RADIATION AND CHEMICAL REACTION EFFECTS ON ISOTHERMAL VERTICAL OSCILLATING PLATE WITH VARIABLE MASS DIFFUSION

by

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The unsteady flow of a viscous incompressible flow past an infinite isothermal vertical oscillating plate, in the presence of thermal radiation and homogeneous chemical reaction of first order has been studied. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised to T_w and the concentration level near the plate is raised linearly with respect to time. An exact solution to the dimensionless governing equations has been obtained by the Laplace transform method, when the plate is oscillating harmonically in its own plane. The effects of velocity, temperature, and concentration are studied for different physical parameters like phase angle, radiation parameter, chemical reaction parameter, Schmidt number, thermal Grashof number, mass Grashof number, and time are studied graphically. It is observed that the velocity increases with decreasing phase angle ωt.

Key words: *chemical reaction, gray, oscillating, radiation, vertical plate, heat and mass transfer, radiation*

Introduction

Thermal radiation effects on heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines, and various propulsion device for aircraft, missiles, satellites, and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. England *et al.* [1] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain *et al*. [2]. Raptis *et al.* [3] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Das *et al*. [4] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate.

The effect of a chemical reaction depend whether the reaction is homogeneous or heterogeneous. This depends on whether they occur at an interface or as a single phase volume reaction. In well-mixed systems, the reaction is heterogeneous, if it takes place at an interface and homogeneous, if it takes place in solution. Chambre *et al.* [5] have analyzed a first order chemical reaction in the neighborhood of a stationary horizontal plate. Das *et al.* [6] have studied the effect of homogeneous first order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Again, mass transfer effects on moving isothermal vertical plate in the presence of chemical reaction studied by Das *et al*. [7]. The dimensionless governing equations were solved by the usual Laplace-transform technique and the solutions are valid only at lower time level.

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar [8]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar *et al.* [9]. Radiation effects on the oscillatory flow past vertical in the presence of uniform temperature analyzed by Mansour [10]. The governing were solved by perturbation technique. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar *et al.* [11]. Muthucumaraswamy [12] studied thermal radiation effects on vertical oscillating plate in the presence of variable temperature and mass diffusion.

It is proposed to study chemical reaction and thermal radiation effects on unsteady flow past infinite isothermal vertical oscillating plate with variable mass diffusion. The dimensionless governing equations are solved using the Laplace transform technique. The solutions are in terms of exponential and complementary error function. The present study will be found useful in the design of spaceships.

Mathematical formulation

Thermal radiation effects on unsteady flow of a viscous incompressible fluid past an infinite isothermal vertical oscillating plate with variable mass diffusion is studied. Here the unsteady flow of a viscous incompressible fluid which is initially at rest and surrounds an infinite vertical plate with temperature T_∞ and concentration C_∞. Here, the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, it is assumed that the plate and the fluid are of the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency ω' and the temperature of the plate is raised to T_w and the concentration level near the plate is raised linearly with respect to time. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. It is also assumed that there exists a homogeneous first order chemical reaction between the fluid and species concentration. Then by usual Boussinesq's approximation, the unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t} = g\beta(T - T_\infty) + g\beta^*(C - C_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C}{\partial t} = D\frac{\partial^2 C}{\partial y^2} - K_1 C \tag{3}$$

In most cases of chemical reactions, the reaction rate depends on the concentration of the species itself. A reaction is said to be of the order *n*, if the reaction rate is proportional to the n^{th} power of the concentration. In particular, a reaction is said to be first order, if the rate of reaction is directly proportional to concentration itself.

With the following initial and boundary conditions:

$$
\begin{aligned}
&t \le 0: \; u = 0, \quad T = T_\infty, \quad C = C_\infty \quad \text{for all } y \\
&t > 0: \; u = u_0 \cos \omega t, \quad T = T_w, \quad C = C_\infty + (C_w - C_\infty)At \quad \text{at } y = 0 \\
&\qquad u \to 0, \quad T \to T_\infty, \quad C \to C_\infty \quad \text{as } y \to \infty
\end{aligned}
\tag{4}
$$

The local radiant for the case of an optically thin gray gas is expressed by:

$$
-\frac{\partial q_\mathrm{r}}{\partial y} = 4a^* \sigma (T_\infty^4 - T^4)
\tag{5}
$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus:

$$
T^4 \cong 4T_\infty^3 T - 3T_\infty^4
\tag{6}
$$

By using eqs. (5) and (6), eq. (2) reduces to:

$$
\rho C_\mathrm{p} \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 (T_\infty - T)
\tag{7}
$$

The dimensionless quantities are defined as:

$$
\begin{gathered}
U = \frac{u}{u_0}, \quad t = \frac{t\,u_0^2}{\nu}, \quad Y = \frac{y u_0}{\nu}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty} \\
\mathrm{Gr} = \frac{g\beta\nu(T_\mathrm{w} - T_\infty)}{u_0^3}, \quad C = \frac{C - C_\infty}{C_\mathrm{w} - C_\infty}, \quad \mathrm{Gc} = \frac{g\beta^*\nu(C_\mathrm{w} - C_\infty)}{u_0^3} \\
\mathrm{Pr} = \frac{\mu C_p}{k}, \quad \mathrm{Sc} = \frac{\nu}{D}, \quad R = \frac{16a^*\nu^2\sigma T_\infty^3}{k u_0^2} \quad K = \frac{\nu K_1}{u_0^2}, \quad \omega = \frac{\omega\,\nu}{u_0^2}
\end{gathered}
\tag{8}
$$

in eqs. (1) to (4), leads to:

$$
\frac{\partial U}{\partial t} = \mathrm{Gr}\theta + \mathrm{Gc}C + \frac{\partial^2 U}{\partial Y^2}
\tag{9}
$$

$$
\frac{\partial \theta}{\partial t} = \frac{1}{\mathrm{Pr}} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\mathrm{Pr}} \theta
\tag{10}
$$

$$
\frac{\partial C}{\partial t} = \frac{1}{\mathrm{Sc}} \frac{\partial^2 C}{\partial Y^2} - KC
\tag{11}
$$

The initial and boundary conditions in non-dimensional form are:

$$
\begin{aligned}
&U = 0, \quad \theta = 0, \quad C = 0, \quad \text{for all } Y, t \le 0 \\
t > 0: \; &U = \cos \omega t, \quad \theta = 1, \quad C = t, \quad \text{at } Y = 0 \\
&U = 0, \quad \theta \to 0, \quad C \to 0 \quad \text{as } Y \to \infty
\end{aligned}
\tag{12}
$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of thermal radiation and chemical reaction.

The eqs. (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta \quad \frac{1}{2}[\exp(2\eta\sqrt{Rt})\operatorname{erfc}(\eta\sqrt{\Pr} \quad \sqrt{at}) \quad \exp(\quad 2\eta\sqrt{Rt})\operatorname{erfc}(\eta\sqrt{\Pr} \quad \sqrt{at})] \tag{13}$$

$$C \quad \frac{t}{2}[\exp(2\eta\sqrt{Kt\mathrm{Sc}})\operatorname{erfc}(\eta\sqrt{\mathrm{Sc}} \quad \sqrt{Kt}) \quad \exp(\quad 2\eta\sqrt{Kt\mathrm{Sc}})\operatorname{erfc}(\eta\sqrt{\mathrm{Sc}} \quad \sqrt{Kt})]$$

$$\frac{\eta\sqrt{\mathrm{Sc}t}}{2\sqrt{K}}[\exp(\quad 2\eta\sqrt{Kt\mathrm{Sc}})\operatorname{erfc}(\eta\sqrt{\mathrm{Sc}} \quad \sqrt{Kt}) \quad \exp(2\eta\sqrt{Kt\mathrm{Sc}})\operatorname{erfc}(\eta\sqrt{\mathrm{Sc}} \quad \sqrt{Kt})] \tag{14}$$

$$U \quad \frac{\exp(i\omega t)}{4}[\exp(2\eta\sqrt{i\omega t})\operatorname{erfc}(\eta \quad \sqrt{i\omega t}) \quad \exp(\quad 2\eta\sqrt{i\omega t})\operatorname{erfc}(\eta \quad \sqrt{i\omega t})]$$

$$\frac{\exp(\quad i\omega t)}{4}[\exp(2\eta\sqrt{\quad i\omega t})\operatorname{erfc}(\eta \quad \sqrt{\quad i\omega t}) \quad \exp(\quad 2\eta\sqrt{\quad i\omega t})\operatorname{erfc}(\eta \quad \sqrt{\quad i\omega t})]$$

$$2(d \quad ce)\operatorname{erfc}(\eta) \quad d\exp(bt)[\exp(2\eta\sqrt{bt})\operatorname{erfc}(\eta \quad \sqrt{bt}) \quad \exp(\quad 2\eta\sqrt{bt})\operatorname{erfc}(\eta \quad \sqrt{bt})]$$

$$e\exp(ct)[\exp(2\eta\sqrt{ct})\operatorname{erfc}(\eta \quad \sqrt{ct}) \quad \exp(\quad 2\eta\sqrt{ct})\operatorname{erfc}(\eta \quad \sqrt{ct})]$$

$$d[\exp(2\eta\sqrt{Rt})\operatorname{erfc}(\eta\sqrt{\Pr} \quad \sqrt{at}) \quad \exp(\quad 2\eta\sqrt{Rt})\operatorname{erfc}(\eta\sqrt{\Pr} \quad \sqrt{at})]$$

$$d\exp(bt)\{\exp[\quad 2\eta\sqrt{\Pr(a \quad b)t}]\operatorname{erfc}[\eta\sqrt{\Pr} \quad \sqrt{(a \quad b)t}]$$

$$\exp[2\eta\sqrt{\Pr(a \quad b)t}]\operatorname{erfc}[\eta\sqrt{\Pr} \quad \sqrt{(a \quad b)t}]\}$$

$$e(1 \quad ct)[\exp(2\eta\sqrt{Kt\mathrm{Sc}})\operatorname{erfc}(\eta\sqrt{\mathrm{Sc}} \quad \sqrt{Kt}) \quad \exp(\quad 2\eta\sqrt{Kt\mathrm{Sc}})\operatorname{erfc}(\eta\sqrt{\mathrm{Sc}} \quad \sqrt{Kt})]$$

$$\frac{ec\eta\sqrt{\mathrm{Sc}t}}{\sqrt{K}}[\exp(\quad 2\eta\sqrt{Kt\mathrm{Sc}})\operatorname{erfc}(\eta\sqrt{\mathrm{Sc}} \quad \sqrt{Kt}) \quad \exp(2\eta\sqrt{Kt\mathrm{Sc}})\operatorname{erfc}(\eta\sqrt{\mathrm{Sc}} \quad \sqrt{Kt})]$$

$$e\exp(ct)\{\exp[\quad 2\eta\sqrt{\mathrm{Sc}(K \quad c)t}]\operatorname{erfc}[\eta\sqrt{\mathrm{Sc}} \quad \sqrt{(K \quad c)t}]$$

$$\exp[2\eta\sqrt{\mathrm{Sc}(K \quad c)t}]\operatorname{erfc}[\eta\sqrt{\mathrm{Sc}} \quad \sqrt{(K \quad c)t}]\} \tag{15}$$

where

$$a \quad \frac{R}{\Pr}, \; b \quad \frac{R}{1 \quad \Pr}, \; c \quad \frac{K\mathrm{Sc}}{1 \quad \mathrm{Sc}}, \; d \quad \frac{\mathrm{Gr}}{2b(1 \quad \Pr)}, \; e \quad \frac{Gc}{2c^2(1 \quad \mathrm{Sc})}, \text{ and } \eta \quad \frac{Y}{2\sqrt{t}}$$

In order to get the physical insight into the problem, the numerical values of U have been computed from eq. (15). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$\operatorname{erf}(a \quad ib) \quad \operatorname{erf}(a) \quad \frac{\exp(\quad a^2)}{2a\pi}[1 \quad \cos(2ab) \quad i\sin(2ab)]$$

$$\frac{2\exp(\quad a^2)}{\pi} \sum_{n \quad 1}^{\infty} \frac{\exp(\quad n^2/4)}{n^2 \quad 4a^2}[f_n(a,b) \quad ig_n(a,b)] \quad \varepsilon(a,b)$$

where

$$f_n \quad 2a \quad 2a\cosh(nb)\cos(2ab) \quad n\sinh(nb)\sin(2ab)$$

$$g_n \quad 2a\cosh(nb)\sin(2ab) \quad n\sinh(nb)\cos(ab)$$

$$|\varepsilon(a,b)| \quad 10^{\quad 16}|\operatorname{erf}(a \quad ib)|$$

Results and discussion

In order to get a physical view of the problem the numerical values of the velocity, temperature and concentration for different values of the phase angle, radiation parameter, magnetic field parameter, Schmidt number, and time. The purpose of the calculations given here is to asses the effect of different ωt, K, R, Sc, and t upon the nature of the flow and transport. The Laplace transform solutions are in terms of exponential and complementary error function.

The temperature profiles are calculated for different values of thermal radiation parameter ($R = 2, 5, 7, 10$) from eq. (13) and these are shown in fig. 1 for air (Pr = 0.71) at time $t =$ $= 0.4$. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter.

Figure 2 demonstrates the effect of the concentration profiles for different values of the chemical reaction parameter ($K = 2, 5, 10$), Sc = 0.6, and time $t = 0.4$. It is observed that the concentration increases with decreasing chemical reaction parameter. Figure 3 represents the effect of concentration profiles at time $t = 0.4$ for different Schmidt number (Sc = 0.16, 0.3, 0.6, 2.01), and $K = 2$. The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number. The concentration profiles for different time ($t = 0.2, 0.4, 0.6, 1$), Sc = 0.6, and $t = 0.2$ are shown in fig. 4. The trend shows that the wall concentration increases with increasing values of the time.

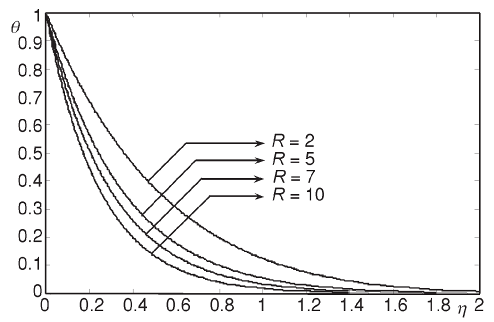

Figure 1. Temperature profiles for different values of *R*

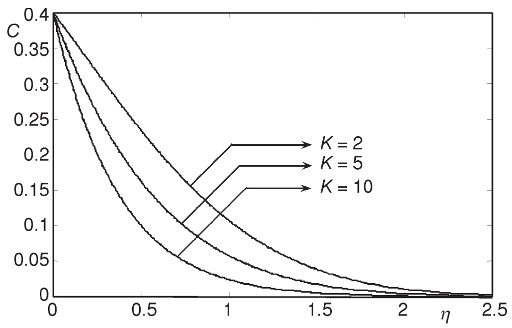

Figure 2. Concentration profiles for different values of *K*

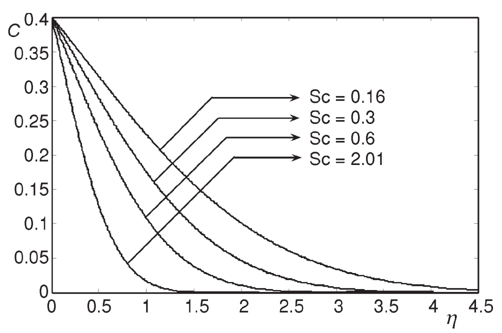

Figure 3. Concentration profiles for different values of Sc

The velocity profiles for different phase angles ($\omega t = 0, \pi/4, \pi/3, \pi/2$), $R = 10$, $K = 4$, Gr = = 2, Gc = 2, Sc = 0.6, Pr = 0.71, and $t = 0.2$ are shown in fig. 5. It is observed that the velocity increases with decreasing phase angle ωt. Figure 6 illustrates the effect of the velocity for different values of the reaction parameter (K = = 0.2, 7, 20), $\omega t = \pi/4$, $R = 5$, Gr = 5, Gc = 5, Sc = 0.6, Pr = 0.71, and $t = 0.4$. The trend shows that the velocity increases with decreasing chemical reaction parameter.

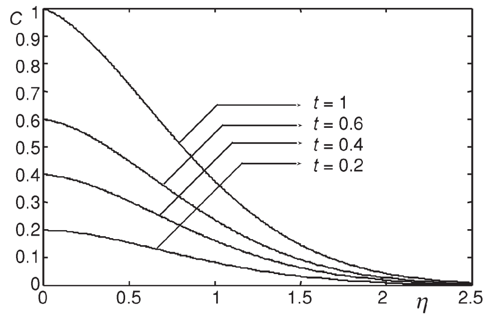

Figure 4. Concentration profiles for different values of *t*

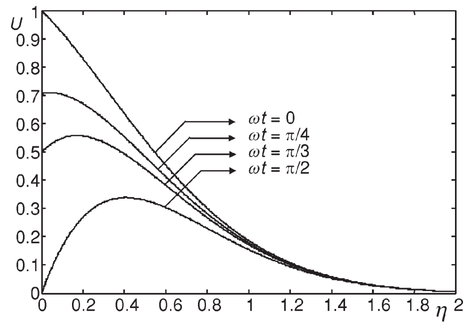

Figure 5. Velocity profiles for different values of *ωt*

The effect of velocity for different values of the radiation parameter ($R = 0.2, 5, 20$), $\omega t = \pi/4$, $K = 4$, Gr = 5, Gc = 2, Pr = 0.71, Sc = 0.6, and $t = 0.2$ are shown in figure 7. The trend shows that the velocity increases with decreasing radiation parameter. It is observed that the velocity decreases in the presence of high thermal radiation.

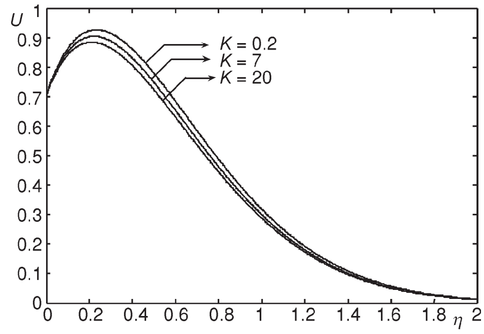

Figure 6. Velocity profiles for different values of *K*

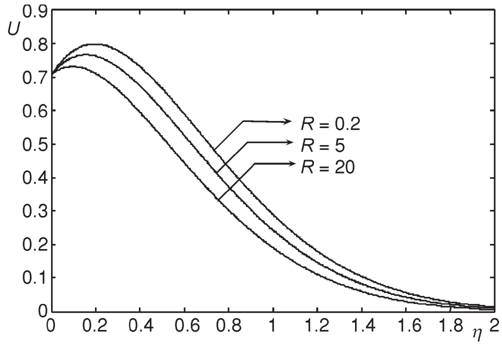

Figure 7. Velocity profiles for different values of *R*

The effect of velocity profiles for different time ($t = 0.2, 0.3, 0.4$), $R = 5$, $K = 2$, $\omega t = \pi/4$, Gr = 5, Gc = 5, Pr = 0.71, and Sc = 0.6 are shown in fig. 8. In this case, the velocity in-

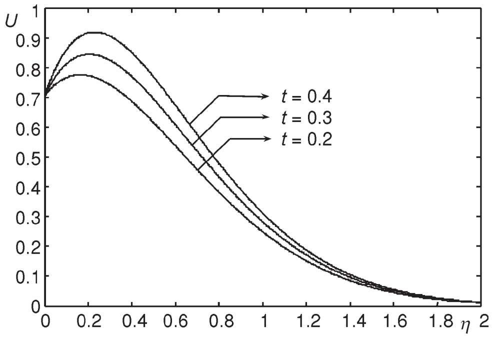

Figure 8. Velocity profiles for different values of *t*

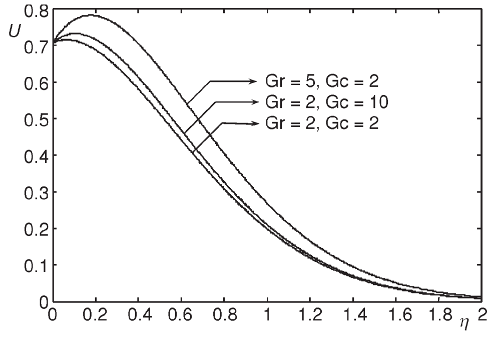

Figure 9. Velocity profiles for different values of Gr and Gc

creases gradually with respect to time t. The velocity profiles for different thermal Grashof number (Gr = 2, 5), mass Grashof number (Gc = 2, 10), $\omega t = \pi/4$, $K = 7$, $R = 2$, Sc = 0.6, Pr = = 0.71, and time $t = 0.2$ are shown in fig. 9. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number.

Conclusions

An exact analysis is performed to study thermal radiation effects on unsteady flow past an infinite isothermal vertical oscillating plate, in the presence of variable wall concentration. The governing equations are solved using Laplace transform technique. The conclusions of the study are:

- the velocity increases with decreasing phase angle ωt and radiation parameter R,
- the wall concentration increases with decreasing Schmidt number,
- the temperature decreases due to high thermal radiation, and
- the leading edge effect is not affected by the oscillation of the plate.

Nomenclature

A – constant, [–]
a^* – absorption coefficient, [m^{-1}]
C' – concentration, [kgm^{-3}]
C – dimensionless concentration
Cp – specific heat at constant pressure, [$Jkg^{-1}K$]
D – mass diffusion coefficient, [m^2s^{-1}]
erfc – complementary error function
g – acceleration due to gravity, [ms^{-2}]
Gr – thermal Grashof number, [–]
Gc – mass Grashof number, [–]
K – dimensionless chemical reaction parameter
K_1 – chemical reaction parameter, [–]
k – thermal conductivity, [$Wm^{-1}K^{-1}$]
Pr – Prandtl number, [–]
q_r – radiative heat flux in the y-direction, [Wm^{-2}]
R – radiation parameter
Sc – Schmidt number, [–]
T – temperature of the plate, [K]
t – dimensionless time
t_0 – time, [s]
U – dimensionless velocity component in x-direction
u – velocity component in x-direction, [ms^{-1}]
u_0 – amplitude of the oscillation, [ms^{-1}]
x – spatial coordinate along the plate, [m]
Y – dimensionless spatial coordinate normal to the plate
y – spatial coordinate normal to the plate, [m]

Greek letters

α – thermal diffusivity, [$m^{-2}s^{-1}$]
β – volumetric coefficient of thermal expansion, [K^{-1}]
β^* – volumetric coefficient of expansion with concentration, [K^{-1}]
μ – coefficient of viscosity, [Ra·s]
ωt – phase angle
ν – kinematic viscosity, [m^2s^{-1}]
ρ – density of the fluid, [kgm^{-3}]
τ – dimensionless skin-friction, [$kgm^{-1}s^2$]
σ – Stefan-Boltzman constant (= $5{\cdot}67{\cdot}10^{-8}$), [$Wm^{-2}K^{-4}$]
θ – dimensionless temperature
η – similarity parameter

Subscripts

ω – conditions on the wall
∞ – free stream conditions

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