumber for all values of magnetic field. Hence the larger value of wavenumber will give the minimum value of growth rate.

In fig. 3, the dimensionless growth rate of R-T instability vs. dimensionless wavenumber is depicted for different values of relaxation frequency of suspended dust particles f_s^* 0.5, 1.0, and 1.5. The values of constant parameters are taken to be $\alpha' = 0.5$, $V_A^* = 1.0$, $\nu^* = 0.5$, and $\eta_2 - \eta_1 = 1.5$. It is obvious from the curves that the growth rate of R-T instability f_s^* increases by increasing the value of f_s^* . The growth rate is also maximum for the larger value of f_s^* . Hence relaxation frequency of suspended dust particles has destabilizing role on the growth rate of R-T instability.

If fig. 4, we have plotted the growth rate of R-T instability against the dimensionless wavenumber for various values of kinematic viscosity parameter $\nu^* = 0.0, 0.6$, and 1.2 . The values of constant parameters are taken to be $\alpha = 0.5$, $V_A^* = 1.0$, $f_s^* = 0.5$, and $\eta_2 - \eta_1 = 1.5$. From the curves it is clear that on increasing the value of ν^* the growth rate decreases. The peak value of growth rate is unaffected by the inclusion of kinematic viscosity. Hence kinematic viscosity has damping effect and causes stabilization on the growth rate of R-T instability.

In fig. 5, we have seen the effect of mass concentration of suspended dust particles on the growth rate of R-T instability. The curves are traced for $\alpha' = 0.5, 1.5$, and 2.5 . The values of constant parameters are taken to be $\nu^* = 0.5$, $V_A^* = 0.5$, $f_s^* = 0.5$, and $\eta_1 - \eta_2 = 1.5$. From the curves it is clear that on increasing the value of α' the growth rate of the system decreases. The peak value also decreases by increasing α' . Hence mass concentration of suspended dust particles has stabilizing influence on the growth rate of the system.

If we neglect the effect of suspended dust particles ($\tau = 0, N = 0$) in eq. (36), we get the dispersion relation

$$n^2 - 2nk^2 \frac{\mu_1}{\rho_1} \frac{\mu_2}{\rho_2} - gk \frac{\rho_1}{\rho_1} \frac{\rho_2}{\rho_2} - \frac{2\mu_e H^2 k_x^2}{4\pi(\rho_1 - \rho_2)} = 0 \quad (42)$$

where $\mu_1 = \nu\rho_1$ and $\mu_2 = \nu\rho_2$.

In dispersion relation (42) if we ignore the effect of magnetic field, then we get the similar result that is obtained by Mikaelian [4]. In absence of viscosity we get the classical result of Chandrasekhar [1]. Thus we find that the results in the present analysis have been modified due to the presence of suspended dust particles, magnetic field, and viscosity of the medium.

The perturbation only in y-direction

In the present case we assume that the perturbation is only in y-direction. We consider eqs. (7)-(11) of the problem. On taking the perturbations of the form of:

$$\exp(ik_y y + nt)$$

where k_y is the wavenumber of perturbation along y-direction ($k_2 = k_y^2$) and n is the growth rate of harmonic disturbance, the linearized perturbation equations are written in the component form. From these equations after simplification we get the differential equation in w as:

$$n(\tau n - 1)[D(\rho Dw) - k^2 \rho w]n\{D[mN(Dw)] - k^2(mN)w\} - \mu(\tau n - 1)(D^2 - k^2)^2 w - \frac{gk^2}{n}[(D\rho)(\tau n - 1)w] - (\tau n - 1)\{D[(D\mu)(D^2 - k^2)w] - 2k^2(D\mu)(Dw)\} = 0 \quad (43)$$

We consider the case where two superposed fluids have uniform densities ρ_1 and ρ_2 and uniform viscosities μ_1 and μ_2 . The fluids are separated by a horizontal boundary $z = 0$, then in each region of constant viscosity and density, eq. (43) reduces to:

$$(D^2 - k^2)(D^2 - K^2) w = 0 \quad (44)$$

where

$$K^2 = k^2 \frac{n}{v} \left(1 - \frac{mN}{\rho(1 - \tau n)} \right) \quad (45)$$

Since w must vanish both when $z \rightarrow -\infty$ (in the lower fluid) and $z \rightarrow +\infty$ (in the upper fluid), the general solutions of the eq. (44) appropriate to the regions are:

$$\begin{aligned} w_1 &= A_1 \exp(kz) + B_1 \exp(K_1 z) \\ w_2 &= A_2 \exp(-kz) + B_2 \exp(-K_2 z) \end{aligned} \quad (46)$$

where $A_1, B_1, A_2,$ and B_2 are arbitrary constants and K_1 and K_2 are the positive square roots of eq. (44) for the two regions, respectively.

The above solutions must satisfy boundary conditions given in eq. (24). Also, integrating eq. (43) across the interface $z = 0$, we have another condition:

$$\begin{aligned} \rho_2 \frac{\mu_2}{n} (D^2 - k^2) Dw_2 \Big|_{z=0} - \rho_1 \frac{\mu_1}{n} (D^2 - k^2) Dw_1 \Big|_{z=0} \\ - \frac{mN}{\tau n} (Dw_2 - Dw_1) \Big|_{z=0} - \frac{gk^2}{n^2} (\rho_2 - \rho_1) w_0 - \frac{2k^2}{n} (\mu_2 - \mu_1) (Dw)_0 = 0 \end{aligned} \quad (47)$$

Applying the boundary conditions stated above and after eliminating constants A_1, B_1, A_2 and B_2 , we get the following dispersion relation:

$$\begin{aligned} (\rho_1 \tau n - \rho_1 - mN)(\rho_2 \tau n - \rho_2 - mN) \tau n^3 - n^2 \left(1 - \frac{2mN}{\rho_1 - \rho_2} \right) 2k^2 v \tau \\ - n g k \tau \frac{\rho_1 - \rho_2}{\rho_1 - \rho_2} - 2k^2 v - gk \frac{\rho_1 - \rho_2}{\rho_1 - \rho_2} = 0 \end{aligned} \quad (48)$$

Equation (48) gives the dispersion relation for the R-T instability of two superposed fluids of density ρ_1 and ρ_2 in the presence of viscosity and suspended dust particles. In this case we have considered only 2-D perturbations (in y-direction) of various physical quantities. We find that due to consideration of perturbation only in y-direction, the effect of magnetic field does not come into the dispersion relation. In the absence of suspended particles ($\tau = 0$) the dispersion relation reduces to Ogbonna *et al.* [3] excluding the effect of vertical magnetic field in their case.

The dispersion relation (48) has three factors. On equating first and second factors to zero, we get:

$$n \frac{\rho_1 - mN}{\rho_1 \tau} = 0 \quad (49)$$

and

$$n \frac{\rho_2}{\rho_1} \frac{mN}{\rho_2 \tau} = 0 \quad (50)$$

These two equations are due to the presence of suspended dust particles, which do not give the R-T instability. Equations (49) and (50) represent stable damped mode whose damping rate depend upon ratio of mass concentration and relaxation time of suspended dust particles. If there are no suspended dust particles then obviously these two modes vanish.

From the third factor of dispersion relation (48) we get:

$$\tau n^3 - n^2 - 1 - \frac{2mN}{\rho_1 \rho_2} - 2k^2 \nu \tau - n g k \tau \frac{\rho_1}{\rho_2} - \frac{\rho_2}{\rho_1} - g k \frac{\rho_1}{\rho_2} - \frac{\rho_2}{\rho_1} = 0 \quad (51)$$

If we neglect the effect of viscosity in dispersion relation (51) then we get:

$$\tau n^3 - n - 1 - \frac{2mN}{\rho_1 \rho_2} - n g k \tau \frac{\rho_2}{\rho_1} - \frac{\rho_1}{\rho_2} - g k \frac{\rho_2}{\rho_1} - \frac{\rho_1}{\rho_2} = 0 \quad (52)$$

This dispersion relation is identical to Sanghvi *et al.* [13] where perturbation is assumed only in y-direction. Thus we find that if we include the effect of viscosity then the dispersion relation is modified as relation (51) in the present case. We also find on comparing eqs. (51) and (52) that no new mode is appearing in the dispersion relation due to viscosity. Also we find that in both the cases *i. e.* with viscosity and without viscosity the term of magnetic field is not appearing in the dispersion relation.

The effect of viscosity on the growth rate of R-T instability is studied numerically by plotting the curves between growth rate of unstable R-T mode and relaxation frequency of suspended dust particles. The results have been also reduced to Sanghvi *et al.* [13] by taking $\alpha' = 0.6$ and $\nu' = 0.0$. We write dispersion relation (51) in dimensionless form by using the dimensionless parameters given in eq. (41). We get from eq. (51):

$$n^{*3} - n^{*2} [(1 - 2\alpha) f_s^* - 2k^{*2} \nu^*] - n^* [(\eta_2 - \eta_1) - 2k^{*2} \nu^* f_s^*] - f_s^* (\eta_2 - \eta_1) = 0 \quad (53)$$

In fig. 6, we have depicted the growth rate of R-T instability against relaxation frequency of suspended dust particles for different values of viscosity parameters. Curve (a) is plotted for $\alpha' = 0.6$, $k^* = \nu^* = 0.0$, and $\eta_2 - \eta_1 = 0.6$ which is identical to Sanghvi *et al.* [13]. Curves (b) and (c) have been depicted for $\alpha' = 0.6$, $k^* = 1.0$, $\eta_2 - \eta_1 = 0.6$, and, $\nu^* = 0.25$ and 0.5 , respectively. From the curves we find that on increasing the value of viscosity the growth rate of R-T instability and peak value of growth rate both decrease. Hence viscosity has damping effect

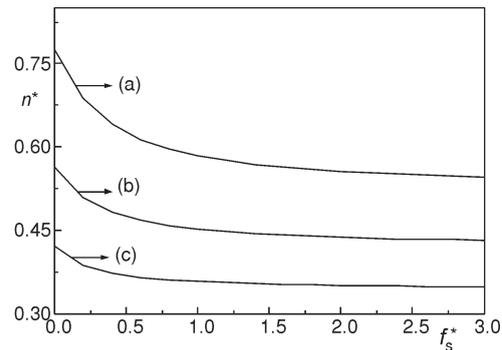


Figure 6. The different curves for the growth rate of unstable R-T mode (positive real roots of n^*) against relaxation frequency of suspended dust particles (f_s^*); curve (a) is for non-viscous fluids ($\nu^* = 0.0$), which is identical to Sanghvi *et al.* [13]; curves (b) and (c) are for $\nu^* = 0.25$ and 0.5 , respectively

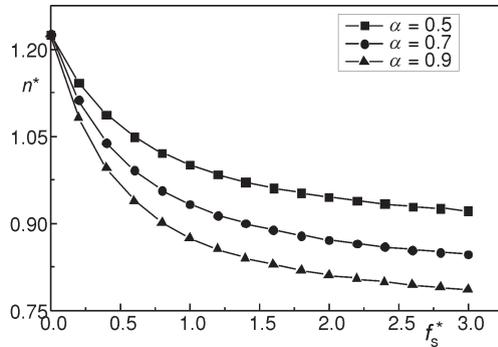


Figure 7. Shows the growth rate of R-T instability vs. relaxation frequency of suspended dust particles for non-viscous non-magnetized system; the values of constant parameters are taken to be $v^* = 0.0$, $k^* = 1.0$, and $\eta_2 - \eta_1 = 1.5$

on the growth rate of the R-T instability. We find that the growth rate gets modified due to the presence of viscosity and it is identical to Sanghvi *et al.* [13] for non-viscous case ($v^* = 0.0$).

In fig. 7 we have made only the effect of mass concentration of suspended dust particles for non-magnetized and non-viscous fluids on the classical R-T instability. For this we have plotted the curves between growth rate of R-T instability and relaxation frequency of suspended dust particles. From the curves it is clear that the growth rate of R-T instability decreases on increasing value of mass concentration of suspended dust particle. The peak value of the growth rate does not depend on mass concentration of suspended dust particles. These results

are identical to the previous one in which the effect of viscosity and magnetic field is considered and we get the same results. Hence the effect of suspended dust particle is to stabilize the system in both presence and absence of viscosity and magnetic field.

Equation (51) is a cubic equation and it gives three modes of propagations. If we put $\tau = 0$ and $N = 0$ in eq. (51) we get:

$$n^2 - 2k^2 v n \frac{\rho_1}{\rho_1} \frac{\rho_2}{\rho_2} - gk \frac{\rho_1}{\rho_1} \frac{\rho_2}{\rho_2} = 0 \tag{54}$$

This is well known dispersion relation for the R-T instability with viscosity. We put $v\rho_1 = \mu_1$ and $v\rho_2 = \mu_2$, we get from eq. (54)

$$n^2 - 2k^2 n \frac{\mu_1}{\rho_1} \frac{\mu_2}{\rho_2} - gk \frac{\rho_1}{\rho_1} \frac{\rho_2}{\rho_2} = 0 \tag{55}$$

This is the same relation as given by Mikaelian [4]. This gives the effect of two viscosities of the two media in dispersion relation.

If we neglect the effect of viscosity we get the classical R-T instability which growth rate depends upon the Atwood number and given as:

$$n^2 - gk \frac{\rho_1}{\rho_1} \frac{\rho_2}{\rho_2} = 0 \tag{56}$$

Now we consider dispersion relation (51) in which the presence of suspended dust particles is considered. This is a cubic equation but from constant term we can deduce the condition of R-T instability. We assume following two cases.

Stable configuration ($\rho_1 > \rho_2$)

We assume $\rho_1 > \rho_2$ and we apply this condition to eq. (51), we find that all the coefficients of eq. (51) are positive, meaning thereby all the roots of the equation are located in

left-hand complex plane *i. e.* have a negative real part (Hurwitz necessary criterion). Hence the interface is stable. We have discussed only necessary condition of instability. For necessary and sufficient condition of stability we should calculate Routh-Hurwitz minors of the eq. (51) and we get:

$$\begin{aligned} \Delta_1 &= 1 - \frac{2mN}{\rho_1 - \rho_2} - 2k^2 v \tau > 0 \\ \Delta_2 &= 2k^2 v - \frac{2mN}{\rho_1 - \rho_2} - 2k^2 v \tau - \frac{gk\tau \rho_1 - \rho_2}{\rho_1 - \rho_2} - 2k^2 v > 0 \\ \Delta_3 &= gk \frac{\rho_1 - \rho_2}{\rho_1 - \rho_2} \Delta_2 > 0 \end{aligned} \quad (57)$$

If we assume $\rho_1 > \rho_2$ then we find all the Δ are positive. Thus on applying the necessary and sufficient condition of stability of the system we find that for $\rho_1 > \rho_2$ the system is always stable.

Therefore, the stable configuration remains stable even in the presence of suspended particles and where there is no effect of suspended particles, we get the same result.

Unstable configuration ($\rho_2 > \rho_1$)

Applying Hurwitz criterion to eq. (51), we find that configuration shall be stable or unstable according as:

$$\rho_1 - \rho_2 < 0, \text{ or } \rho_1 - \rho_2 > 0 \quad (58)$$

Evidently the unstable configuration remains unaffected under the condition $\rho_1 > \rho_2$ in the presence of suspended dust particles. Thus, we may conclude that the R-T instability of two superposed conducting viscous fluids remains uninfluenced by the presence of suspended dust particles for the case of potentially stable arrangement.

We also note that there is no effect of horizontal magnetic field on the condition of instability. The magnetic field term does not appear in the dispersion relation and we get the dispersion relation as if there is no magnetic field.

The perturbation only in x-direction

In this case we assume the perturbation only in x-direction. We take the eqs. (7)-(11) for this problem. On taking the perturbation of the form:

$$\exp(ik_x x + nt)$$

where k_x is the wavenumber of perturbation along x-direction ($k^2 = k_x^2$) and n is the growth rate of harmonic disturbance. The linearized perturbation equations written in the component form and after eliminating variables between these equations we obtain following differential equations in w :

$$\begin{aligned} n(\tau n - 1)[D(\rho Dw) - k^2 \rho w] - n\{D[mN(Dw)] - k^2(mN)w\} - \mu(\tau n - 1)(D^2 - k^2)^2 w \\ - \frac{gk^2}{n} [(D\rho)(\tau n - 1)w] - \frac{\mu_e H^2 k^2}{4\pi n} (\tau n - 1)(D^2 - k^2)w \\ (\tau n - 1)\{D[(D\mu)(D^2 - k^2)w] - 2k^2(D\mu)(Dw)\} = 0 \end{aligned} \quad (59)$$

In this section we consider the case where two superposed fluids have uniform densities ρ_1 and ρ_2 and uniform viscosities μ_1 and μ_2 . The fluids are separated by horizontal boundary $z = 0$, then in each region of constant viscosity and density, eq. (59) reduces to:

$$(D^2 - k^2)(D^2 - K^2)w = 0 \quad (60)$$

where

$$K^2 = k^2 \frac{n}{v} \left[1 - \frac{mN}{\rho(1 - \tau n)} - \frac{\mu_e H^2 k^2}{4\pi n^2 \rho} \right]$$

Since w must vanish both when $z \rightarrow -\infty$ (in the lower fluid) and $z \rightarrow +\infty$ (in the upper fluid), the general solutions of the eq. (60) appropriate to the regions are:

$$\begin{aligned} w_1 &= A_1 \exp(kz) + B_1 \exp(K_1 z) \quad (z < 0) \\ w_2 &= A_2 \exp(-kz) + B_2 \exp(-K_2 z) \quad (z > 0) \end{aligned} \quad (61)$$

where $A_1, B_1, A_2,$ and B_2 are arbitrary constants and K_1 and K_2 are the positive square roots of eq. (60) for the two regions, respectively.

Integrating eq. (59) across the interface $z = 0$, we have another condition:

$$\begin{aligned} \rho_2 \frac{\mu_2}{n} (D^2 - k^2) Dw_2 \Big|_{z=0} - \rho_1 \frac{\mu_1}{n} (D^2 - k^2) Dw_1 \Big|_{z=0} - \frac{mN}{\tau n} (Dw_2 - Dw_1) \Big|_{z=0} \\ - \frac{\mu_e k_x^2 H^2}{4\pi n^2} (Dw_2 - Dw_1) \Big|_{z=0} - \frac{gk^2}{n^2} (\rho_2 - \rho_1) w_0 - \frac{2k^2}{n} (\mu_2 - \mu_1) (Dw)_0 = 0 \end{aligned} \quad (62)$$

Applying the boundary condition stated above and after eliminating constants $A_1, B_1, A_2,$ and B_2 , we get the following dispersion relation:

$$\begin{aligned} \rho_1 n^2 (\tau n - 1) - n^2 mN - \frac{\mu_e k_x^2 H^2}{4\pi} (\tau n - 1) - \rho_2 n^2 (\tau n - 1) - n^2 mN - \frac{\mu_e k_x^2 H^2}{4\pi} (\tau n - 1) \\ \tau n^3 - n^2 \left[1 - \frac{2mN}{\rho_1 - \rho_2} - \frac{2k^2 \nu \tau}{n} - \frac{gk\tau}{n} \frac{\rho_1 - \rho_2}{\rho_1 - \rho_2} - \frac{2\mu_e k_x^2 H^2}{4\pi(\rho_1 - \rho_2)} - 2k^2 \nu \right] \\ - gk \frac{\rho_1 - \rho_2}{\rho_1 - \rho_2} - \frac{2\mu_e k_x^2 H^2}{4\pi(\rho_1 - \rho_2)} = 0 \end{aligned} \quad (63)$$

Thus eq. (63) gives the dispersion relation for the R-T instability of two superposed fluids of different densities in the presence of horizontal magnetic field, viscosity, and suspended dust particles. In this study we have considered only 1-D perturbation (only in x-direction) of various quantities. In the absence of suspended particles ($\tau = 0$) the dispersion relation agrees with the earlier one obtained by Ogbonna *et al.* [3] when vertical magnetic field is not considered in the dispersion relation.

Now on equating each factor of eq. (63) to zero, we get three dispersion relations:

$$\tau n^3 - n^2 \left[1 - \frac{mN}{\rho_1} - \frac{\mu_e k_x^2 H^2 \tau n}{4\pi \rho_1} - \frac{\mu_e k_x^2 H^2}{4\pi \rho_1} \right] = 0 \quad (64)$$

$$\tau n^3 - n^2 - 1 \frac{mN}{\rho_2} - \frac{\mu_e k_x^2 H^2 \tau n}{4\pi\rho_2} - \frac{\mu_e k_x^2 H^2}{4\pi\rho_2} = 0 \quad (65)$$

$$\tau n^3 - n^2 - 1 \frac{2mN}{\rho_1 - \rho_2} - 2k^2 \nu \tau - n g k \tau \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} - \frac{2\mu_e k_x^2 H^2 \tau}{4\pi(\rho_1 - \rho_2)} - 2k^2 \nu - g k \frac{\rho_1 - \rho_2}{\rho_1 \rho_2} - \frac{2\mu_e k_x^2 H^2}{4\pi(\rho_1 - \rho_2)} = 0 \quad (66)$$

We find that these three dispersion relations given by eqs. (64)-(66) are similar to eqs. (34)-(36) of the case for 3-D perturbations and in the present case we have considered only 2-D perturbation (in y-direction). On comparing dispersion relation (64) and (65), we find that Alfvén velocity has no effect in y-directional perturbation, but it comes into picture when we consider 3-D perturbations or perturbation only in x-direction. In the case of dispersion relation (66), we find that in the 3-D case the wavenumber k is $(k_x^2 + k_y^2)^{1/2}$ and in 2-D case k , is k_y . Thus there is change in k due to the consideration of type of perturbation. Following the previous case for 3-D perturbations, we may obtain condition of instability as well as stability for this case also.

From the above discussion in x-y, y and x-direction perturbation we get some conclusion about the effect of magnetic field on R-T instability. In discussing R-T instability some authors [13, 18, 20] have taken perturbation only in y-direction and they could not get the effect of magnetic field on the R-T condition. If they could have taken the perturbation in x-direction, the effect of magnetic field on the R-T instability could be obtained as in our present case. At the same time if perturbations are in both x-y-directions the condition of R-T instability is influenced by external magnetic field.

Conclusions

In the present paper, we have discussed the R-T instability of two superposed magnetized fluids in the presence of suspended dust particles and viscosity. The magnetic field is assumed in x-direction and gravitational fields is assumed perpendicular to the direction of magnetic field (in z-direction). The condition of R-T instability as well as stability is obtained for 3-D and 2-D perturbation cases. The stability of the system is discussed by applying Routh-Hurwitz criterion. We find that there is no effect of suspended dust particles and viscosity on the condition of R-T instability.

We find that when we consider perturbation in y-direction (*i. e.* perpendicular to the direction of the magnetic field), there is no effect of magnetic field on the dispersion relation as well as condition of R-T instability. In the case, when we consider perturbation in x-direction (*i. e.* parallel to the direction of magnetic field), the condition of R-T instability is identical to that as obtained in 3-D perturbations case. We conclude that the effect of magnetic field is appearing when perturbations are taken along the magnetic field. We conclusively say that if there is either 3-D perturbations or perturbation along the direction of magnetic field, the presence of magnetic field is predominantly in the dispersion relation and condition of R-T instability. We notice that the Alfvén mode is unaffected by the presence of suspended dust particles when we consider perturbation only in y-direction while it modifies in the remaining two cases.

In the graphical presentations we have plotted dimensionless growth rate of unstable R-T mode against dimensionless wavenumber for different values of relaxation frequency of

suspended dust particles, magnetic field, mass concentration of dust particles, viscosity, and density difference of the fluids. It is found that the presence of magnetic field and relaxation frequency of suspended dust particles destabilizes the growth rate of R-T instability. But the effect of kinematic viscosity and mass concentration of suspended dust particles is found to have stabilized the growth rate of R-T instability. The peak value of growth rate of R-T instability does not depend upon kinematic viscosity but it is affected by the presence of magnetic field, relaxation frequency, and mass concentration of suspended dust particles.

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Nomenclature

a	– suspended dust particle radius, [m]	t	– time, [s]
f_s	– relaxation frequency of suspended dust particles, [s^{-1}]	\bar{u}	– fluid velocity, [ms^{-1}]
g	– acceleration due to gravity, [ms^{-2}]	\bar{v}	– suspended dust particle velocity, [ms^{-1}]
\bar{g}	– gravity field, [ms^{-2}]	<i>Greek letters</i>	
H	– magnetic filed, [Am^{-1}]	μ	– dynamic viscosity, [$kgm^{-1}s^{-1}$]
k_x, k_y	– horizontal wavenumbers, [m^{-1}]	μ_c	– magnetic permeability, [Hm^{-1}]
m	– mass of suspended dust particles, [kg]	ν	– kinematic viscosity, [m^2s^{-1}]
N	– number density, [m^{-3}]	ρ	– density, [kgm^{-3}]
n	– growth rate, [s^{-1}]	τ	– relaxation time of suspended dust particles, [s]
p	– fluid pressure, [Pa]		

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