ANALYTIC SOLUTION OF HEAT AND MASS TRANSFER OVER A PERMEABLE STRETCHING PLATE AFFECTED BY CHEMICAL REACTION, INTERNAL HEATING, DUFOUR-SORET EFFECT AND HALL EFFECT

by

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The analytical solution is derived for the steady MHD mixed convection, laminar, heat and mass transfer over an isothermal, inclined permeable stretching sheet, immersed in a uniform porous medium in the presence of chemical reaction, thermal radiation, Dufour and Soret effects, an external transverse magnetic field, and internal heating. The governing equations are transformed into a dimensionless coupled system of non-linear ordinary differential equations and then solved analytically by the homotopy analysis method. A parametric study illustrating the influence of the chemical reaction, magnetic field, porous medium inertia parameter, and the Dufour and Soret numbers on the fluid velocity, temperature, and concentration are investigated through the obtained analytic solution. As well as the local Nusselt and the Sherwood numbers is conducted. The obtained results are presented graphically and the physical aspects of the problem are discussed. The obtained solution has been tested numerically for some values of the system parameters. Comparison with previously reported numerical results is tabulated and agreement is recorded. Analytic form of some characteristic parameters, e. g. the local skin-friction coefficient, the local Nusselt number, and the local Sherwood number, stress at the stretching surface, local mass transfer coefficient, the local wall mass flux, the local heat transfer coefficient and the local heat flux, are given due to the obtained analytic solution.

Key words: *heat mass transfer, chemical reaction, Soret-Dufore effect, internal heating, permeable stretching sheet, homotopy analysis method*

Introduction

In last few years there has been a great interest to investigate the boundary layer flows of viscous fluids due to a uniformly stretching sheet because of its technological applications importance in metallurgical and polymer sheet extrusion from a die. The study of boundary layer flows over flat surfaces have been amply investigated numerically by some researchers since Sakiadis [1, 2] who was the first invented such problem. Crane [3], and Andersson [4], have been treated the problem from different aspects. Recently, a great attention has been directed to investigate the mixed free-forced convective and mass transfer boundary layer MHD fluid flow from an inclined permeable stretching plate in a porous medium. This problem has many industrial applications in the reactor safety, oil reservoirs, geothermal systems, energy-storage units, heat insulation, heat exchangers, drying technology, catalytic reactors, and nuclear waste repository. The mixed free-forced convective and mass transfer problem had been reviewed by Dey *et al.* [5], De Hoog *et al*. [6], Schneider [7], Chamkha, *et. al*. [8] and Merkin *et al.* [9].

The MHD fluid flow from a vertical plate with chemical reaction has been studied by many authors [10-13]. The effects of thermal-diffusion and diffusion-thermo on mixed convection boundary layer flows had been considered by Kafoussias *et al.* [14]. Afify [15] studied the influence of temperature-dependent viscosity with Soret-Dufour effects on non-Darcy MHD free convective heat and mass transfer. Seddeek [16] used the finite element method to study the effect of various injection parameters on heat transfer for a power-law non-Newtonian fluid over a stretched surface with thermal radiation.

Afify [17] discussed the effects of variable viscosity on non-Darcy MHD free convection along a non-isothermal vertical surface in a porous medium. Ghaly *et al.* [18] investigated the effects of chemical reaction, heat and mass transfer on laminar flow with temperature dependent viscosity using finite difference method. The above mentioned studies and others were restricted to the numerical solution, considering some effects while neglecting the others. For example Postelnicu in [19] analyzed the effect of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret-Dufour effects, and in [20] studied the effect of chemical reaction and Soret-Dufour effects. Seddeek [21] discussed the Soret-Dufour effects effect on mixed free-forced convective flow and mass transfer over a stretching surface with a heat source the case of constant viscosity. The more recent study by Abd El-Aziz [22] discussed the effect of Ohmic heating by using the shooting method and in [23] he discussed the effect of Soret-Dufour effect on MHD three-dimensional free convection heat and mass transfer for a temperature dependent viscosity fluid with radiation flows over a permeable stretching surface.

In spite of all these investigations reported in the literatures; no one discussed all the effects actually exist in the applications. Although, the analytical solutions are more economical for the industrial purposes, all the above investigations are numerical. This was the motivation to do the present paper.

In this paper the analytical solution is provided and discussed for mixed free-forced convective and mass transfer problem of a viscous, incompressible, electrically conducting MHD fluid flow past an inclined permeable stretching surface in a porous medium and considering simultaneously the additional effects which arise by porosity, Hall effect, chemical reaction, heat generation or absorption due to the chemical reaction, thermal-diffusion and diffusion-thermo, which cannot be neglected in several practical cases. The reaction features of the obtained solution are analyzed under different conditions by varying the key parameters.

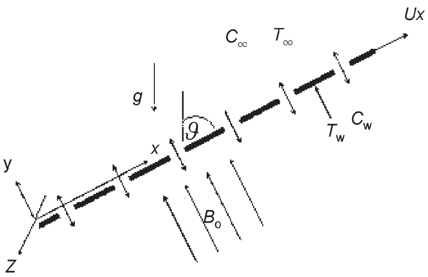

Figure 1. The stretching plate of the problem and coordinates

Problem formulation

Consider a steady viscous incompressible laminar three-dimensional mixed free-forced convective boundary-layer electrically conducting fluid flow with heat and mass transfer over an inclined permeable stretching flat plate embedded in a saturated porous medium influenced by chemical reaction, porosity, internal heat generation/absorption, Hall effect, Soret and Dufour effects (fig. 1).

The problem is performed based on the following assumptions: (1) the flow is laminar, steady-state and three-dimensional; (2) the sheet is inclined to the vertical, continuously stretching in the x-direction in the plane $y = 0$ with a velocity $u = Ux$; (3) the sheet is permeable to allow possible blowing or suction; (4) the porous medium is isotropic, homogeneous and non-magnetic therefore there is no magnetic induction; (5) the effect of compressibility and viscosity heating are neglected; (6) the Boussinesq approximation is valid and the boundary-layer approximation is applicable; (7) the fluid and the porous medium are in local thermodynamic equilibrium; (8) the fluid is well-mixed systems, considering the chemical reactions to be first order; (9) the coordinate origin is located at the surface of the sheet $y = 0$; (10) the x- and z-axis are taken parallel to the sheet and the y-axis is normal to it; (11) the magnetic field is uniform and applied parallel to the direction and there is no electric field; (12) the velocity components, temperature, and concentration are functions of x and y variables; (13) there is no slip flow at walls; (14) the physical properties of the fluid and porous medium are constant. Therefore the governing equations:

– continuity equation

$$\frac{\partial u}{\partial x} \quad \frac{\partial W}{\partial z} \quad 0 \tag{1}$$

– momentum equations

$$u\frac{\partial u}{\partial x} \quad W\frac{\partial u}{\partial z} \quad \nu\frac{\partial^2 u}{\partial z^2} \quad \mathrm{g}[\beta_\mathrm{T}(T \quad T_\infty) \quad \beta_C(C \quad C_\infty)]\cos\vartheta \quad \frac{\sigma B_0^2(u \quad mW)}{\rho(1 \quad m^2)} \quad \frac{\nu}{K}u \tag{2}$$

$$u\frac{\partial W}{\partial x} \quad W\frac{\partial W}{\partial z} \quad \nu\frac{\partial^2 W}{\partial z^2} \quad \mathrm{g}[\beta_\mathrm{T}(T \quad T_\infty) \quad \beta_C(C \quad C_\infty)]\sin\vartheta \quad \frac{\sigma B_0^2(mu \quad W)}{\rho(1 \quad m^2)} \quad \frac{\nu}{K}W \tag{3}$$

– energy equation

$$u\frac{\partial T}{\partial x} \quad W\frac{\partial T}{\partial z} \quad \frac{k}{\rho c_\mathrm{p}}\frac{\partial^2 T}{\partial z^2} \quad \frac{D_\mathrm{m}K_\mathrm{T}}{c_\mathrm{p}C_\mathrm{s}}\frac{\partial^2 C}{\partial z^2} \quad \frac{Q}{\rho c_p}T \quad q \tag{4}$$

– diffusion equation

$$u\frac{\partial C}{\partial x} \quad W\frac{\partial C}{\partial z} \quad D_\mathrm{m}\frac{\partial^2 C}{\partial z^2} \quad k_1 C\frac{D_\mathrm{m}K_\mathrm{T}}{T_\mathrm{m}}\frac{\partial^2 T}{\partial z^2} \tag{5}$$

subjected to the boundary conditions

$$\begin{aligned} &y \quad 0, \quad u_\mathrm{w} \quad Ux, \quad v \quad 0, \quad W \quad 0, \quad C \quad C_\mathrm{w} \\ &y \quad \infty, \quad u \quad 0, \quad W \quad 0, \quad T \quad T_\infty, \quad C \quad C_\infty \end{aligned} \tag{6}$$

The internal heat to generation/absorption term q''' is modeled according to the following equation:

$$q \quad [A^*(T_\mathrm{w} \quad T_\infty)\mathrm{e}^{\quad \eta} \quad B^*(T_\mathrm{w} \quad T_\infty)]\frac{kU}{\nu} \tag{7}$$

We considered the interaction between diffusion of (heat and mass) $(D_\mathrm{m}K_\mathrm{T}/T_\mathrm{m})(\quad T \quad z^2)$ and $(D_\mathrm{m}K_\mathrm{T}/c_\mathrm{p}C_\mathrm{s})(\quad ^2C/ \quad z^2)$, namely the Soret (Sr) and Dufour ($\mathrm{D_u}$) effects. The x-axis is taken in the plate whose inclination angle ϑ is with the horizontal. The z-axis is taken in the plate and parallel to the horizon. The y-axis is taken normal to the plate. The Hall effect as a result of applied external magnetic field transverse to the plate parallel to y-axis is considered, under the assumption of neglecting the induced magnetic field compared with the applied field; this is characterized by a small Reynolds number. By using the following dimensionless variables:

$$\eta \quad \sqrt{\frac{U}{x\nu}}y, \quad \psi(x, y) \quad \sqrt{U\nu x}f(\eta), \quad \Theta(\eta) \quad \frac{T \quad T_\infty}{T_\mathrm{w} \quad T_\infty}, \quad \Phi(\eta) \quad \frac{C \quad C_\infty}{C_\mathrm{w} \quad C_\infty} \tag{8}$$

and

$$u \quad \frac{\partial\psi}{\partial y} \quad xUf(\eta), \quad v \quad \frac{\partial\psi}{\partial x} \quad \sqrt{U\nu}f(\eta), \quad W \quad UxH(\eta),$$
$$A_1 \quad \frac{M}{1 \quad m^2} \quad \frac{\nu}{Uk}, \quad A_2 \quad \frac{mM}{1 \quad m^2} \tag{9}$$

The governing eqs. (1)-(5) can be reduced to the following dimensionless equations where eq. (1) is satisfied identically:

$$f''' + ff'' - f'^2 + \mathrm{Gr}\Theta + \mathrm{Gc}\Phi - A_1f' - A_2H = 0 \tag{10}$$

$$H'' + fH' - f'H + A_2f' - A_1H = 0 \tag{11}$$

$$\Theta'' + \mathrm{Pr}f\Theta' + B^*\Theta + D_u\Phi'' + A^*\mathrm{e}^{-\eta} = 0 \tag{12}$$

$$\Phi'' + \mathrm{Sc}(f\Phi' - \mathrm{g}\Phi) + \mathrm{Sr}\,\Theta'' = 0 \tag{13}$$

where the prime denotes the differentiation $\mathrm{d}f/\mathrm{d}\eta$. The boundary conditions (6) can be reduced to the following dimensionless form:

$$\eta \quad 0: \quad f(0) \quad 0, \quad f'(0) \quad 1, \quad H(0) \quad 0, \quad \Theta(0) \quad 1, \quad \Phi(0) \quad 1,$$
$$\eta \quad \infty \quad f(\infty) \quad 0, \quad H(\infty) \quad 0, \quad \Theta(\infty) \quad 0, \quad \Phi(\infty) \quad 0 \tag{14}$$

The reduced dimension less system (10)-(14) describes the considered boundary value problem. This set of equations is coupled non-linear non-homogeneous system of differential equations. It is not so simple to solve by using the traditional methods either analytical or numerical which is noticed for most realistic problems. This is one of our motivation by work in this paper, and our second motivation is to solve one of the realistic problem arise in the reactors.

Analytic solution

The homotopy analysis method (HAM) invented by Laio [24], proved its power to solve several non-linear problems [25, 26]. To overcome the difficulty appears with the problem under investigation in this paper related to the non-linearity and the non-homogeneity of the system of differential equations and to get a uniformly analytic solution for this complicated problem we shall use the HAM. By means of the traditional homotopy method; using p as the homotopy embedding parameter, $\hbar$ is stands for, $\hbar_\mathrm{f}$, $\hbar_\mathrm{H}$, $\hbar_\Theta$, and $\hbar_\Phi$ as the auxiliary parameter their values which control the convergence of the series can be determined. L stands for L_f, L_H, L_Θ, and L_Φ as the auxiliary linear operators, and $\pi_0(\eta)$ stands for $f_0(h)$, $H_0(\eta)$, $\Theta_0(\eta)$, and $\Phi_0(\eta)$ as an initial guess, and $\pi(\eta)$ for $f(\eta)$, $H(\eta)$, $\Theta(\eta)$, and $\Phi(\eta)$ as a solution, respectively, where:

$$L_f \quad (\partial^3 \quad \partial^2), \quad J_1(\eta, 0) \quad f_0(\eta) \quad 1 \quad \mathrm{e}^{\ \eta}, \quad L_\mathrm{f}[C_1 \quad C_2\eta \quad C_3\mathrm{e}^\eta] \quad 0 \tag{15}$$

$$L_\mathrm{H} \quad \frac{1}{2}(\partial^2 \quad \partial), \quad J_2(\eta, 0) \quad H_0(\eta) \quad \mathrm{e}^{\ \eta} \quad \mathrm{e}^{\ 2\eta}, \quad L_\mathrm{H}[C_1 \quad C_2\mathrm{e}^\eta] \quad 0 \tag{16}$$

$$L_\Theta = (\partial^2 + \partial), \quad J_3(\eta, 0) = \Theta_0(\eta) = \mathrm{e}^{-\eta}, \quad L_\Theta[C_1 + C_2\mathrm{e}^{-\eta}] = 0 \tag{17}$$

$$L_\Phi = (\partial^2 + \partial), \quad J_4(\eta, 0) = \Phi_0(\eta) = \mathrm{e}^{-\eta}, \quad L_\Phi[C_1 + C_2\mathrm{e}^{-\eta}] = 0 \tag{18}$$

Using the rules of solution expression for the unknown functions:

$$f(\eta) = \sum_{m=0}^{\infty} f_m(\eta), \quad H(\eta) = \sum_{m=0}^{\infty} H_m(\eta), \quad \Theta(\eta) = \sum_{m=0}^{\infty} \Theta_m(\eta), \quad \Phi(\eta) = \sum_{m=0}^{\infty} \Phi_m(\eta), \tag{19}$$

$$\begin{aligned} f_m(\eta) &= \sum_{k=0}^{2m+1} f_{mk}\mathrm{e}^{-k\eta}, \quad H_m(\eta) = \sum_{k=1}^{2m+2} f_{mk}\mathrm{e}^{-k\eta}, \\ \Theta_m(\eta) &= \sum_{k=1}^{2m+2} \theta_{mk}\mathrm{e}^{-k\eta}, \quad \Phi_m(\eta) = \sum_{k=1}^{2m+2} \phi_{mk}\mathrm{e}^{-k\eta}, \end{aligned} \tag{20}$$

where the constant coefficient, $f_{\mathrm{m\,k}}$, $h_{\mathrm{m\,k}}$, $\theta_{\mathrm{m\,k}}$, and $\phi_{\mathrm{m\,k}}$, have to be determined.

The zero-order deformation equation:

$$(1 - p)L[J_i(\eta, p) - \pi_{i0}(\eta)] = p\hbar N_i[J_i(\eta, p)] \tag{21}$$

For the homotopy mapping the initial guess is $J_i(\eta, 0) = \pi_{i\,0}(\eta)$ and $J_i(\eta, 1) = \pi_i(\eta)$ is the solution, where $J_i(\eta, p)$, $i = 1, 2, 3, 4$ stand for the functions $f(\eta)$, $H(\eta)$, $\Theta(\eta)$, and $\Phi(\eta)$, respectively. Following the known steps of the HAM [24] and assuming the Taylor series expansion in the power of p:

$$J_i(\eta, p) = \pi_{i0}(\eta) + \sum_{m=1}^{\infty} \pi_{i\,m}(\eta)p^m, \quad \pi_{i\,m}(\eta) = \frac{1}{m!}\left.\frac{\partial^m J_i(\eta, p)}{\partial p^m}\right|_{p=0} \tag{22}$$

where for $i = 1, 2, 3, 4$, $\pi_{i\,m}(\eta)$ is stands for $f_m(\eta)$, $H_m(\eta)$, $\Theta_m(\eta)$, and $\Phi_m(\eta)$, respectively.

Therefore $m \geq 1$; the m-th order deformation equation reads:

$$L[\pi_{im}(\eta) - \chi_m \pi_{i,m-1}(\eta)] = \hbar I_{im}(\pi_{i,m-1}(\eta)) \tag{23}$$

$$\chi_m = \begin{cases} 1 & m \quad 1 \\ 0 & m \quad 0 \end{cases} \tag{24}$$

where $I_{i\,m}[\pi_{i,m-1}(\eta)]$

$$I_{im}[\pi_{i,m-1}(\eta)] = \frac{1}{(m-1)!}\left.\frac{\partial^{m-1} N_i[J_i(\eta, p)]}{\partial p^{m-1}}\right|_{p=0} \tag{25}$$

Then the f-equation in the system:

$$N_1[J_1, J_2, J_3, J_4](\eta, p) = \frac{\partial^3 J_1}{\partial\eta^3} + J_1\frac{\partial^2 J_1}{\partial\eta^2} - \left(\frac{\partial J_1}{\partial\eta}\right)^2 + \mathrm{Gr}J_3 + \mathrm{Gc}J_4 - A_1\frac{\partial J_1}{\partial\eta} - A_2 J_2 \tag{26}$$

the H-equation in the system;

$$N_2[J_1, J_2](\eta, p) = \frac{\partial^2 J_2}{\partial\eta^2} + J_1\frac{\partial J_2}{\partial\eta} - J_2\frac{\partial J_1}{\partial\eta} - A_2\frac{\partial J_1}{\partial\eta} - A_1 J_2 \tag{27}$$

the Θ-equation:

$$N_1[J_1, J_3, J_4](\eta, p) = \frac{\partial^2 J_3}{\partial\eta^2} + \mathrm{Pr}\, J_1\frac{\partial J_3}{\partial\eta} + \left(\frac{\partial J_1}{\partial\eta}\right)^2 + B^* J_3 + D_{\mathrm{u}}\frac{\partial^2 J_4}{\partial\eta^2} + A^*\mathrm{e}^{-\eta} \tag{28}$$

the Φ-equation:

$$N_4[J_1, J_3, J_4](\eta, p) = \frac{\partial^2 J_4}{\partial\eta^2} + \mathrm{Sc}J_1\frac{\partial J_4}{\partial\eta} - \gamma\mathrm{Sc}J_4 + \mathrm{Sr}\frac{\partial^2 J_3}{\partial\eta^2} \tag{29}$$

where $J_1(\eta, p)$, $J_2(\eta, p)$, $J_3(\eta, p)$, and $J_4(\eta, p)$ are the homotopy mappings related to the unknown functions $f(\eta)$, $J(\eta)$, $\Theta(\eta)$, and $\Phi(\eta)$, respectively, which grew by the homotopy parameter

$0 \le p \le 1$ from the initial guess solutions $f_0(\eta)$, $H_0(\eta)$, $\Theta_0(\eta)$, and $\Phi_0(\eta)$ at $p = 0$ to the solutions $f(\eta)$, $H(\eta)$, $\Theta(\eta)$, and $\Phi(\eta)$ at $p = 1$ *i. e.*;

$$J_1(\eta,1) = f(\eta), \quad J_2(\eta,1) = H(\eta), \quad J_3(\eta,1) = \Theta(\eta), \quad J_4(\eta,1) = \Phi(\eta) \tag{30}$$

Therefore by eqs. (26), and (29)-(36) we get:

$$L_{\mathrm{f}}[f_m(\eta, p) - \chi_m f_{m-1}(\eta)] = \hbar_{\mathrm{f}} F_m(\eta)$$
$$F_m(\eta) = f'''_{m-1} + \sum_{n=0}^{m-1} f_n f''_{m-1-n} - \sum_{n=0}^{m-1} f'_{\mathrm{n}} f'_{m-1-n} + \mathrm{Gr}\Theta_{m-1} + \mathrm{Gc}\Phi_{m-1} - A_1 f'_{m-1} - A_2 H_{m-1} \tag{31}$$

the H-equation in the system:

$$L_{\mathrm{H}}[H_m(\eta) - \chi_m H_{m-1}(\eta)] = \hbar_{\mathrm{H}} P_m(\eta)$$
$$P_m(\eta) = H''_{m-1} + \sum_{n=0}^{m-1} f_n H'_{m-1-n} - \sum_{n=0}^{m-1} f'_n H_{m-1-n} + A_2 f'_{m-1} - A_1 H_{m-1} \tag{32}$$

the Θ-equation:

$$L_{\Theta}[\Theta_m(\eta, p) - \chi_m \Theta_{m-1}(\eta)] = \hbar_{\Theta} Q_m(\eta)$$
$$Q_m(\eta) = \Theta''_{m-1} + \mathrm{Pr}\sum_{n=0}^{m-1} f_n \Theta'_{m-1-n} + B^*\Theta_{m-1} + D_{\mathrm{u}}\Phi''_{m-1} + A^* \mathrm{e}^{-\eta} \tag{33}$$

the Φ-equation:

$$L_{\Phi}[\Phi_m(\eta, p) - \chi_m \Phi_{m-1}(\eta)] = \hbar_{\Phi} R_m(\eta)$$
$$R_m(\eta) = \Phi''_{m-1} + \mathrm{Sc}\sum_{n=0}^{m-1} f_{\mathrm{n}} \Phi'_{m-1-n} - \gamma\,\mathrm{Sc}\Phi_{m-1} + \mathrm{Sr}\Phi''_{m-1} \tag{34}$$

The equations (38), (40), (42), and (44) are subject to the conditions:

$$f_m(0) = 0, \quad f'_m(0) = 1, \quad f'_m(\infty) = 0, \quad H_m(0) = 0, \quad H_m(\infty) = 0,$$
$$\Theta_m(0) = 1, \quad \Theta_m(\infty) = 0, \quad \Phi_m(0) = 0, \quad \Phi_m(\infty) = 0, \qquad 1 \le m$$

then the coefficients $f_{m\,k}$, $h_{m\,k}$, $\theta_{m\,k}$, and $\phi_{m\,k}$ can be determined, and as a consequence the solution of the system of the coupled non-linear non-homogenous differential equations which described the considered problem. We have from the expressions of the initial guess functions as a zero approximation of the solution that the coefficients:

$$f_{00} = 1, \quad f_{01} = -1, \quad h_{01} = 1, \quad h_{02} = -1, \quad \theta_{01} = 1, \quad \text{and} \quad \phi_{01} = 1 \tag{35}$$

Therefore the other coefficients in the expression of the solution can be easyly determined from the following recurrence relations:

$$f_{m0} = -\sum_{k=1}^{2m+1} f_{mk}$$
$$f_{mk} = \frac{\hbar}{k^2 - k^3} S_{mk} + \chi_m \lambda_{m-1,k} f_{m-1,k} \quad 1 \le k \le 2m+1 \tag{36}$$

$$h_{m,2} = -\sum_{k=1}^{2m+2} h_{mk}$$
$$h_{m\,k} = \frac{2\hbar}{k^2 - k} T_{m,k-1} + \chi_m \lambda_{m-1,k} h_{m-1,k} \quad 2 \le k \le 2m+2 \tag{37}$$

$$\theta_{m,1} = -\sum_{k=2}^{2m+2} \theta_{m\,k}$$
$$\theta_{m\,k} = \frac{\hbar}{k^2 - k} U_{m,k-1} + \chi_m \lambda_{m-1,k} \theta_{m-1,k}, \quad 2 \le k \le 2m+2 \tag{38}$$

$$\phi_{m,1} \quad {}^{2m\ \ 2}_{k\ \ 2} \phi_{m\,k}$$
$$\phi_{m\,k} \quad \frac{\hbar}{k^2 \quad k} V_{m,\,k\ \ 1} \quad \chi_m \lambda_{m\ \ 1,\,k} \phi_{m\ \ 1,\,k}, \quad 2 \quad k \quad 2m \quad 2 \tag{39}$$

$$S_{m\,k} \quad \alpha_{m\,k} \quad \beta_{m\,k}, \quad T_{m\,k} \quad \varepsilon_{m\,k} \quad \delta_{m\,k},$$
$$U_{m\,k} \quad \rho_{m\,k} \quad \sigma_{m\,k}, \quad V_{m\,k} \quad \gamma_{m\,k} \quad \mu_{m\,k}, \quad 2 \quad k \quad 2m \quad 2 \tag{40}$$

$$\alpha_{mk} \quad A_1(k \quad k^3)\lambda_{m\ \ 1,\,k} f_{m\ \ 1,\,k} \quad A_2\lambda_{m\ \ 1,\,k} h_{m\ \ 1,\,k}$$
$$\mathrm{Gr}\lambda_{m\ \ 1,\,k}\theta_{m\ \ 1,\,k} \quad \mathrm{Gc}\lambda_{m\ \ 1,\,k}\phi_{m\ \ 1,\,k}, \quad 1 \quad k \quad 2m \quad 1 \tag{41}$$

$$\beta_{m1} \quad 0, \quad \beta_{m,2m\ \ 1} \quad 0$$
$$\beta_{m\,k} \quad {}^{m\ \ 1}_{n\ \ 0} \quad {}^{\min[2m\ \ 2n\ \ 1,k\ \ 1]}_{s\ \ \max[1,k\ \ 2n\ \ 1]} [s^2 \quad s(s \quad k)] f_{m,k\ \ s} f_{m\ \ 1\ \ n,s} \quad 2 \quad k \quad 2m \tag{42}$$

$$\varepsilon_{mk} \quad A_2(k)\lambda_{m\ \ 1,k} \quad 1 f_{m\ \ 1,k} \quad (k^2 \quad A_1)\lambda_{m\ \ 1,k} h_{m\ \ 1,k} \quad 1 \quad k \quad 2m \quad 1 \tag{43}$$

$$\delta_{mk} \quad {}^{m\ \ 1}_{n\ \ 0} \quad {}^{\min[2m\ \ 2n,k]}_{s\ \ \max[1,k\ \ 2n\ \ 1]} (k \quad 2s) f_{m,k\ \ s} h_{m\ \ 1\ \ n,s} \quad 1 \quad k \quad 2m \quad 1 \tag{44}$$

$$\rho_{mk} \quad (k^2 \quad B^{*})\lambda_{m\ \ 1,k}\theta_{m\ \ 1,k} \quad k^2\lambda_{m\ \ 1,k}\phi_{m\ \ 1,k} \quad A^{*}\delta_k^1 \quad 1 \quad k \quad 2m \quad 1 \tag{45}$$

$$\sigma_{mk} \quad \mathrm{Pr} \ {}^{m\ \ 1}_{n\ \ 0} \quad {}^{\min[2m\ \ 2n,k]}_{s\ \ \max[1,k\ \ 2n\ \ 1]} (\ \ s) f_{m,k\ \ s}\theta_{m\ \ 1\ \ n,s} \quad 1 \quad k \quad 2m \quad 1 \tag{46}$$

$$\gamma_{mk} \quad [\mathrm{Sr}k^2]\lambda_{m\ \ 1,k}\theta_{m\ \ 1,k} \quad (k^2 \quad \mathrm{Sc}\gamma)\lambda_{m\ \ 1,k}\phi_{m\ \ 1,k} \quad 1 \quad k \quad 2m \quad 1 \tag{47}$$

$$\mu_{mk} \quad \mathrm{Sr} \ {}^{m\ \ 1}_{n\ \ 0} \quad {}^{\min[2m\ \ 2n,k]}_{s\ \ \max[1,k\ \ 2n\ \ 1]} (\ \ s) f_{m,k\ \ s}\phi_{m\ \ 1\ \ n,s} \quad 1 \quad k \quad 2m \quad 1 \tag{48}$$

where

$$\delta_k^1 \quad \begin{matrix} 1 & k & 1 \\ 0 & k & 1 \end{matrix} \quad \text{and} \quad \lambda_{\mathrm{m,k}} \quad \begin{matrix} 1 & 1 \quad k \quad 2m \quad 2 \\ 0 & \text{otherwise} \end{matrix}$$

Substituting in eq. (19) one gets the analytic solution expressions for the dimensionless velocity components profiles *f*, and *H*, also the heat distribution Θ and mass transfer Φ.

Results and discussion

The obtained analytic solutions (19)-(22) for the velocity components profiles $f'(\eta)$ and $H(\eta)$, also the temperature distribution $\Theta(\eta)$, and the concentration distribution $\Phi(\eta)$ are plotted for some specific values of the parameters to show the behavior of the solution and to depict the effect of some parameters. We have used the Mathematica package in our calculations and to plot the solutions up 30-order approximation. The chosen value of the auxiliary parameter $\hbar$ 1 is determined in fig. 2 by plotting the $\hbar$-curves for $f''(0)$, $H'(0)$, $\Theta'(0)$, and $\Phi(0)$ *vs.* $\hbar$ are performed at the values of: $\gamma = 0$, Sr = 0.01, $M = 0$, Pr = 0.7, $m = 0.0$, $A^* = B^* = 0.01$, Da = 0, Gr = Gc = 1, Sc = 0.22, Re = 3, and $D_u = 0.01$.

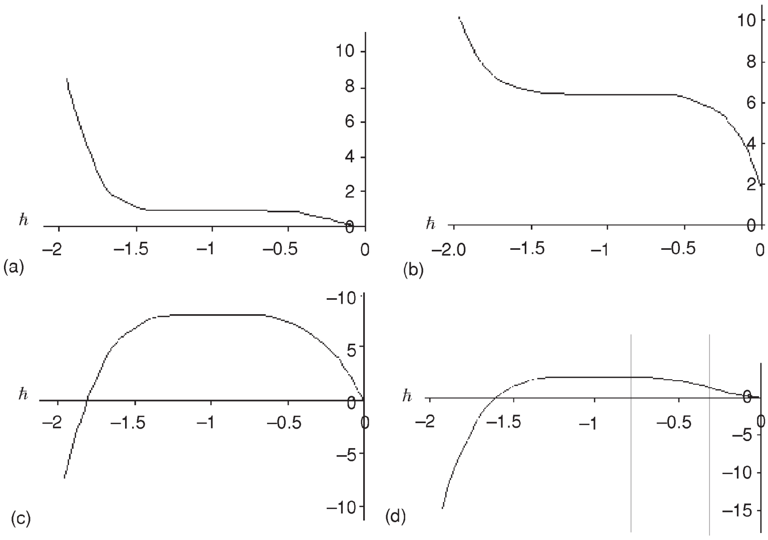

Figure 2. The ħ curves for the (a) $f''(0)$, (b) $H'(0)$, (c) $\Theta'(0)$, and (d) $\Phi'(0)$, up to 30th order approximation at Pr = 0.7 with Gr = 1, Gc = 1, Sc = 0.22, Da = 0, m = 0.0, D_u = 0.3, Sr = 0.1; Re = 3, $A^* = B^*$ = 0.01, M = 0.01, and γ = 0; it show that the value ħ = –1 is the control key of the series convergence

The influence of variation in the fluid parameters, Pr, M, A^*, B^*, Sr, D_u, and γ on the velocity profiles, the temperature profile, and the mass transfer profile are displayed in figs. (3)-(11). In order to get a clear insight of the physical problem, the velocities $f(\eta)$ and $H(\eta)$, the temperature $\Theta(\eta)$, and concentration $\Phi(\eta)$ have been discussed by assigning numerical values to the problem parameters. Realistic values of Sc = 0.22, 0.62, and 0.78 are chosen for hydrogen, water vapor, and ammonia at temperature 25 °C and on atmospheric pressure, Pr = 0.7 for the air at temperature 20 °C and on atmospheric pressure, Gr_x = 1 for heat transfer, and Gc_x = 3 for mass transfer. Re_x = 3, and γ = –0.5, –0.3, 0.7, 1.0, and 3.0.

Figures (3)-(4) depict the effect of the rate of internal heat generation $A^* > 0$, $B^* > 0$, and absorption $A^* < 0$, $B^* < 0$, due to the space-dependent coefficient $A^* > 0$, and due to the tem-

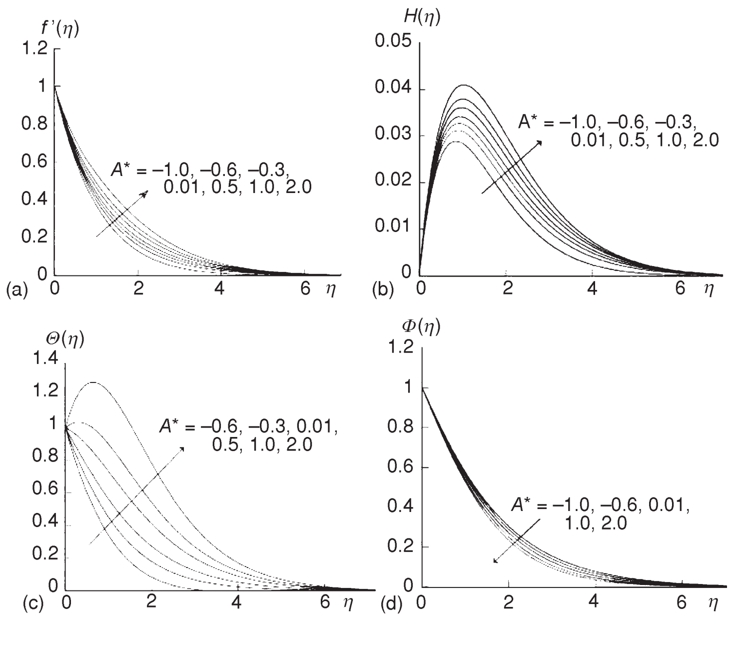

Figure 3. The influence of internal heat generation $A^* > 0$, and absorption $A^* < 0$, coefficients of space-dependent on the analytic solution for the velocity profiles $f'(\eta)$ in x-direction and $H(\eta)$ in z-direction, the temperature $\Theta(\eta)$, and concentration $\Phi(\eta)$

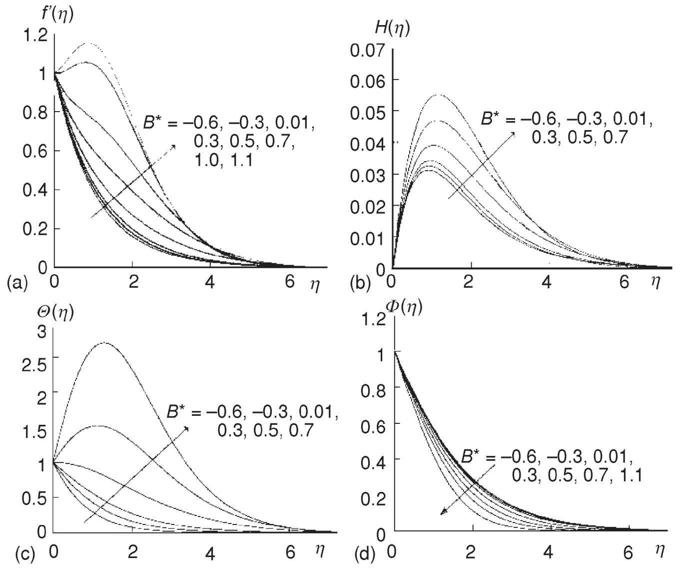

Figure 4. The influence of internal heat generation $B^* > 0$, and absorption $B^* < 0$, coefficients of space-dependent on the analytic solution for the velocity profiles $f'(\eta)$ and $H(\eta)$, the temperature $\Theta(\eta)$, and concentration $\Phi(\eta)$

perature-dependent coefficient $B^* > 0$. It shows that the solution respond to the increasing in these parameters by increasing for the cases of the velocities components profiles $f'(\eta)$, $H(\eta)$, and on the temperature $\Theta(\eta)$, and by decreasing for the case of concentration $\Phi(\eta)$.

Figure 5 illustrates the effect of the chemical reaction parameter for mass generation $\gamma < 0$/destruction $\gamma > 0$. It is seen, that the velocities $f'(\eta)$, and $H(\eta)$ and concentration $\Phi(\eta)$ decreases by increasing of γ, and the temperature $\Theta(\eta)$ is responds by increasing with increasing in γ.

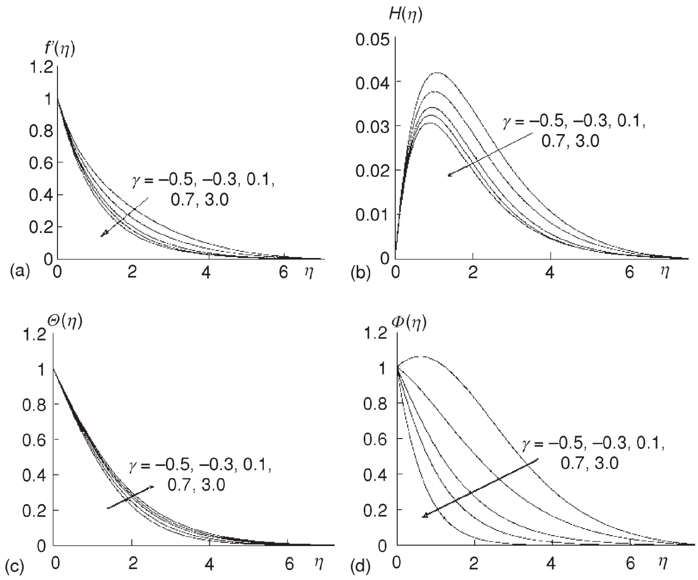

Figure 5. The effect of the chemical reaction parameter for mass generation $\gamma < 0$/ destruction on internal heat generation on the velocity profiles $f'(\eta)$ and $H(\eta)$, and on the temperature $\Theta(\eta)$, and on the concentration $\Phi(\eta)$

Figure 6 shows the effect interaction of the magnetic parameter M. The velocity profiles $f'(\eta)$ and $H(\eta)$, respond by decreasing *vs*. to the increasing in the magnetic parameter, while the temperature $\Theta(\eta)$ and the concentration $\Phi(\eta)$ responds by increasing *vs*. to the increasing in the magnetic parameter.

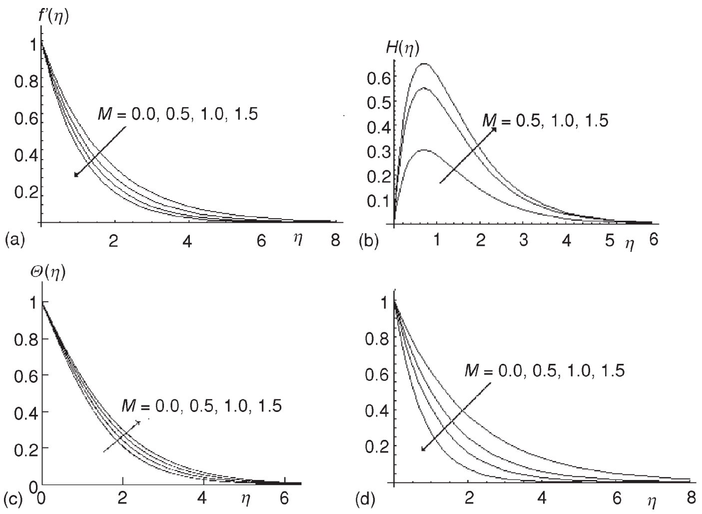

Figure 6. The effect of the magnetic parameter *M* on internal heat generation on the velocity profiles $f'(\eta)$ and $H(\eta)$ and on the temperature $\Theta(\eta)$, and the concentration $\Phi(\eta)$

Figure 7 shows also that in each group with constant magnetic parameter response to the increasing of Soret parameter by increasing the velocity profiles $f'(\eta)$ and $H(\eta)$, and decreasing both the temperature $\Theta(\eta)$ and the concentration $\Phi(h)$. The opposite response by the velocity profiles $f'(\eta)$ and $H(\eta)$, and the temperature $\Theta(\eta)$ and the concentration $\Phi(\eta)$; is noticed for the increasing variation in the Dufour parameter D_u. We can use the obtained analytic solution expressions given by (19) to calculate $f''(0)$, $H'(0)$, $\Theta'(0)$, and $\Phi'(0)$.

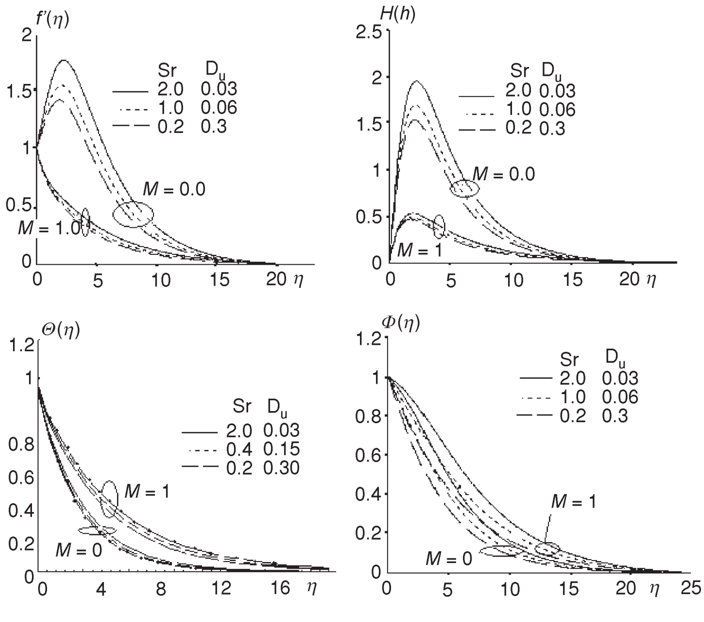

Figure 7. The effect of the Soret parameter (Sr), Dufour parameter (D_u), on internal heat generation on the velocity profiles $f'(\eta)$ and $H(\eta)$, and on the temperature $\Theta(\eta)$, and the concentration $\Phi(\eta)$

Therefore one can calculate the characteristic numbers of engineering interest and give a series expression for each as follow, respectively: the Shear stress in the x-direction and in the z-direction are:

$$\tau_{wx} \quad \mu\sqrt{\frac{U}{\nu}}Uxf\ (0), \qquad \tau_{wz} \quad \mu\sqrt{\frac{U}{\nu}}UxH\ (0) \tag{49}$$

The local skin-friction coefficients are:

$$C_{fx} \quad \frac{2}{\sqrt{Re_x}} f\ (0), \qquad C_{fz} \quad \frac{2}{\sqrt{Re_x}} H\ (0) \tag{50}$$

The local Nusselt number is:

$$Nu_x \quad \frac{xh_w}{k} \quad \sqrt{Re_x}\Theta\ (0) \tag{51}$$

where the heat transfer coefficient is

$$h_w \quad \frac{q_w}{T_w \quad T_\infty} \quad k\sqrt{\frac{U}{\nu}}\ {}^{\infty\ \ 2m\ \ 2}_{m\ \ 0\ \ \ell\ \ 1}(\ \ \ell)\theta_{m\ell} \tag{52}$$

and the mass flux is

$$q_w \quad k(T_w \quad T_\infty)\sqrt{\frac{U}{\nu}}\Theta\ (0) \quad k(T_w \quad T_\infty)\sqrt{\frac{U}{\nu}}\ {}^{\infty\ \ 2m\ \ 2}_{m\ \ 0\ \ \ell\ \ 1}(\ \ \ell)\theta_{m\ell} \tag{53}$$

The local Sherwood number is:

$$Sh_x \quad \frac{xh_m}{D_m} \quad \sqrt{Re_x}\Phi\ (0) \tag{54}$$

where the mass transfer coefficient is

$$h_m \quad \frac{m_w}{C_w \quad C_\infty} \quad D_m\sqrt{\frac{U}{\nu}}\ {}^{\infty\ \ 2m\ \ 2}_{m\ \ 0\ \ \ell\ \ 1}(\ \ \ell)\phi_{m\ell} \tag{55}$$

and the mass flux is

$$m_w \quad D_m(C_w \quad C_\infty)\sqrt{\frac{U}{\nu}}\Phi\ (0) \quad D_m(C_w \quad C_\infty)\sqrt{\frac{U}{\nu}}\ {}^{\infty\ \ 2m\ \ 2}_{m\ \ 0\ \ \ell\ \ 1}(\ \ \ell)\phi_{m\ell} \tag{56}$$

Therefore we can find for the first time an analytic series expression for the characteristic numbers: skin friction coefficients, Nusselt number, local Sherwood number, stress at the stretching surface, local mass transfer coefficient, local wall mass flux, local heat transfer coefficient, and the local heat flux, due to the obtained analytic solution. The behavior of the characteristic numbers is plotted in figs. (8)-(12).

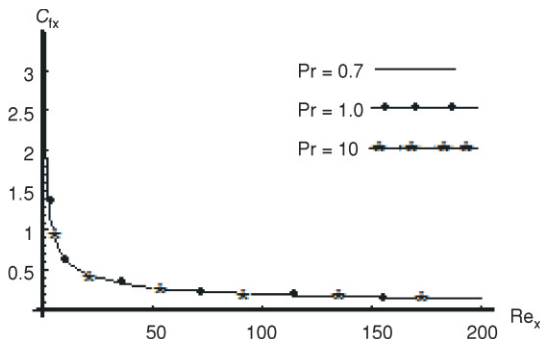

Figure 8. Local skin-friction coefficients *vs.* local Reynolds number for different values of Pr = 0.7, 1.0, and 10.0

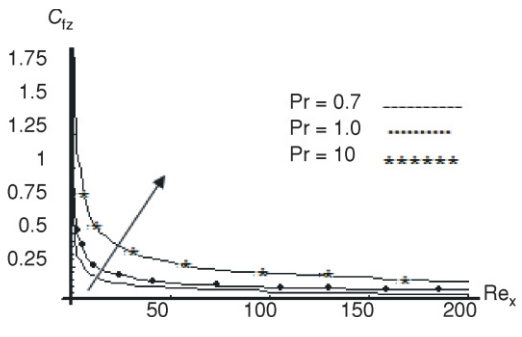

Figure 9. Local skin-friction coefficients *vs.* local Reynolds number for different values of Pr = 0.7, 1.0, and 10.0

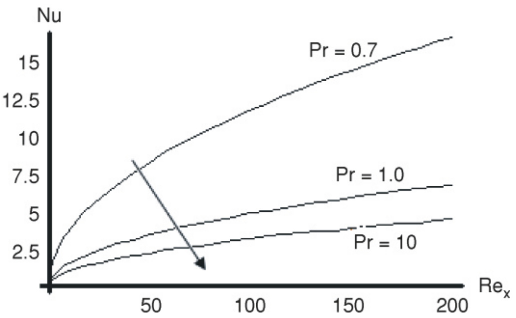

Figure 10. The Nusselt number *vs.* Reynolds number for different values of the Pr = 0.7, 1.0, and 10

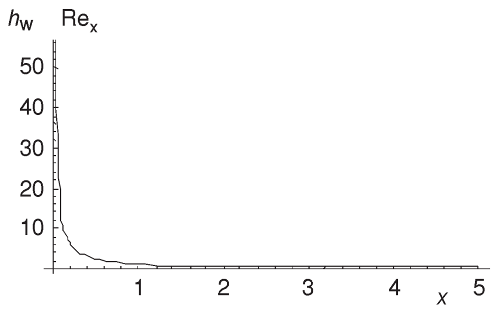

Figure 11. The heat transfer coefficients over the square root of local Reynolds number *vs. x*

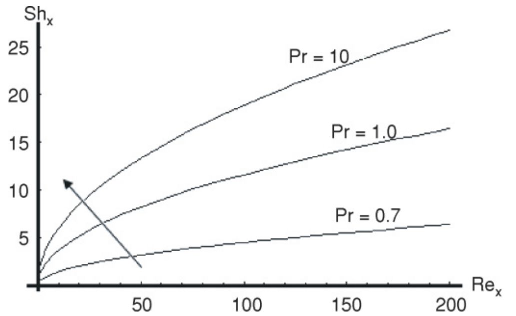

Figure 12. Local Sherwood number *vs.* local Reynolds number for different values of Pr = 0.7, 1.0, and 10.0

For the sake of comparison numerical results have also been computed just in our analytic solution for parameter values and listed in tabs. 1 and 2 beside the previously computed results by the others used numerical solutions; we reported agreement with these results as shown in the tables.

Table 1. Comparison of the present analytic calculation of $f''(0)$, $H'(0)$, $\Theta'(0)$, and $\Phi'(0)$ by the calculated values [22]

Pr	Present results				Abd El-Aziz [22]			
	$f''(0)$	$H'(0)$	$\Theta'(0)$	$\Phi'(0)$	$f''(0)$	$H'(0)$	$\Theta'(0)$	$\Phi'(0)$
0.71	–1.000	0.9732	1.1874	0.4493	–1.0000	0.9465	1.2056	0.4540
3.0	–1.000	0.4387	0.4883	1.1598	–1.0000	0.4285	0.4885	1.1652
7.0	–1.000	0.2933	0.3192	1.8896	–1.0000	0.2828	0.3092	1.8954

Table 2. The values of $f''(0)$, $H'(0)$, $\Theta'(0)$, and $\Phi'(0)$ for different values of M result by the analytic solution evaluated at: Pr = 0.71, Gr = 1, Gc = 1, Sc = 0.22, D_u = 0.06, Sr = 1, A* = B* = 0.01, and γ = 0.1

M	$f''(0)$	$H'(0)$	$\Theta'(0)$	$\Phi'(0)$
0.01	0.80613	0.32114	–0.39936	–0.39748
0.1	0.95711	0.37794	–0.37286	–0.38768
0.3	1.10667	0.53926	–0.32118	–0.35101
0.5	1.23667	0.57365	–0.31227	–0.33656
1.0	1.35667	0.58926	–0.25118	–0.31107

Conclusions

In this paper the boundary layer problem is formulated for the mixed free-forced convective MHD fluid flow with heat and mass transfer over an inclined permeable stretching plate in a porous medium influenced by: chemical reaction, thermal-diffusion and diffusion-thermo, internal heat generation/absorption, porosity, and Hall effect. The highly non-linear coupled system of partial differential equations characterizing flow, heat and mass transfer has been converted to a coupled system of non-linear ordinary differential equations by applying suitable similarity transformations. The resulting system solved by homotopy analysis method to obtain a uniformly analytic solution. The obtained solution is examined and analyzed by verification and applying to a realistic case with specific parameter values.

A graphical verification of the obtained analytic solution for the velocities $f'(\eta)$ and $H(\eta)$, the temperature $\Theta(\eta)$ and concentration $\Phi(\eta)$ given by solution (19) is examined graphically for a set of parameter values of the problem. A good agreement with the previous special cases discussed numerically in the literatures are noticed and reported in tables. The derived analytic solution (19) is serve as a direct tool (for the first time) to introduce analytic series expressions for the important engineering characteristic numbers: skin friction coefficients, Nusselt number, local Sherwood number, stress at the stretching surface, local mass transfer coefficient, the local wall mass flux, the local heat transfer coefficient, and the local heat flux eqs. (49)-(56). Acording to the knowledge of the author, this is not reported before in the literature. The main advantage of this analytic solution for such highly complicated problems *vs.* the numerical solution is that we can investigate such problems for a wide class of parameters values more economically. The analytic solution serves as a tool to avoid the expensive numerical solution and difficulties that arise for some specific parameter values during the numerical computations. The power of the homotopy analysis method to treat such non-linear problems provides a direct method to treat the MHD problems with additional effects appears in the realistic problems. The important advantage to treat analytically such realistic problems is to provide the ability to build a control system on the parameters set of the problems related to the quality production.

Nomenclature

A^* – coefficients of space-dependent internal heat generation/absorption, $[\mathrm{Ks^{-1}}]$
B^* – coefficients of temperature-dependent internal heat generation/absorption, $[\mathrm{Ks^{-1}}]$
B_0 – the magnetic induction, [Tesla]
C – concentration in fluid layer, $[\mathrm{kmolm^{-3}}]$
C_{fx} – local skin friction coefficient, [–]
C_{fz} – local skin friction coefficient, [–]
C_s – concentration susceptibility, [–]
C_w – concentration at sheet surface, $[\mathrm{kmolm^{-3}}]$
C_∞ – concentration in free stream, $[\mathrm{kmolm^{-3}}]$
Da – Darcy number $[=(\mathrm{Gr})^{1/2}K/\nu]$
D_m – the coefficient of mass diffusivity, $[\mathrm{m^2s^{-1}}]$
$\mathrm{D_u}$ – Dufour number $[=D_m k_T(C_w - C_\infty)/c_p C_s(T_w - T_\infty)]$, [–]
$f'(\eta)$, $H(\eta)$ – dimensionless velocities
$\mathrm{Gc_x}$ – local mas Grashof number $[g\beta_C(C_w - C_\infty)x^3\cos\vartheta\ \nu^2$, [–]
$\mathrm{Gr_x}$ – local heat Grashof number $[g\beta_T(T_w - T_\infty)x^3\cos\vartheta/\nu^2$, [–]
h_m – mass transfer coefficient, [–]
h_w – heat transfer coefficient, [–]
K – permeability of the porous medium, $[\mathrm{m^2}]$
K_T – thermal diffusion ratio, [–]
k – thermal conductivity, $[\mathrm{Wm^{-1}K^{-1}}]$
k_m – thermal diffusivity $(=k/\rho c_p)$, $[\mathrm{m^2s^{-1}}]$
k_1 – chemical reaction parameter of mass generation/destructive, $[\mathrm{s^{-1}}]$
M – magnetic parameter $(=\sigma B_0^2/\rho U)$, [–]
m – Hall parameter, [–]
m_w – mass flux at sheet surface, $[\mathrm{kmolm^{-2}s^{-1}}]$
$\mathrm{Nu_x}$ – local Nusselt number, $[kh_w\mathrm{K^{-1}}]$
Pr – Prandtl number $(=\rho\nu c_p\ \mathrm{K^{-1}})$
Q – coefficient of heat generation/absorption, [W]
q_w – heat flux at the surface, $[\mathrm{Wm^{-2}}]$
q''' – rate of internal heat generation, $[\mathrm{Ws^{-1}}]$
$\mathrm{Ra_x}$ – local Darcy-Rayleigh number $[(g\beta_T(T_w - T_\infty)Kx/\nu\beta]$, [–]
$\mathrm{Re_x}$ – local Reynolds number $[=(Ux/\nu)]$, [–]

Sc – Schmidt number (= ν/D_m), [–]
Sh_x – local Sherwood number (=x_{hm}/D_m), [–]
Sr – Soret number [= $D_m k_T(T_w - T_\infty)/T_m(C_w - C_\infty)$], [–]
T – temperature inside the boundary layer, [K]
T_m – mean fluid temperature, [K]
T_w – temperature at sheet surface, [K]
T_∞ – fluid temperature in the free stream, [K]
x, y, z – rectangular Cartesian co-ordinates, [m]

Greek letters

β_C – expansion coefficient with concentration, [m^3kmol^{-1}]
β_T – coefficient expansion with temperature, [K^{-1}]
γ – dimensionless chemical reaction parameter (= $\nu k_1/U^2$)
μ – dynamic viscosity [$kgm^{-1}s^{-1}$]
ν – kinematic viscosity (= μ/ρ), [m^2s^{-1}]
ρ – density of the fluid, [kgm^{-3}]
σ – electrical conductivity, [$^{-1}m^{-1}$]
ϑ – inclination angle, [rad]
τ_{wx} – shear stress in the x-direction [Nm^{-2}]
τ_{wz} – shear stress in the z-direction [Nm^{-2}]

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References

[1] Sakiadis, B. C., Boundary Layer Behavior on Continuous Solid Surface: I. Boundary Layer Equations for Two-Dimensional and Axisymmetric Flow, *AIChE J., 7* (1961), 1, pp. 26-28
[2] Sakiadis, B. C., Boundary-Layer Behavior on Continuous Solid Surface: The Boundary Layer on a Continuous Flat Surface, *AIChE J., 7* (1961), 2, pp. 221-225
[3] Crane, L. J., Flow Past a Stretching Plate, *Z. Angew. Math. Phys., 21* (1970), 4, pp. 645-647
[4] Andersson, H. I., MHD Flow of a Visco-Elastic Fluid Past a Stretching Surface, *Acta. Mech., 95* (1992), 1-4, pp. 227-230
[5] Dey, J., Nath, G., Mixed Convection Flow on Vertical Surface, *Wärme Stoffübertragung, 15* (1981), 1, pp. 279-283
[6] De Hoog, F. R., Laminger, B., Weiss, R., A Numerical Study of Similarity Solutions for Combined Forced and Free Convection, *Acta Mechanica, 51* (1984), 3-4, pp. 139-149
[7] Schneider, W., Laminar Mixed Convection Flows on Horizontal Surfaces, *Third Caribbean Congress on Fluid Dynamics,* Caracas, Venezuela, 1995, vol. 1, pp. 1-7
[8] Chamkha, A. J., Hameed, Al-Naser, Hydro-Magnetic Double-Diffusive Convection in a Rectangular Enclosure with Opposing Temperature and Concentration Gradients, *International Journal of Heat and Mass Transfer, 45* (2002), 12, pp. 2465-2483
[9] Merkin, J. H., Pop, I., Mixed Convection along a Vertical Surface: Similarity Solutions for Uniform Flow, *Fluid Dynamics Research, 30* (2002), 4, pp. 233-250
[10] Seddeek, M. A., Finite Element Method for the Effects Chemical Reaction, Variable Viscosity, Thermo-Phoresis and Heat Generation/Absorption on a Boundary-Layer Hydromagnetic Flow with Heat and Mass Transfer over a Heat Surface, *Acta Mechanica, 177* (2005), 1-4, pp. 1-18
[11] Kandasamy, P., Periasamy, K., Sivagnana Prabhu, K. K., Effects of Chemical Reaction, Heat and Mass Transfer along a Wedge with Heat Source and Concentration in the Presence of Suction or Injection, *Int. J. Heat and Mass Transfer, 48* (2005), 7, pp. 1388-1394
[12] Anjalidevi, S. P., Kandasamy, R., Effect of Chemical Reaction, Heat and Mass Transfer on Laminar Flow along a Semi Infinite Horizontal Plate, *Heat and Mass Transfer, 35* (1999), 6, pp. 465-472
[13] Cortell, R., MHD Flow and Mass Transfer of an Electrically Conducting Fluid of Second Grade in a Porous Medium over a Stretching Sheet with Chemically Reactive Species, *Ch. Eng. and Proc., 46* (2007), 8, pp. 721-728
[14] Kafoussias, N. G., Williams, E. W., Thermal-Diffusion and Diffusion-Thermo Effects on Mixed Free-Forced Convective and Mass Transfer Boundary Layer Flow with Temperature Dependent Viscosity, *International Journal of Engineering Science, 33* (1995), 9, pp. 1369-1384
[15] Afify, A., Effects of Temperature-Dependent Viscosity with Soret and Dufour Numbers on Non-Darcy MHD Free Convective Heat and Mass Transfer Past a Vertical Isothermal Surface Embedded in a Porous Medium, *Transport in Porous Media, 66* (2007), 3, pp. 391-401

[16] Seddeek, M. A., Finite Element Method for the Effect of Various Injection Parameter on Heat Transfer for a Power-Law Non-Newtonian Fluid over a Stretched Surface with Thermal Radiation, *Comp. Mater. Sci., 37* (2006), 4, pp. 624-627
[17] Afify, A., Effects of Variable Viscosity on Non-Darcy MHD Free Convection along a Non-Isothermal Vertical Surface in a Thermally Stratified Porous Medium, *Appl. Math. Modeling, 31* (2007), 8, pp. 1621-1634
[18] Ghaly, A.Y., Seddeek, M. A., Chebyshev Finite Difference Method for the Effects of Chemical Reaction, Heat and Mass Transfer on Laminar Flow along a Semi Infinite Horizontal Plate with Temperature Dependent Viscosity, *Chaos Solitons and Fractals, 19* (2004), 1, pp. 61-70
[19] Postelnicu, A., Influence of a Magnetic Field on Heat and Mass Transfer by Natural Convection from Vertical Surfaces in Porous Media Considering Soret and Dufour Effects, *Int. J. Heat Mass Transfer, 47* (2004), 6-7, pp. 1467-1472
[20] Postelnicu, A., Influence of Chemical Reaction on Heat and Mass Transfer by Natural Convection from Vertical Surfaces in Porous Media Considering Soret and Dufour Effects, *Heat Mass Transfer, 43* (2007), 6, pp. 595-602
[21] Seddeek, M. A., Thermal-Diffusion and Diffusion-Thermo Effects on Mixed Free-Forced Convective Flow and Mass Transfer over an Accelerating Surface with a Heat Source in the Presence of Suction and Blowing in the Case of Variable Viscosity, *Acta Mechanica, 172* (2004), 1-2, pp. 83-94
[22] Abd El-Aziz, M., Temperature Dependent Viscosity and Thermal Conductivity Effects on Combined Heat and Mass Transfer in MHD Three-Dimensional Flow over a Stretching Surface with Ohmic Heating, *Meccanica, 42* (2007), 2, pp. 375-386
[23] Liao, S.-J., Beyond Perturbation: Introduction to the Homotopy Analysis Method, Chapman and Hall/CRC Press, Boca Raton, Fla., USA, 2003
[24] Xu, H., Liao, S.-J., Dual Solutions of Boundary Layer Flow over an Upstream Moving Plate, *Comm. Nonlin. Sci. Num. Sim., 13* (2008), 2, pp. 350-358.
[25] Abdallah, I. A., Homotopy Analytical Solution of MHD Fluid Flow and Heat Transfer Problem, *Applied Mathematics and Information Sciences*, 3 (2009), 2, pp. 223-233
[26] Liao, S.- J., An Analytic Solution of Unsteady Boundary-Layer Flows Caused by an Impulsively Stretching Plate, *Commun. Non-Linear Sci. Numer. Simul., 11* (2006), 3, pp. 326-339

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