HEAT EXCHANGER OPERATING POINT DETERMINATION

by

Dušan D. GVOZDENAC

Original scientific paper
UDC: 66.045.1:66.011
DOI: 10.2298/TSCI0904151G

This paper indicates 21 possible tasks for the calculation of heat exchangers and specifies in particular the procedure for determining heat exchanger operating point. Features of heat exchanger energy micro-balance are contained in its mathematical model, and features of its macro-balance hold in relations for heat flow rate. Operating point of heat exchanger is defined by satisfying micro and macro balances. The paper presents basic relations for determining operating points for some types of tasks and algorithms of certain procedures. A special case in which two, one or none non-trivial solutions appear within two of 21 tasks is analyzed and discussed separately. Presented procedures are very suitable for the preparation of own software for the calculation of operating parameters of any heat exchanger and analysis of heat exchangers network.

Key words: *heat exchanger, operating point*

Introduction

Within traditional assumptions for the calculation of heat exchangers [1], a series of task may occur depending on primary data which are known for a given heat exchanger.

Thus, for example, Bosnjakovic [2] classifies basic tasks of heat exchangers calculations into seven groups whereas, Novikov *et al.* [3] and Levin *et al*. [4] indicate that totally 21 problems can be grouped into six types of tasks. It is natural to keep the division of totally 21 tasks as it directly follows from possible combinations of two unknowns each in a set of seven starting values relevant for determining the operating point. Gvozdenac *et al*. [5], Bačlić [6], and Shah *et al.* [7] specified review of all known 21 possible problems. Also, they precisely define traditional assumptions for the calculation of heat exchanger and procedurally indicate specificities of calculations for certain tasks.

Although there are many papers which deal with calculations for various types of heat exchangers, only few systematically analyze all possible types of problems which can occur in practice. This paper provides systematic approach for resolving all 21 defined types of tasks. Two of all these tasks are particularly analyzed. Those are tasks in which unknowns are *fluid stream heat capacity rate of one fluid stream* and *inlet temperature of the other fluid stream*. In these cases, two non-trivial solutions, one or none for given data can appear. This behavior of a heat exchanger occurs only in case of those heat exchangers which have in their flow arrangement counter or cross flow component, irrespective of the fact how complicated they are. In case of pure parallel flow there are no two non-trivial solutions.

The software package based on procedures presented in this paper is given in [8].

Problem definition

At the beginning of the analysis, it is natural to distinguish *hot* and *cold* fluid streams which are designated with (h) and (c), respectively.

The first law of thermodynamic is to be satisfied in any exchanger design procedure both at *macro* and *micro* levels. The overall energy balance for any *two-fluid heat exchanger* is in explicit form as [5, 6]:

$$m_{h}c_{ph}(T_{h\,in} - T_{h\,out}) = m_{c}c_{pc}(T_{c\,out} - T_{c\,in}) \tag{1}$$

This equation certainly satisfies the macro energy balance under the assumptions usual for the basic design theory of heat exchangers [1]. The main assumptions assume that overall heat transfer coefficient (U) and isobaric specific heat of fluids (c_p) are constant. Next important assumptions are that heat losses are negligible and flow rates of fluids are constant. However, it is often not very obvious that the *ε–NTU–ω* relation in general form:

$$\varepsilon = \mathrm{f}(NTU, \omega, \text{flow arrangement}) \tag{2}$$

is the statement expressing the micro energy balance for the particular two-fluid heat exchanger under the same assumptions. This particularity is due to the uniqueness of the solution of the governing differential equations and boundary conditions for a particular flow arrangement. These differential equations, describing the fluid-temperature fields in the heat exchanger core are the statements of micro energy balances for an arbitrary differential control volume of that particular core. The boundary conditions specify where the fluids at temperatures $T_{h\,in}$ and $T_{c\,in}$ enter the core in a particular flow arrangement. The solution of such a mathematical model, which introduces the overall heat transfer coefficient U and the total heat transfer surface A, gathered in the overall conductance UA, enables the evaluation of both fluid outlet temperatures ($T_{h\,out}$ and $T_{c\,out}$) for the particular flow arrangement. Due to the simplifying classical assumptions underlying the theory, the mathematical model is linear and tractable by available methods of calculus. This means that the effectiveness relationship of eq. (2) can be derived for any heat exchanger no matter how complicated the flow arrangement is. This fact makes the ε–NTU–ω method universal. This method will be used consequently in this paper. The intention of this paper is not to polemize about other methods used [7].

The second important feature of the ε–NTU–ω method is the thermodynamic significance of the dimensionless groups appearing in the analysis. They are: (1) heat capacity rate ratio, (2) number of transfer units, and (3) heat exchanger effectiveness.

The fluid heat capacity rate ratio is defined as:

$$\omega \quad \frac{W_{min.}}{W_{max.}} \tag{3}$$

In this equation, $W_{min.}$ means lower heat capacity of two streams ($W_{min.}$ = min.(W_h, W_c). Simply, the ratio of smaller and larger heat capacity rates for two fluid streams is in closed range [0, 1] and represents the dimensionless group suitable for understanding overall fluid temperature changes. The condition $\omega = 0$ indicates the tendency of the strong stream towards the isothermal change, while $\omega = 1$, the trend of each stream to undergo the same temperature change from the exchanger's inlet to outlet (balanced streams).

Similarly, thermodynamic reasoning can be associated with the second dimensionless group, the number of heat transfer units:

$$NTU \quad \frac{UA}{W_{min.}} \tag{4}$$

It is the ratio of overall conductance *UA* and smaller heat capacity rate ($W_{min.}$). The range $0 \le NTU < \infty$ in practice has finite upper limit, but thermodynamically speaking, the higher *NTU* (higher overall conductance and smaller $W_{min.}$) the smaller local temperature differences across the heat transfer surface area and consequently lower irreversibility. This means that better heat exchanger flow arrangements must have higher monotonically increasing effectiveness with *NTU*.

The effectiveness (ε) of any two-fluid heat exchanger is essentially dimensionless measure of the heat quantity which is actually transferred between two streams normalized with maximum possible fluid enthalpy change in the system. This hypothetical quantity of heat can be seen as the enthalpy change of the weak stream (stream with lower heat capacity) undergoing the maximum possible temperature change ($T_{h\,in} - T_{c\,in}$) without any losses. The heat exchanger effectiveness is then simply defined as:

$$\varepsilon = \frac{Q_{act}}{Q_{max.}} = \frac{W_h(T_{h\,in} - T_{h\,out})}{W_{min.}(T_{h\,in} - T_{c\,in})} = \frac{W_c(T_{c\,out} - T_{c\,in})}{W_{min.}(T_{h\,in} - T_{c\,in})} \tag{5}$$

and it is a unique measure of its thermal performance. Uniqueness in this context means that the same effectiveness is obtained by writing Q_{act} either in terms of hot fluid parameters or in terms of cold fluid parameters.

The effectiveness is to be obtained from the solution of the mathematical model mentioned above and will, thus, depend on two dimensionless groups which are the heat exchanger parameters *NTU* and ω.

The operating point of an exchanger is the set of ε, *NTU*, and ω values that identically satisfy both its macro and micro energy balance. The flow arrangement as an argument of the ε–NTU–ω relation makes the heat exchanger operating point unique for the particular flow arrangement. Different flow arrangement has different operating points even for the same values of two arbitrarily chosen out of three corresponding parameters (ε, *NTU*, and ω). If this is not the case, the flow arrangements are said to be equivalent.

In practice, a designer is faced with the problem of seven physical entities (for a specific flow arrangement and for $0 < \omega \le 1$) that have to satisfy just two equations, namely eqs. (1) and (2). These equations state an unambiguous relation of the type:

$$f(T_{h\,in}, T_{h\,out}, T_{c\,in}, T_{c\,out}, UA, (mc_p)_h, (mc_p)_c, \text{flow arrangement}) = 0 \tag{6}$$

For an arbitrary, but specified flow arrangement, any five of seven variables must be known for heat exchanger operating point determination. Depending on the combination of two unknowns that have to be determined in order to satisfy eqs. (1) and (2), there are $21 = \binom{7}{5}$ possible problems for determining heat exchanger operating point. They are shown in tab. 1 classified in six groups.

It can be stated that data on mass flow rates and fluid types are included in $W_i = (mc_p)_i$ ($i = h, c$) or so cold *strongness* of fluid streams. Units of these heat capacities are the same as for *UA* [WK^{-1}]. Dimensionless heat exchanger groups: *NTU* and ω are combinations of these dimension values. As overall heat transfer coefficient (*U*) can be defined independently of the size of heat transfer surface area (*A*), complex *UA* has not to be divided into constituents. But, complexes W_i have different nature. Known W_i assumes that both mass flow rate (m_i) and isobaric specific heat of fluid ($c_{p\,i}$) are known. If one of these two values is not known, this means that heat capacity is not known. In this paper we are using only fluid heat capacity (W_i).

By defining all seven basic heat exchanger parameters according to the energy balance, it is possible to define the heat exchanger operating point (HEOP).

Table 1. Twenty-one problems to determine the heat exchanger operating point

	Group no.	Problem no.	UA [WK^{-1}]	$m_h c_{ph}$ [WK^{-1}]	$m_c c_{pc}$ [WK^{-1}]	$T_{h\ in}$ [°C]	$T_{h\ out}$ [°C]	$T_{c\ in}$ [°C]	$T_{c\ out}$ [°C]
Sizing proglems	I	1	???			???			
		2	???				???		
		3	???					???	
		4	???						???
	II	5	???	???					
		6	???		???				
Rating problems	III	7		???	???				
	IV	8				???	???		
		9				???		???	
		10				???			???
		11					???	???	
		12					???		???
		13						???	???
Regime problems	V	14		???		???			
		15		???			???		
		16			???			???	
		17			???				???
	VI	18			???	???			
		19			???		???		
		20		???				???	
		21		???					???

The *sizing* problems in groups I and II, and the *rating* problems in groups III and IV can readily be recognized. However, the problems in groups V and VI may be termed as the *regime* problems [5-7]. They are most difficult to solve because there is no possibility to identify fluid streams according to their relative strongness and eqs. (1) and (2) must be treated and resolved simultaneously based upon a guess made for the $W_{min.}$ stream. Also, problems 14-17, 19, and 21 always have one solution, but problems 18 and 20 have two or one non-trivial solutions or none.

It should be noted that in all tasks (groups V and VI), except tasks 18 and 20, it is possible to determine heat flow rate *without* determination of two unknown variables. Practically, this means that in all tasks, except in tasks (18) and (20), the heat flow rate is also given and the problem of determining operating point is reduced to one unknown value. *The tasks 18 and 20 are specific because two independent equations have to be resolved simultaneously and therefore, it is possible to obtain two, one, or none solutions.*

In the case when $\omega = 0$, there are two special types of heat exchangers named condensers and evaporators. The remaining are only five variables and known flow arrangement and total number of tasks is, thus, equal to $10 = {5 \atop 3}$. Five of them belong to condensers and five to evaporators. Effectiveness of any flow arrangement of evaporators and condensers is:

$$\varepsilon \quad 1 \quad \mathrm{e}^{\quad NTU} \tag{7}$$

Basic relations covering all 21 tasks

In the index of variables given in tab. 1, there are letters 'h' and 'c' standing for 'hot' and 'cold' fluid, respectively. However, for the calculation that follows, it is crucial to make a distinction only between weaker and stronger fluid streams. The fluid stream in which the product of mass flow and isobaric specific heat is smaller (weaker fluid) will be designated as min., and the other one with max. All basic relations that follow are written recognizing only fluids according to which one is min. and which one is max.

If effectiveness, heat capacity rate ratio and two or three inlet-outlet temperatures are known, unknown temperature can be calculated using one of the following equations:

$$\begin{aligned} T_{\text{min. in}} \quad & T_{\text{min. out}} \quad \frac{1}{\omega}(T_{\text{max. out}} \quad T_{\text{max. in}}) \\ & T_{\text{min. out}} \quad \frac{\varepsilon}{1 \quad \varepsilon}(T_{\text{min. out}} \quad T_{\text{max. in}}) \\ & T_{\text{max. in}} \quad \frac{1}{\omega\varepsilon}(T_{\text{max. out}} \quad T_{\text{max. in}}) \\ & T_{\text{max. out}} \quad \frac{1 \quad \omega\varepsilon}{1 \quad (1 \quad \omega)\varepsilon}(T_{\text{min. out}} \quad T_{\text{max. out}}) \end{aligned} \tag{8}$$

$$\begin{aligned} T_{\text{min. out}} \quad & T_{\text{max. out}} \quad [1 \quad (1 \quad \omega)\varepsilon](T_{\text{min. in}} \quad T_{\text{max. in}}) \\ & T_{\text{max. in}} \quad (1 \quad \varepsilon)(T_{\text{min. in}} \quad T_{\text{max. in}}) \\ & T_{\text{max. in}} \quad \frac{1 \quad \varepsilon}{\omega\varepsilon}(T_{\text{max. out}} \quad T_{\text{max. in}}) \\ & T_{\text{min. in}} \quad \frac{\varepsilon}{1 \quad \omega\varepsilon}(T_{\text{min. in}} \quad T_{\text{max. out}}) \end{aligned} \tag{9}$$

$$\begin{aligned} T_{\text{max. in}} \quad & T_{\text{max. out}} \quad \omega(T_{\text{min. in}} \quad T_{\text{min. out}}) \\ & T_{\text{max. out}} \quad \frac{\omega\varepsilon}{1 \quad \omega\varepsilon}(T_{\text{min. in}} \quad T_{\text{max. out}}) \\ & T_{\text{min. out}} \quad \frac{1}{1 \quad \dfrac{\omega\varepsilon}{1 \quad \varepsilon}}(T_{\text{min. out}} \quad T_{\text{max. out}}) \\ & T_{\text{min. in}} \quad \frac{1}{\varepsilon}(T_{\text{min. in}} \quad T_{\text{min. out}}) \end{aligned} \tag{10}$$

$$\begin{aligned} T_{\text{max. out}} \quad & T_{\text{min. out}} \quad [1 \quad (1 \quad \omega)\varepsilon](T_{\text{min. in}} \quad T_{\text{max. in}}) \\ & T_{\text{min. in}} \quad (1 \quad \omega\varepsilon)(T_{\text{min. in}} \quad T_{\text{max. in}}) \\ & T_{\text{min. in}} \quad \frac{1 \quad \omega\varepsilon}{\varepsilon}(T_{\text{min. in}} \quad T_{\text{max. out}}) \\ & T_{\text{max. in}} \quad \frac{\omega\varepsilon}{1 \quad \varepsilon}(T_{\text{min. out}} \quad T_{\text{max. in}}) \end{aligned} \tag{11}$$

If any three of four input-output fluid temperatures ($T_{min.\,in}$, $T_{min.out}$, $T_{max.in}$, and $T_{max.out}$) and heat capacity rate ratio (ω) are known, heat exchanger effectiveness can be calculated using one of the following equations:

$$\varepsilon \quad \frac{1}{1 \quad \omega \dfrac{T_{min.\,out} \quad T_{max.\,in}}{T_{max.\,out} \quad T_{max.\,in}}} \qquad (T_{min.\,in} \quad \text{unknown}) \tag{12}$$

$$\varepsilon \quad \frac{1}{\omega} \frac{T_{max.\,out} \quad T_{max.\,in}}{T_{min.\,in} \quad T_{max.\,in}} \qquad (T_{min.\,out} \quad \text{unknown}) \tag{13}$$

$$\varepsilon \quad \frac{1}{\omega \dfrac{T_{min.\,in} \quad T_{max.\,out}}{T_{min.\,in} \quad T_{min.\,out}}} \qquad (T_{max.\,in} \quad \text{unknown}) \tag{14}$$

$$\varepsilon \quad \frac{T_{min.\,in} \quad T_{min.\,out}}{T_{min.\,in} \quad T_{max.\,in}} \qquad (T_{max.\,out} \quad \text{unknown}) \tag{15}$$

In the case when three input-output fluid temperatures and heat exchanger effectiveness are known, the heat capacity rate ratio can be found using one of the following equations:

$$\omega \quad \frac{1}{\varepsilon} \quad 1 \quad \frac{T_{max.\,out} \quad T_{max.\,in}}{T_{min.\,out} \quad T_{max.\,in}} \qquad (T_{min.\,in} \quad \text{uknown}) \tag{16}$$

$$\omega \quad \frac{1}{\varepsilon} \frac{T_{max.\,out} \quad T_{max.\,in}}{T_{min.\,in} \quad T_{max.\,in}} \qquad (T_{min.\,out} \quad \text{uknown}) \tag{17}$$

$$\omega \quad \frac{1}{\varepsilon} \quad \frac{T_{min.\,in} \quad T_{max.\,out}}{T_{min.\,in} \quad T_{min.\,out}} \qquad (T_{max.\,in} \quad \text{uknown}) \tag{18}$$

Heat capacity rate ratio cannot be found without knowing $T_{max.out}$ (for this group of problems).

Heat exchanger heat flow rate (Q) can be determined using the following relations:

$$Q \quad \frac{UA\varepsilon}{NTU(1 \quad \varepsilon)} \left| T_{min.\,out} \quad T_{max.\,in} \right| \qquad (T_{min.\,in} \quad \text{unknown}) \tag{19}$$

$$Q \quad \frac{UA\varepsilon}{NTU} \left| T_{min.\,in} \quad T_{max.\,in} \right| \qquad (T_{min.\,out} \quad \text{unknown or } T_{max.\,out} \quad \text{unknown}) \tag{20}$$

$$Q \quad \frac{UA}{NTU} \left| T_{min.\,in} \quad T_{min.\,out} \right| \qquad (T_{max.\,in} \quad \text{unknown}) \tag{21}$$

$$Q = UA \cdot \Delta T \tag{22}$$

Values for mean fluid temperatures in the stream space of the exchanger are usually estimated on the basis of arithmetic means between inlet and outlet temperatures:

$$T_{min.\,ar} \quad \frac{T_{min.\,in} \quad T_{min.\,out}}{2} \tag{23}$$

$$T_{\text{max. ar}} = \frac{T_{\text{max. in}} + T_{\text{max. out}}}{2} \tag{24}$$

Mean fluid temperatures are important for determining thermo-physical properties of fluids. They can be determined in a different way, for example, as mean integral temperatures [6, 7].

Heat flow rate of heat exchanger can be calculated by using one of the following equations:

$$\dot{Q} = W_{\text{min.}}(T_{\text{min. in}} - T_{\text{min. out}}) \tag{25}$$

$$\dot{Q} = W_{\text{max.}}(T_{\text{max. out}} - T_{\text{max. in}}) \tag{26}$$

$$\dot{Q} = \varepsilon W_{\text{min.}}(T_{\text{min. in}} - T_{\text{max. in}}) \tag{27}$$

$$\dot{Q} = \frac{\varepsilon}{1-\varepsilon} W_{\text{min.}}(T_{\text{min. out}} - T_{\text{max. in}}) \tag{28}$$

$$\dot{Q} = \frac{\varepsilon}{1-\omega\varepsilon} W_{\text{min.}}(T_{\text{min. in}} - T_{\text{max. out}}) \tag{29}$$

$$\dot{Q} = \frac{\varepsilon}{1-(1-\omega)\varepsilon} W_{\text{min.}}(T_{\text{min. out}} - T_{\text{max. out}}) \tag{30}$$

Procedures for resolving problem tasks per certain groups

Group I (sizing problems)

As heat capacities of both fluid streams are known, it is possible to designate fluid streams as min. and max. (*weak* and *strong*) and calculate heat capacity rate ratio. The effectiveness is calculated by using one of eqs. (12)-(15). In the next step, the number of transfer units (eq. 4) is determined and then unknown temperatures using one of eqs. (8)-(11).

The size of overall exchanger is given by the product (*UA*) which can now be easily found (= $NTU\, W_{\text{min.}}$). The heat flow rate is calculated from one of eqs. (19)-(21) and (25)-(30).

Finally, it is necessary to define mean fluid temperatures and their specific heats. With known specific heats, mass flow rates can be calculated.

In the case when fluid mass flow rates are given, specific heats have to be assumed, fluid heat capacities calculated and whole calculation performed. On the end, the specific heats are calculated and if there are significant deviations between assumed and calculated specific heats, the calculation has to be repeated.

Group II (sizing problems)

In this group of tasks, all four temperatures are known. By using the eq. (1), it is possible to calculate heat capacity rate ratio in such a way to satisfy the condition $\omega \le 1$. Accordingly, it is possible to designate fluid stream which carries the mark min. and which carries the mark max. and by so doing definitely designate fluid streams.

The effectiveness of heat exchanger can be calculated using eq. (15). Number of heat transfer units is calculated using eq. (4). Isobaric specific heat of fluids is calculated for corresponding referent temperatures.

Unknown mass flow rates are determined according to:

$$W_{\text{max.}} = c_{\text{p max.}} m_{\text{max.}}; \quad W_{\text{min}} = \omega W_{\text{max.}}; \quad m_{\text{min.}} = \frac{W_{\text{min.}}}{c_{\text{p min.}}} \tag{31}$$

$$W_{\text{min.}} = c_{\text{p min.}} m_{\text{min.}}; \quad W_{\text{max.}} = \frac{W_{\text{min.}}}{\omega}; \quad m_{\text{max.}} = \frac{W_{\text{max.}}}{c_{\text{p max.}}} \tag{32}$$

Group III (rating problems)

There is only one problem here. The calculation is completely identical with previous one up to the moment when it is necessary to determine mass flow rate. Here, mass flow rates are equal to:

$$m_{\text{min.}} = \frac{W_{\text{min.}}}{c_{\text{p min.}}} \tag{33}$$

$$m_{\text{max.}} = \frac{W_{\text{max.}}}{c_{\text{p max.}}} \tag{34}$$

Group IV (rating problems)

Designation of fluid stream strongness can be simply done if specific heats of fluids are known. If strongnesses are approximately equal and when specific heats are substantially changed with temperature of fluids, it may happen that fluid streams will have to be re-designated after calculations have been completed. This mostly refers to tasks 8 and 13 when both inlet and outlet temperatures of the same fluid are unknown.

The calculation of heat capacity rate ratio and the number of heat transfer units is performed in the same way as for previous groups of problems.

If ω and NTU are known, the heat exchanger effectiveness is determined from known relations for selected heat exchanger.

Unknown temperatures are determined under eqs. (8)-(11).

Group V and VI (regime problems)

When the group V is in question, from tab. 1, we can see that the heat flow rate can be directly determined. This simplifies the problem and, as already explained, one unknown is determined without using the other one.

For the group of tasks VI it is necessary to point out to two important facts. First, it is not possible to designate weak and strong stream according to a certain criterion and secondly, for these tasks it is characteristic to perform simultaneous resolving of eqs. (1) and (2). In the eq. (1), one of below relations is available:

$$\varepsilon = f\left(\frac{UA}{W_{\text{min.}}}, \omega, \text{flow arrangement}\right) \quad (W_{\text{max.}} = \text{unknown}) \tag{35}$$

$$\varepsilon = f\left(\frac{UA}{W_{\text{max.}}}, \omega, \text{flow arrangement}\right) \quad (W_{\text{min.}} = \text{unknown}) \tag{36}$$

From eq. (2), the following relations can be obtained:

$$\frac{\omega\varepsilon(NTU,\omega)}{1 \quad \varepsilon(NTU,\omega)} \quad \frac{T_{\text{max. out}} \quad T_{\text{max. in}}}{T_{\text{min. out}} \quad T_{\text{max. in}}} \quad (T_{\text{min. in}} \quad \text{unknown}) \tag{37}$$

$$\frac{\varepsilon(NTU,\omega)}{1 \quad \varepsilon(NTU,\omega)} \quad \frac{T_{\text{min. in}} \quad T_{\text{min. out}}}{T_{\text{min. in}} \quad T_{\text{max. out}}} \quad (T_{\text{max. in}} \quad \text{unknown}) \tag{38}$$

$$\omega\varepsilon(NTU,\omega) \quad \frac{T_{\text{max. out}} \quad T_{\text{max. in}}}{T_{\text{min. in}} \quad T_{\text{max. in}}} \quad (T_{\text{min. out}} \quad \text{unknown}) \tag{39}$$

$$\varepsilon(NTU,\omega) \quad \frac{T_{\text{min. in}} \quad T_{\text{min. out}}}{T_{\text{min. in}} \quad T_{\text{max. in}}} \quad (T_{\text{max. out}} \quad \text{unknown}) \tag{40}$$

Depending on the concrete task, it is necessary to use above equation in which the right side is known. If the right side of eqs. (37-40) is marked as K = const., the characteristic equation can be written in the following way:

$$\Phi = \mathrm{f}(\varepsilon,\omega) - K \tag{41}$$

Those values of ω for which the characteristic function is zero are solutions of the task. For tasks 19 and 21, the characteristic function is monotonously ascending but for tasks 18 and 20, this function has minimum and two solutions may appear (fig. 1); one solutions (when abscissa is tangented) and none solution when $\Phi > 0$ in the whole interval $0 \le W < \infty$. The example in fig. 1 refers to the counter flow heat exchanger where given data are: UA = = 4.57 kW/K, W_h = 3.00 kW/K, $T_{\text{h in}}$ = 105.1 °C, $T_{\text{c in}}$ = 15.0 °C, and $T_{\text{c out}}$ = 54.4 °C (task 19). In fig. 1, it is clear that the solution is W_c = 4.575 kW/K. The unknown outlet temperature of hot fluid is $T_{\text{h out}}$ = 45 °C (one of eq. 9). However, for the same exchanger but for the task 18, in case of unknown inlet temperature of hot fluid ($T_{\text{h in}}$ = ?) and known outlet temperature of the same fluid $T_{\text{h out}}$ = 45 °C, there are two real solutions for the heat capacity of cold fluid: $W_{\text{c 1}}$ = = 4.575 kW/K, the same as in the task 19 and $W_{\text{c 2}}$ = 0.736 kW/K. In the first case, it is obtained that the value of unknown temperature $T_{\text{h in}}$ = 105.1 °C and in the second case, 54.7. Of course, if this last temperature in the task 19 is taken as known, a completely new solution will be obtained for this task.

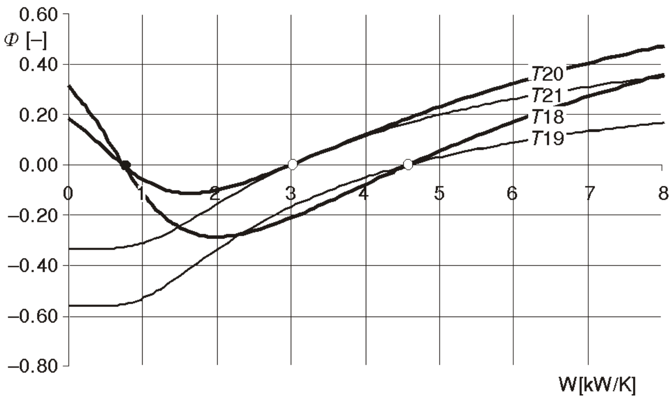

Figure 1. Characteristic function Φ versus heat capacity (W) for counter flow heat exchanger (tasks 18, 19, 20, and 21). Given data for example 19 are: UA = 4.57 kW/K; W_h = 3.00 kW/K; $T_{\text{h in}}$ = 105.1 °C; $T_{\text{c in}}$ = 15.0 °C; $T_{\text{c out}}$ = 54.4 °C

Very similar analysis can be carried out for tasks 20 and 21 (fig. 1).

In fig. 2, concrete example is presented (counter flow heat exchanger) for four different values of W_h (2.0; 3.0; 4.408, and 6.0) when resolving the problem 18. Other parameters are given in the figure itself. The abscissa provides all possible values for heat capacity rate ratio and the ordinate provides values for characteristic functions. As in this task W_c is not known and W_h is known, it is necessary to investigate values

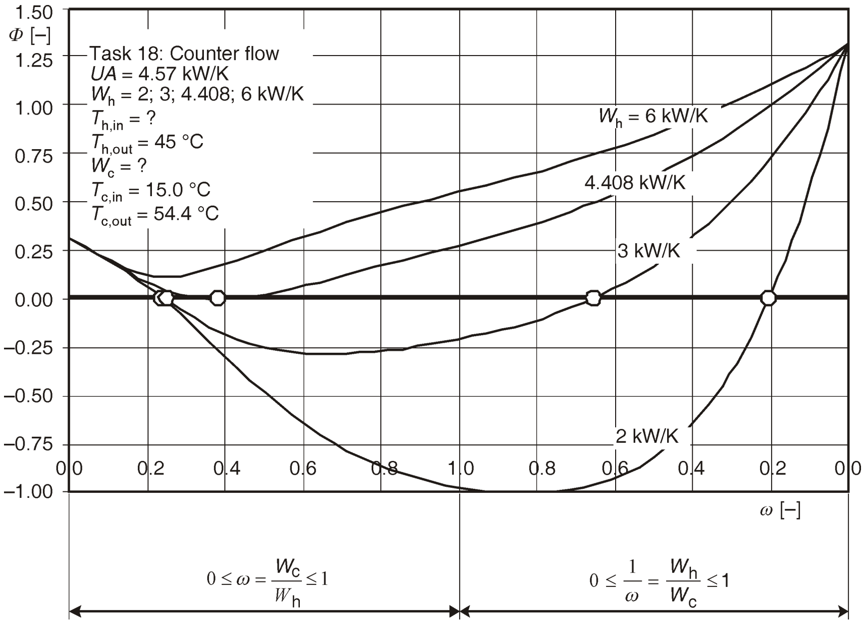

Figure 2. Dependence of changes in values of the characteristic function for counter flow heat exchanger *vs.* the capacity rate ratio (ω) for task 18 (W_h is presented as parameter)

of characteristic function in the interval 0 $W_c < \infty$. In this interval, it is also inevitable to re-designate streams due to the use of necessary equations in which clear distinction is made between $W_{min.}$ and $W_{max.}$. For the left part of fig. 2, $W_{min.} = W_c$ and $W_{max.} = W_h$ is valid and for the right part, $W_{min.} = W_h$ and $W_{max.} = W_c$ is valid. It is clear from the figure that for the case $W_h = 2$ and 3 kW/K there are two solutions, for the case $W_h = 4.408$ kW/K one and for the case $W_h = 6.0$ kW/K none.

The calculation results for this example are presented in tab. 2.

For task 20, the calculation procedure is similar and it is not necessary to be especially analyzed.

If the calculation for the task 18 is carried out for parallel flow heat exchanger of the same size and similar input data only one solution is obtained (fig. 3). Here, only hot fluid outlet temperature is higher than in the previous case. This is physical limitation of parallel flow heat exchanger or $T_{h\,out} > T_{c\,out}$.

Table 2. Task 18 (counter flow)

Input data						
UA	[kWK^{-1}]	4.57	4.57	4.57	4.57	4.57
W_h	[kWK^{-1}]	2.00	2.00	3.00	3.00	4.408
$T_{h\,out}$	[°C]	45.0	45.0	45.0	45.0	45.0
$T_{c\,in}$	[°C]	15.0	15.0	15.0	15.0	15.0
$T_{c\,out}$	[°C]	54.4	54.4	54.4	54.4	54.4
Calculated values						
W_c	[kWK^{-1}]	0.48	9.92	0.74	4.57	1.68
ω	[–]	0.239	0.202	0.246	0.656	0.380
NTU	[–]	9.557	2.285	6.200	1.523	2.728
ε	[–]	0.999	0.867	0.993	0.667	0.877
$T_{h\,in}$	[°C]	54.4	240.5	54.7	105.0	60.0
Q	[kW]	18.8	391.0	29.0	180.1	66.0

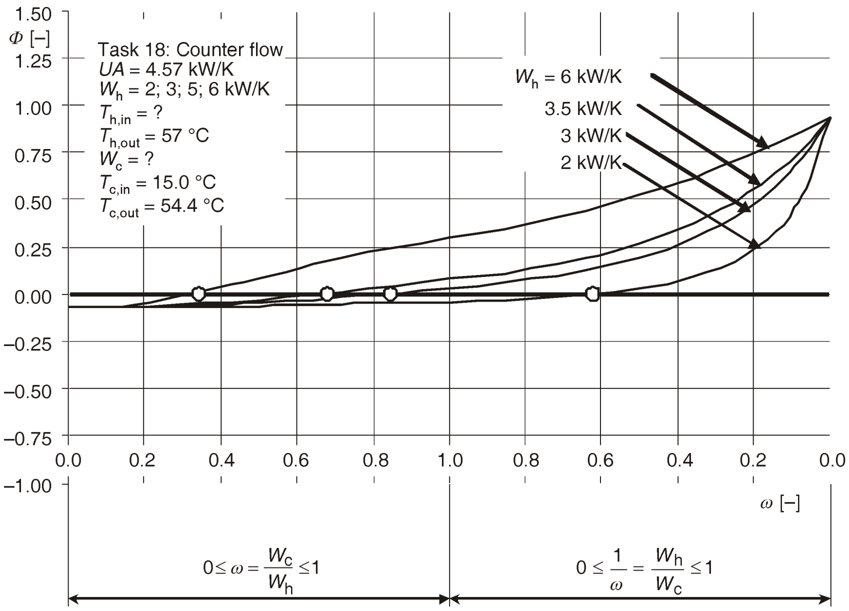

Figure 3. Dependence of changes in values of the characteristic function for parallel flow heat exchanger *vs.* the capacity rate ratio ω for task 18 (W_h is presented as parameter)

The calculation results for this example are presented in tab. 3.

Table 3. Task 18 (parallel flow)

Input data					
UA	[kWK^{-1}]	4.57	4.57	4.57	4.57
W_h	[kWK^{-1}]	2.00	3.00	3.50	6.00
$T_{h\ out}$	[°C]	57.0	57.0	57.0	57.0
$T_{c\ in}$	[°C]	15.0	15.0	15.0	15.0
$T_{c\ out}$	[°C]	54.4	54.4	54.4	54.4
Calculated values					
W_c	[kWK^{-1}]	3.23	2.49	2.33	1.99
ω	[–]	0.620	0.832	0.665	0.332
NTU	[–]	2.285	1.832	1.962	2.298
ε	[–]	0.602	0.527	0.578	0.716
$T_{h\ in}$	[°C]	120.5	89.8	83.2	70.1
Q	[kW]	127.1	98.3	91.8	78.4

The calculation for task 18 will be presented in the case of cross flow (both fluids unmixed). In this case, as well as in the case of counter flow heat exchanger, it is possible to obtain two solutions, one or none. The results of this calculation and relevant input data and calculation results are presented in fig. 4 and in tab. 4.

Such a behavior of different flow arrangements of heat exchanger can be explained in nature over micro energy balance, the resulting governing differential equations for different

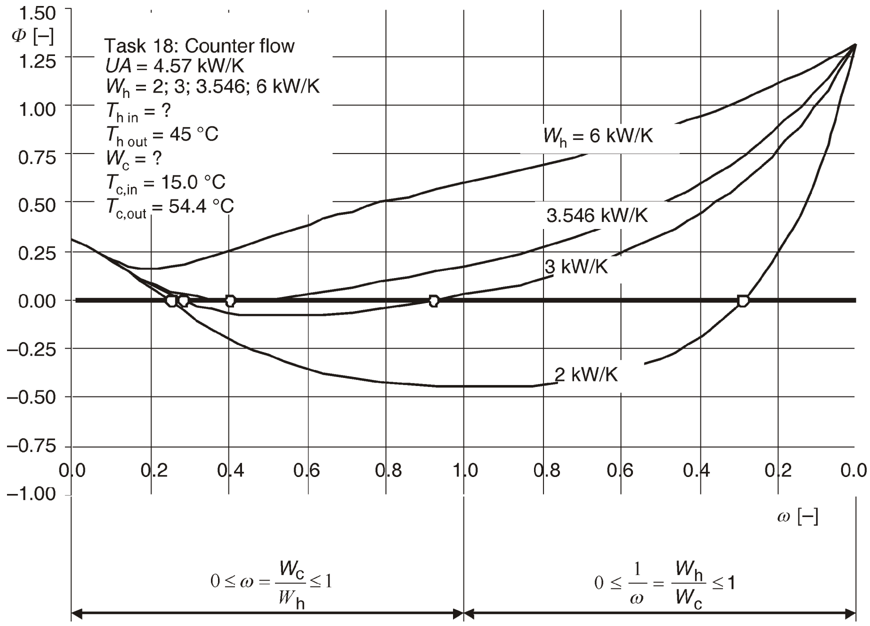

Figure 4. Dependence of changes in values of the characteristic function for cross flow heat exchanger *vs*. the capacity rate ratio for task 18 (W_h is presented as parameter)

Table 4. Task 18 (cross flow – both fluids unmixed)

Input data						
UA	[kWK^{-1}]	4.57	4.57	4.57	4.57	4.57
W_h	[kWK^{-1}]	2.00	2.00	3.00	3.00	3.546
$T_{h\ out}$	[°C]	45.0	45.0	45.0	45.0	45.0
$T_{c\ in}$	[°C]	15.0	15.0	15.0	15.0	15.0
$T_{c\ out}$	[°C]	54.4	54.4	54.4	54.4	54.4
Calculated values						
W_c	[kWK^{-1}]	0.49	6.96	0.83	2.78	1.42
ω	[–]	0.245	0.287	0.278	0.926	0.400
NTU	[–]	9.315	2.285	5.486	1.645	3.222
ε	[–]	0.993	0.821	0.962	0.593	0.861
$T_{h\ in}$	[°C]	54.7	182.1	55.9	81.5	60.8
Q	[kW]	19.3	274.3	32.8	109.4	55.9

flow arrangements and different boundary conditions. Typical case of counter flow arrangement is when boundary conditions of one fluid are given for one end of the heat exchanger and the other for the other end. It is similar for cross flow, but not for parallel flow. For it, boundary conditions are given from the same end of the heat exchanger.

The same applies to complex multi-pass flow arrangements where fluids flow in counter or cross directions. In that flow arrangements two, one or none solutions can be expected for tasks 18 and 20.

Conclusions

The calculation of heat exchangers for all possible 21 cases is presented in a systematic way. In so doing, usual assumptions for the analysis of heat exchangers are used [1]. The presented calculation procedures are very simple and suitable for the preparation of own software which can also be used in complex calculations for the network of heat exchangers. Two of the 21 tasks are particularly interesting and analyzed in details as they can produce two, one or none solutions. These are the tasks 18 and 20 in tab. 1.

The calculations are performed for heat capacity of both fluids and mass flow rate of fluids is not specially considered. As mass flow rates are usually known (or found in calculations), it is necessary to assume mean fluid temperatures and determine mean specific heats and calculate heat capacities. Upon the completion of calculations, final values of specific heat of the fluids are determined and mass flow rates calculated for concrete fluids and, if necessary, the calculation is repeated depending on deviations between assumed and calculated values.

Nomenclature

A – total heat transfer surface area, [m^2]
c_p – specific heat of fluids at constant pressure, [Jkg^{-1}°C^{-1}]
m – mass flow rate, [kgs^{-1}]
NTU – number of heat transfer units, [–]
T – temperature, [°C]
T_m – mean fluid stream temperature, [°C]
ΔT – temperature difference between streams, [°C]
U – overall heat transfer coefficient, [Wm^{-2}°C^{-1}]
UA – overall heat transfer conductance, [W°C^{-1}]
W – fluid stream heat capacity rate, [W°C^{-1}]

Greek letter

ε – heat exchanger effectiveness, [–]
ω – heat capacity rate ratio, [–]

References

[1] Kays, W. M., London, A. L., Compact Heat Exchangers, McGraw-Hill, Inc., New York, USA, 1984
[2] Bošnjaković, F., Working Parameters of Heat Exchangers (in German), Abschnit N des VDI – Wärmeatlas, Düsseldorf, Germany, 2. Auflage, 1974
[3] Novikov, I. I., Voskresenskij, K. D., Applied Thermodynamics and Heat Transfer (in Russian), Atomizdat, Moscow, 1988
[4] Levin, B. I., Shubin, E. P., Heat Transfer Apparatus of District Heating Systems (in Russian), Energiya, Moscow-Leningrad, 1965
[5] Gvozdenac, D. D., Bačlić, B. S., Sekulić, D. P., Procedures for Heat Exchanger Operating Point Definition (in Serbian), *Proceedings* , 7th Symposium of Thermal Engineers, Ohrid, 1984, pp. 368-379
[6] Bačlić, B. S., ε–N_{tu} Analysis of Complicated Flow Arrangements, in: Compact Heat Exchangers: A Festschrift for A. L. London (Eds. R. K. Shah, A. D. Kraus, D. E. Metzger), Hemisphere Publishing, Company, Washington DC, 1990

[7] Shah, R. K., Sekulić, D. P., Fundamentals of Heat Exchanger Design, John Wiley & Sons, New York, USA, 2003
[8] Morvay, Z. K., Gvozdenac, D. D., Applied Industrial Energy and Environmental Management, John Wiley & Sons Inc., UK, 2008

Author´s affiliation:

D. Gvozdenac
Department of Energy and Process Engineering,
Faculty of Technical Sciences,
University of Novi Sad
6, Trg Dositeja Obradovica
21125 Novi Sad, Serbia
E-mail: gvozden@uns.ac.rs

Paper submitted: January 5, 2009
Paper revised: January 10, 2009
Paper accepted: May 2, 2009