ELASTIC-PLASTIC TRANSITION STRESSES IN A TRANSVERSELY ISOTROPIC THICK-WALLED CYLINDER SUBJECTED TO INTERNAL PRESSURE AND STEADY-STATE TEMPERATURE

by

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Elastic-plastic transitional stresses in a transversely isotropic thick-walled cylinder subjected to internal pressure and steady-state temperature have been derived by using Seth's transition theory. The combined effects of pressure and temperature has been presented graphically and discussed. It has been observed that at room temperature, thick-walled cylinder made of isotropic material yields at a high pressure at the internal surface as compared to cylinder made of transversely isotropic material. With the introduction of thermal effects isotropic/transversely isotropic cylinder yields at a lower pressure whereas cylinder made of isotropic material requires less percentage increase in pressure to become fully-plastic from its initial yielding as compared to cylinder made of transversely isotropic material.

Key words: *cylinder, pressure, temperature, elastic-plastic, transition stress*

Introduction

Thick-walled circular cylinders are used commonly either as pressure vessels intended for storage of gases or as media transportation of high pressurized fluids. The problems of thick-walled cylinder under internal pressure for isotropic material were discussed by many authors [1-4]. In analyzing the problem, these authors used some simplifying assumptions. First, the deformation is small enough to make infinitesimal strain theory applicable. Second, simplifications were made regarding the constitutive equations of a material like incompressibility and yield criterion. Incompressibility is one of the most important assumptions, which simplifies the problem. In fact, in most of the cases, it is not possible to find a solution in closed form without this assumption. Seth's transition theory does not require these assumptions and thus can be used to solve a more general problem. Seth's transition theory utilizes the concept of generalized strain measure and asymptotic solution through the transition points of differential system defining the deformed field and has successfully been applied to a large number of problems in plasticity, [5-17]. Gupta *et al.* [8] solved the problem for transversely isotropic cylinder under internal pressure by using finite strain theory and Seth's transition concept, which not only gives the results as obtained by classical theory as a particular case but also includes the effect of compressibility of the material.

Seth [16] has defined the generalized principal strain measure as:

$$e_{ii} = \int_0^{e_{ii}^A} \left[1 - 2e_{ii}^A\right]^{\frac{n}{2} - 1} \mathrm{d}e_{ii}^A = \frac{1}{n}\left[1 - \left(1 - 2e_{ii}^A\right)^{\frac{n}{2}}\right], \quad (i = 1, 2, 3) \tag{1}$$

where n is the measure and e_{ii}^{A} are the Almansi finite strain components.

In this paper, the problem of elastic-plastic stresses in a transversely isotropic thick-walled cylinder under internal pressure and steady-state temperature is investigated. Results have been presented graphically and discussed.

Governing equations

Consider a thick-walled circular cylinder of internal and external radii a and b respectively, subjected to internal pressure p and temperature Θ_0 applied at the internal surface. The components of displacement in cylindrical polar co-ordinates are given [18] by:

$$u = r(1 - \beta);\ \ v = 0;\ \ w = dz \tag{2}$$

where β is position function, depending on $r = (x^2 + y^2)^{1/2}$ only, and d is a constant.

The finite strain components are given by Seth [16] as:

$$\begin{aligned} &e_{rr}^{A} \quad \frac{1}{2}[1 \quad (\beta \quad r\beta\ )^2] \\ &e_{\theta\theta}^{A} \quad \frac{1}{2}(1 \quad \beta^2) \\ &e_{zz}^{A} \quad \frac{1}{2}[1 \quad (1 \quad d)^2] \\ &e_{r\theta}^{A} \quad e_{\theta z}^{A} \quad e_{zr}^{A} \quad 0 \end{aligned} \tag{3}$$

where $\beta' = \mathrm{d}\beta/\mathrm{d}r$ and meaning of superscripts "A" is Almansi.

By substituting eq. (3) into eq. (1), the generalized components of strain are:

$$\begin{aligned} &e_{rr} \quad \frac{1}{n}[1 \quad (\beta \quad r\beta\ )^n] \\ &e_{\theta\theta} \quad \frac{1}{n}(1 \quad \beta^n) \\ &e_{zz} \quad \frac{1}{n}[1 \quad (1 \quad d)^n] \\ &e_{r\theta} \quad e_{\theta z} \quad e_{zr} \quad 0 \end{aligned} \tag{4}$$

The stress-strain relations for transversely isotropic material are given by [19]:

$$\begin{aligned} &T_{rr} \quad C_{11}e_{rr} \quad (C_{11} \quad 2C_{66})e_{\theta\theta} \quad C_{13}e_{zz} \quad \beta_1\Theta \\ &T_{\theta\theta} \quad (C_{11} \quad 2C_{66})e_{rr} \quad C_{11}e_{\theta\theta} \quad C_{13}e_{zz} \quad \beta_2\Theta \\ &T_{zz} \quad C_{13}e_{rr} \quad C_{13}e_{\theta\theta} \quad C_{33}e_{zz} \quad \beta_2\Theta \\ &T_{zr} \quad T_{\theta z} \quad T_{r\theta} \quad 0 \end{aligned} \tag{5}$$

where $\beta_1 = C_{11}\alpha_1 + 2C_{12}\alpha_2$, $\beta_2 = C_{12}\alpha_1 + (C_{22} + C_{33})\alpha_2$, C_{ij} are elastic parameters = temperature change, α_1 is the coefficient of linear thermal expansion along the axis of symmetry, and α_2 – the corresponding quantities orthogonal to axis of symmetry.

By substituting eqs. (4) into eqs. (5), one gets:

$$T_{rr} \quad \frac{C_{11}}{n}[1 \quad (\beta \quad r\beta\ )^n] \quad \frac{C_{11} \quad 2C_{66}}{n}(1 \quad \beta^n) \quad C_{13}e_{zz} \quad \beta_1\Theta$$

$$T_{\theta\theta} \quad \frac{C_{11} \quad 2C_{66}}{n} [1 \quad (\beta \quad r\beta\,)^n] \quad \frac{C_{11}}{n}(1 \quad \beta^n) \quad C_{13}e_{zz} \quad \beta_2\Theta$$

$$T_{zz} \quad \frac{C_{13}}{n}[1 \quad (\beta \quad r\beta\,)^n] \quad \frac{C_{13}}{n}(1 \quad \beta^n) \quad C_{33}e_{zz} \quad \beta_2\Theta$$

$$T_{r\theta} \quad T_{\theta z} \quad T_{zr} \quad 0 \tag{6}$$

Equations of equilibrium are all satisfied except:

$$\frac{\mathrm{d}}{\mathrm{d}r}(T_{rr}) \quad \frac{T_{rr} \quad T_{\theta\theta}}{r} \quad 0 \tag{7}$$

The temperature field satisfying Fourier heat equation $\;^2\Theta$ and

$$\Theta = \Theta_0 \quad \text{at} \quad r = a,$$

$$\Theta = 0 \quad \text{at} \quad r = b,$$

where Θ_0 is constant, given by [19]:

$$\Theta \quad \frac{\Theta_0 \log\dfrac{r}{b}}{\log\dfrac{a}{b}} \tag{8}$$

By substituting eqs. (6) and (8) into eq. (7), one gets a non-linear differential equation with respect to β:

$$nPC_{11}\beta^{n\ 1}(1 \quad P)^{n\ 1}\frac{\mathrm{d}P}{\mathrm{d}\beta} \quad nPC_{11}\beta^n(1 \quad P)^n \quad (C_{11} \quad 2C_{66})nP\beta^n \quad 2C_{66}[1 \quad \beta^n(1 \quad P)^n]$$

$$2C_{66}(1 \quad \beta^n) \quad n\beta_1\overline{\Theta}_0 \quad (\beta_2 \quad \beta_1)n\log\frac{r}{b}\overline{\Theta}_0 \tag{9}$$

where $r\beta' = \beta P$ and $\overline{\Theta}_0 = (\Theta_0)/\log(a/b)$ and $r\beta' = \beta P$ (P is function of β and β is function of r).

The transition points of β in eq. (9) are P $\;\;-1$ and P $\;\;\infty$.The boundary conditions are given by:

$$\begin{aligned} T_{rr} &\quad -p \ \text{ at } \ r \quad a \\ T_{rr} &\quad 0 \ \text{ at } \ r \quad b \end{aligned} \tag{10}$$

The resultant force normally applied to the ends of cylinder is:

$$2\pi \int_a^b rT_{zz}\,\mathrm{d}r \quad \pi a^2 p \tag{11}$$

Solution through the principal stress

It has been shown that the asymptotic solution through the principal stress [5-17] leads from elastic to plastic state at the transition point . If the transition function is defined as:

$$R \quad 2(C_{11} - C_{66}) + nC_{13}e_{zz} - nT_{rr} - n\beta_1\Theta = \beta^n[C_{11} - 2C_{66} + C_{11}(1 + P)^n] \tag{12}$$

and by taking the logarithmic differentiation of eq. (12) with respect to r, one gets:

$$\frac{\mathrm{d}}{\mathrm{d}r}(\log R) \quad \frac{n\beta^{n} P[C_{11} \quad 2C_{66} \quad C_{11}(1 \quad P)^{n}] \quad nC_{11}P(1 \quad P)^{n \quad 1}\beta^{n \quad 1}\dfrac{\mathrm{d}P}{\mathrm{d}\beta}}{r\beta^{n}[C_{11} \quad 2C_{66} \quad C_{11}(1 \quad P)^{n}]} \tag{13}$$

By substituting the value of dP/dβ from eq. (9) into eq. (13), one gets:

$$\frac{\mathrm{d}}{\mathrm{d}r}(\log R) \quad \frac{2C_{66}\beta^{n}[1 \quad (1 \quad P)^{n}] \quad n\overline{\Theta}_{0}[\beta_{1} \quad (\beta_{1}\beta_{2})\log\dfrac{r}{b}}{r\beta^{n}[C_{11} \quad 2C_{66} \quad C_{11}(1 \quad P)^{n}]} \tag{14}$$

Asymptotic value of eq. (14) as P $\quad\infty$ is:

$$\frac{\mathrm{d}}{\mathrm{d}r}(\log R) \quad \frac{2C_{66}}{rC_{11}} \tag{15}$$

By integrating eq. (15), one gets:

$$R \quad K_{1} r^{\quad C_{1}} \tag{16}$$

where K_1 is a constant of integration, which can be determined by the boundary condition and $C_1 = 2C_{66}/C_{11}$. By substituting eq. (16) into eq. (12), one gets:

$$T_{rr} \quad C_{3} \quad \beta_{0}\log\frac{r}{b} \quad \frac{K_{1}}{n} r^{\quad C_{1}} \tag{17}$$

where $C_3 = [2(C_{11} - C_{66}) + nc_{13}e_{zz}]/n$ and $\beta_0 = \beta_1\overline{\beta}_0$.

By applying boundary conditions (10) in eq. (17), one gets:

$$K_{1} \quad nb^{C_{1}} \frac{p \quad \beta_{0}\log\dfrac{a}{b}}{\left(\dfrac{b}{a}\right)^{C_{1}} \quad 1} \quad \text{and} \quad C_{3} \quad \frac{p \quad \beta_{0}\log\dfrac{a}{b}}{\left(\dfrac{b}{a}\right)^{C_{1}} \quad 1} \tag{18}$$

By substituting the value of K_1 and C_3 into eq. (17), one gets:

$$T_{rr} \quad \frac{p \quad \beta_{0}\log\dfrac{a}{b}}{\left(\dfrac{b}{a}\right)^{C_{1}} \quad 1}\, 1 \quad \left(\frac{b}{r}\right)^{C_{1}} \quad \beta_{0}\log\frac{r}{b} \tag{19}$$

By substituting eq. (19) in eq. (7), one gets:

$$T_{\theta\theta} \quad \frac{p \quad \beta_{0}\log\dfrac{a}{b}}{\left(\dfrac{b}{a}\right)^{C_{1}} \quad 1}\, 1 \quad (1 \quad C_{1})\left(\frac{b}{r}\right)^{C_{1}} \quad \beta_{0}\; 1 \quad \log\frac{r}{b} \tag{20}$$

The axial stress is obtained from eq. (6) as:

$$T_{zz} \quad \frac{C_{13}}{2(C_{11} \quad C_{66})}(T_{rr} \quad T_{\theta\theta}) \quad \frac{C_{33}(C_{11} \quad C_{66}) \quad C_{13}^{2}}{C_{11} \quad C_{66}} e_{zz} \\ \frac{\Theta}{2(C_{11} \quad C_{66})}[C_{13}(\beta_{1} \quad \beta_{2}) \quad 2\beta_{2}(C_{11} \quad C_{66})] \tag{21}$$

By applying the condition (11) into eq. (21), the axial strain is given by:

$$e_{zz} \quad \frac{C_{11} \quad C_{66}}{C_{33}(C_{11} \quad C_{66}) \quad C_{13}^{2}} \quad \frac{a^{2}p}{b^{2} \quad a^{2}} \quad \frac{a^{2}C_{13} \quad p \quad \beta_{0}\log\frac{a}{b}}{b^{2} \quad a^{2}} \quad \frac{\beta_{0}C_{13}a^{2}\log\frac{b}{a}}{(C_{11} \quad C_{66})(b^{2} \quad a^{2})} \quad \frac{\beta_{0}}{2}C_{13} \quad 1 \quad \frac{\beta_{2}}{\beta_{1}} \quad \frac{2\beta_{2}}{\beta_{1}}(C_{11} \quad C_{66}) \quad \frac{a^{2}\log\frac{b}{a}}{b^{2} \quad a^{2}} \quad \frac{1}{2} \tag{22}$$

By substituting eq. (22) into eq. (21), one gets:

$$T_{zz} \quad \frac{C_{13}}{C_{11}(2 \quad C_{2})} \quad \frac{p \quad \beta_{0}\log\frac{a}{b}}{\frac{b}{a}^{C_{1}} \quad 1} \quad 2 \quad (2 \quad C_{1}) \quad \frac{b}{r}^{C_{1}} \quad \frac{a^{2}p}{b^{2} \quad a^{2}} \quad \frac{C_{13}\beta_{0}}{C_{11}(2 \quad C_{2})} \quad 1 \quad 2\log\frac{r}{b} \quad \frac{2a^{2}C_{13}p}{C_{11}(2 \quad C_{2})(b^{2} \quad a^{2})} \quad \frac{\beta_{0}}{C_{11}(2 \quad C_{2})} \quad C_{13} \quad 1 \quad \frac{\beta_{2}}{\beta_{1}} \quad \frac{2\beta_{2}}{\beta_{1}}(C_{11} \quad C_{66}) \quad \frac{a^{2}\log\frac{a}{b}}{b^{2} \quad a^{2}} \quad \frac{1}{2} \quad \log\frac{r}{b} \tag{23}$$

From eq. (19) and (20), one gets:

$$T_{\theta\theta} \quad T_{rr} \quad \frac{p \quad \beta_{0}\log\frac{a}{b}}{\frac{b}{a}^{C_{1}} \quad 1} \quad C_{1} \quad \frac{b}{r}^{C_{1}} \quad \beta_{0} \tag{24}$$

Initial yielding

It is found that the value of $|T_{\theta\theta} \quad T_{rr}|$ is maximum at $r = a$, which means that yielding of the cylinder will take place at the internal surface. Therefore:

$$|T_{\theta\theta} \quad T_{rr}|_{r \quad a} \quad \left| \frac{p \quad \beta_{0}\log\frac{a}{b}}{\frac{b}{a}^{C_{1}} \quad 1} \quad C_{1} \quad \frac{b}{a}^{C_{1}} \quad \beta_{0} \right| \quad Y \tag{25}$$

where Y is the yielding stress. The relation between pressure and temperature for initial yielding is given by:

$$P_1 \quad \frac{p}{Y} \quad \frac{1 \quad \frac{\beta_0}{Y}}{C_1 \quad {\frac{b}{a}}^{C_1}} \quad {\frac{b}{c}}^{C_1} \quad 1 \quad \frac{\beta_0}{Y}\log\frac{a}{b} \tag{26}$$

By substituting eq. (26) into eqs. (19-21), one gets the transitional stresses as:

$$\sigma_r \quad \frac{T_{rr}}{Y} \quad \frac{P_1 \quad \frac{\beta_0}{Y}\log\frac{a}{b}}{{\frac{b}{a}}^{C_1} \quad 1} \quad 1 \quad {\frac{b}{r}}^{C_1} \quad \frac{\beta_0}{Y}\log\frac{r}{b}$$

$$\sigma_\theta \quad \frac{T_{\theta\theta}}{Y} \quad \frac{P_1 \quad \frac{\beta_0}{Y}\log\frac{a}{b}}{{\frac{b}{a}}^{C_1} \quad 1} \quad 1 \quad (1 \quad C_1) \quad {\frac{b}{r}}^{C_1} \quad \frac{\beta_0}{Y} \quad 1 \quad \log\frac{r}{b}$$

$$\sigma_z \quad \frac{T_{zz}}{Y} \quad \frac{C_{13}}{C_{11}(2 \quad C_1)} \quad \frac{P_1 \quad \frac{\beta_0}{Y}\log\frac{a}{b}}{{\frac{b}{a}}^{C_1} \quad 1} \quad 2 \quad (2 \quad C_1) \quad {\frac{b}{r}}^{C_1} \quad \frac{a^2 p}{b^2 \quad a^2}$$

$$\frac{C_{13}\frac{\beta_0}{Y}}{C_{11}(2 \quad C_1)} \quad 1 \quad 2\log\frac{r}{b} \quad \frac{2a^2 C_{13} P_1}{C_{11}(2 \quad C_1)(b^2 \quad a^2)} \tag{27}$$

$$\frac{\frac{\beta_0}{Y}}{C_{11}(2 \quad C_1)} \quad c_{13} \quad 1 \quad \frac{\beta_2}{\beta_1} \quad 2\frac{\beta_2}{\beta_1}(C_{11} \quad C_{66}) \quad \frac{a^2\log\frac{b}{a}}{b^2 \quad a^2} \quad \frac{1}{2} \quad \log\frac{r}{b}$$

where $P_1 = p/Y$ and $\beta_0/Y = \beta_1\overline{\Theta}_0/Y$.

Equations (27) define elastic-plastic transitional stresses in a thick-walled cylinder under internal pressure and temperature Θ_0.

For fully plastic state (C_2 0), eq. (24) becomes:

$$\left|T_{\theta\theta} \quad T_{rr}\right|_{r \quad b} \quad \left|\frac{p \quad \beta_0\log\frac{a}{b}}{\log\frac{b}{a}} \quad \beta_0 \quad Y^*(\text{say})\right| \tag{28}$$

From eq. (28), one gets:

$$p \quad Y^*\log\frac{b}{a} \tag{29}$$

By substituting eq. (29) into eqs. (19-21), one gets stresses for fully plastic state:

$$\sigma_r^* \quad \frac{T_{rr}}{Y^*} \quad \log\frac{b}{r}$$

$$\sigma_\theta^* \quad \frac{T_{\theta\theta}}{Y^*} \quad 1 \quad \log\frac{b}{r}$$

$$\sigma_z^* \quad \frac{T_{zz}}{Y^*} \quad \frac{C_{13}}{2C_{11}} \quad 1 \quad 2\log\frac{r}{b} \quad \frac{a^2\log\frac{b}{a}}{b^2 \quad a^2} \quad 1 \quad \frac{C_{13}}{C_{11}}$$

$$\frac{\beta_0}{2Y^*}\,\frac{C_{13}}{C_{11}} \quad 1 \quad \frac{\beta_2}{\beta_1} \quad 2\frac{\beta_2}{\beta_1}\,\frac{a^2\log\frac{b}{a}}{b^2 \quad a^2} \quad \frac{1}{2} \quad \log\frac{r}{b} \tag{30}$$

Isotropic case: For isotropic materials, the material constants reduce to two only [21], *i. e.* $C_{11}=C_{22}=C_{33}$, $C_{12}=C_{21}=C_{13}=C_{31}=C_{23}=C_{32}=C_{11}-2C_{66}$, and $\alpha_1=\alpha_2=\alpha_3=\alpha$. In term of constants λ and μ, these can be written as:

$$C_{12} \quad \lambda, \quad C_{66} \quad \frac{1}{2}(C_{11} \quad C_{12}) \quad \mu \text{ and } C_{11} \quad \lambda \quad 2\mu \tag{31}$$

For isotropic materials eqs. (19-21) become:

$$T_{rr} \quad \frac{p \quad \beta_0\log\frac{a}{b}}{{\frac{b}{a}}^{c} \quad 1} \quad 1 \quad {\frac{b}{r}}^{c} \quad \beta_0\log\frac{r}{b}$$

$$T_{\theta\theta} \quad \frac{p \quad \beta_0\log\frac{a}{b}}{{\frac{b}{a}}^{c} \quad 1} \quad 1 \quad (1 \quad c)\,{\frac{b}{r}}^{c} \quad \beta_0 \quad 1 \quad \log\frac{r}{b}$$

$$T_{zz} \quad \frac{1 \quad c}{2 \quad c}\,\frac{p \quad \beta_0\log\frac{a}{b}}{{\frac{b}{a}}^{c} \quad 1} \quad 2 \quad (2 \quad c)\,{\frac{b}{r}}^{c}$$

$$\frac{a^2 p}{b^2 \quad a^2} \quad 1 \quad \frac{2(1 \quad c)}{2 \quad c} \quad \frac{1 \quad c}{2 \quad c}\beta_0 \quad 1 \quad 2\log\frac{r}{b} \tag{32}$$

$$\frac{\beta_0}{2 \quad c}\,(1 \quad c)\;1 \quad \frac{\beta_2}{\beta_1} \quad \frac{\beta_2}{\beta 1}(2 \quad c) \quad \frac{a^2\log\frac{b}{a}}{b^2 \quad a^2} \quad \frac{1}{2} \quad \log\frac{r}{b}$$

where $c = 2\mu/(1+2\mu)$.

From eq. (32), one gets:

$$T_{\theta\theta} \quad T_{rr} \quad \frac{p \quad \beta_0 \log \dfrac{a}{b}}{\dfrac{b}{a}^{c} \quad 1} \; c \; \frac{b}{r}^{c} \quad \beta_0 \tag{33}$$

It is found that the value of $\left| T_{\theta\theta} \quad T_{rr} \right|$ is maximum at $r = a$, which means that yielding of the cylinder will take place at the internal surface. Therefore:

$$\left| T_{\theta\theta} \quad T_{rr} \right|_{r \;\; a} \quad \left| \frac{p \quad \beta_0 \log \dfrac{a}{b}}{\dfrac{b}{a}^{c_2} \quad 1} \; c \; \frac{b}{a}^{c} \quad \beta_0 \right| \quad Y \,(\text{say}) \tag{34}$$

The relation between pressure and temperature for initial yielding is given by:

$$P_1 \quad \frac{p}{Y} \quad \frac{1 \quad \dfrac{\beta_0}{Y}}{c \quad \dfrac{b}{a}^{c}} \quad \frac{b}{a}^{c} \quad 1 \quad \frac{\beta_0}{Y} \log \frac{a}{b} \tag{35}$$

By substituting eq. (35) into eq. (32), one gets the transitional stresses:

$$\sigma_r \quad \frac{T_{rr}}{Y} \quad \frac{P_1 \quad \dfrac{\beta_0}{Y} \log \dfrac{a}{b}}{\dfrac{b}{a}^{c} \quad 1} \quad 1 \quad \frac{b}{r}^{c} \quad \frac{\beta_0}{Y} \log \frac{r}{b}$$

$$\sigma_\theta \quad \frac{T_{\theta\theta}}{Y} \quad \frac{P_1 \quad \dfrac{\beta_0}{Y} \log \dfrac{a}{b}}{\dfrac{b}{a}^{c} \quad 1} \quad 1 \quad (1 \quad c) \frac{b}{r}^{c} \quad \frac{\beta_0}{Y} \; 1 \quad \log \frac{r}{b}$$

$$\sigma_z \quad \frac{T_{zz}}{Y} \quad \frac{1 \quad c}{2 \quad c} \; \frac{P_1 \quad \dfrac{\beta_0}{Y} \log \dfrac{a}{b}}{\dfrac{b}{c}^{c} \quad 1} \quad 2 \quad (2 \quad c) \frac{b}{r}^{c} \quad \frac{a^2 P_1}{b^2 \quad a^2} \; 1 \quad \frac{1 \quad c}{2 \quad c}$$
$$\frac{(1 \quad c)\dfrac{\beta_0}{Y}}{2 \quad c} \; 1 \quad 2 \log \frac{r}{b} \quad \frac{c \dfrac{\beta_0}{Y}}{2 \quad c} \; \frac{a^2 \log \dfrac{b}{a}}{b^2 \quad a^2} \quad \frac{1}{2} \quad \log \frac{r}{b} \tag{36}$$

For fully plastic state (c   0), eq. (33) becomes:

$$\left|T_{\theta\theta} \quad T_{rr}\right|_{r \quad b} \quad \left| \frac{p \quad \beta_0 \log \dfrac{a}{b}}{\log \dfrac{b}{a}} \quad \beta_0 \right| \quad Y^* \,(\text{say}) \tag{37}$$

By substituting eq. (37) into eq. (32), one gets stresses and pressure for fully plastic state (c   0):

$$\begin{aligned} &\frac{T_{rr}}{Y^*} \quad \log\frac{b}{r} \\ &\frac{T_{\theta\theta}}{Y^*} \quad 1 \quad \log\frac{b}{r} \\ &\frac{T_{zz}}{Y^*} \quad \frac{1}{2} \quad \log\frac{b}{r} \\ &p \quad Y^* \log\frac{b}{a} \end{aligned} \tag{38}$$

These equations are the same as obtained by Nadia [16] and Hill [20].

Results and discussion

As an numerical illustration, the values of pressure P required for initial yielding P_i and fully plastic state P_f at different temperature has been given in tab. 1 and fig. 1. It can be seen that at the room temperature thick-walled cylinder made of isotropic material having thickness ratio $b/a = 4$ yields at the internal surface at high pressure as compared to cylinder made of transversely isotropic material where as thick-walled cylinder having smaller thickness ratio yields at a lower pressure. With the introduction of thermal effects, cylinders made of both isotropic transversely isotropic material yields at a lower pressure. From tab.1, it has been observed that a thick-walled cylinder made of transversely isotropic material requires larger increase in pressure to become fully-plastic from its initial yielding, proportional to the increase in temperature ratio β_0/Y. This increase in pressure increases with the increase in temperature and thickness ratio, as well. It means that at room temperature, thick- -walled cylinder made of isotropic mate-

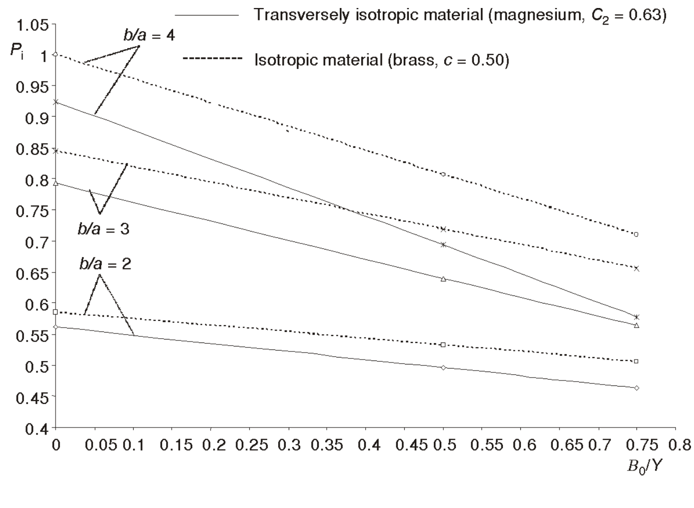

Figure 1. Pressure required for initial yielding of thick-walled cylinder at different temperature

Table 1. The pressure required for initial yielding (P_i) and fully plastic state (P_f) at different temperatures (β_0/Y)

b/a	P	Transversely isotropic material (C_2 = 0.63, magnesium)					
		$\frac{\beta_0}{Y}$ 0	$\frac{\beta_0}{Y}$ 0.5	$\frac{\beta_0}{Y}$ 0.75	$P \quad \frac{P_f \quad P_i}{P_i} 100\,[\%]$ increase in pressure required from initial yielding to fully-plastic state at different temperatures		
					$\frac{\beta_0}{Y}$ 0	$\frac{\beta_0}{Y}$ 0.5	$\frac{\beta_0}{Y}$ 0.75
2	P_i	0.5616	0.4958	0.4629	23.418	39.785	49.713
	P_f	0.6931	0.6931	0.6931			
3	P_i	0.7928	0.6399	0.5635	38.567	71.671	94.459
	P_f	1.098	1.098	1.098			
4	P_i	0.9245	0.6937	0.5782	49.945	99.854	139.76
		Isotropic material (σ = 0.33/C = 0.50, brass)					
2	P_i	0.5857	0.5321	0.5053	19.116	32.477	40.581
	P_f	0.6931	0.6931	0.6931			
3	P_i	0.8453	0.7186	0.6553	31.950	59.375	78.667
	P_f	1.098	1.098	1.098			
4	P_i	1.000	0.8069	0.7103	41.783	83.535	116.92

rial is to withstand a greater pressure to initiate yielding at the internal surface as compared to thick- -walled cylinder made of transversely isotropic material and with the introduction of thermal effect, they yield at a lower pressure whereas cylinder made of isotropic material requires smaller increase in pressure to become fully-plastic from its initial yielding. In figs. 2 and 3,

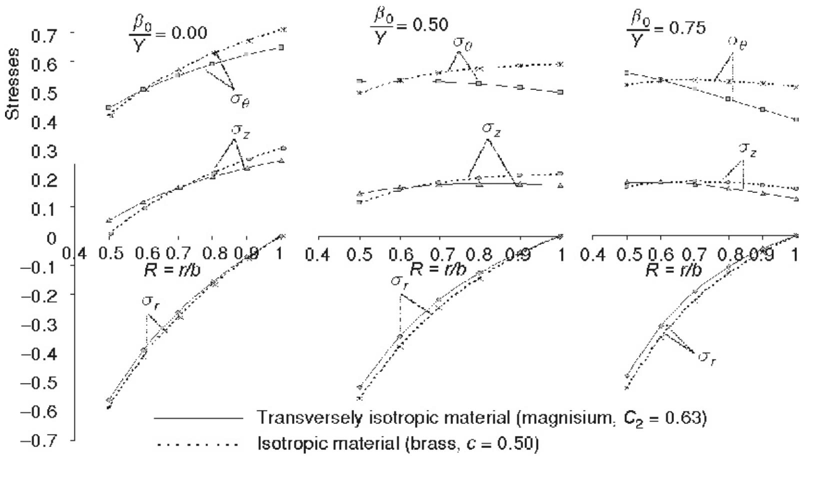

Figure 2. Elastic-plastic transitional stresses for a thick-walled cylinder under internal pressure at different temperatures

elastic-plastic transitional stresses and stresses for fully-plastic state have been drawn with radii ratio $R = r/b$. It has been observed from fig. 3 that for fully-plastic state, radial and circumferential stresses are independent of thermal effects.

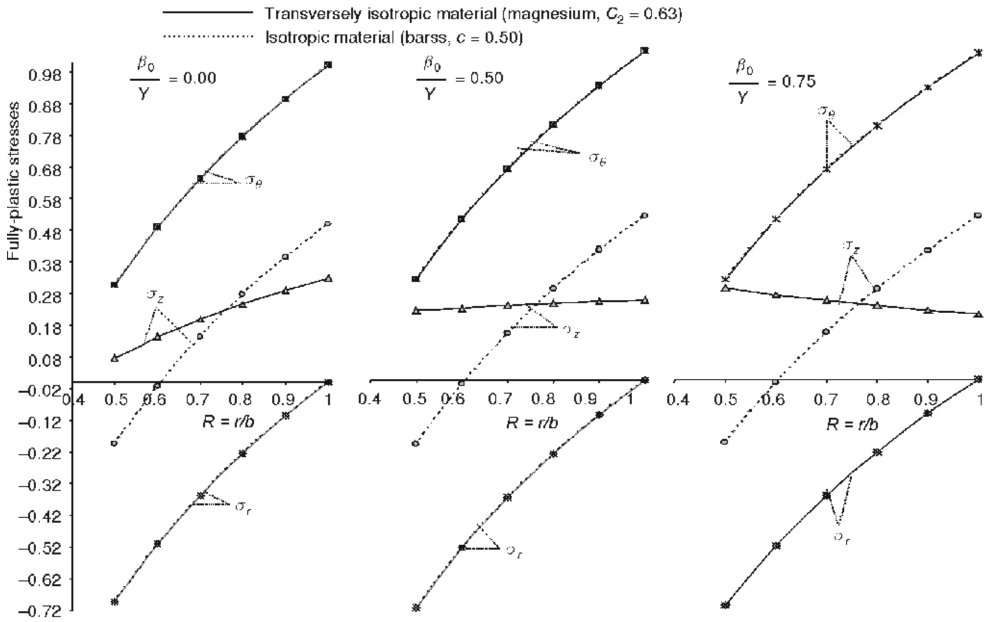

Figure 3. Fully plastic stresses for a thick walled cylinder under internal pressure at different temperatures

Conclusions

It has been observed that at room temperature, thick-walled cylinder made of isotropic material yields at a high pressure at the internal surface as compared to cylinder made of transversely isotropic material whereas thick-walled cylinder having smaller thickness ratio yields at a lower pressure. With the introduction of thermal effects isotropic/transversely isotropic cylinder yields at a lower pressure whereas cylinder made of isotropic material requires less percentage increase in pressure to become fully-plastic from its initial yielding as compared to cylinder made of transversely isotropic material. Radial and circumferential stresses are independent of thermal effects for fully-plastic state.

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Nomenclature

- *a, b* – internal and external radii of the circular cylinder, [m]
- *c* – compressibility factor, [–]
- K_1 – constants of integration, [–]
- *p* – internal pressure [Pa]
- *R* – radii ratio (= *r/b*), [–]

R_o – radii ratio (= *a/b*), [–]
T_{ij}, e_{ij} – stress strain rate tensors, [kgm^{-1}s^{-2}]
u, v, w – displacement components, [m]
Y – yield stress, [kg]

Greek letters

Θ, θ – temperature, [K]
σ_r – radial stress component (= T_{rr}/Y), [–]
σ_θ – circumferential stress component (=$T_{\theta\theta}/Y$), [–]
σ_z – axial stress component (=T_{zz}/Y),

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