GEOMETRIC OPTIMIZATION OF CROSS-FLOW HEAT EXCHANGER BASED ON DYNAMIC CONTROLLABILITY

by

Sorour ALOTAIBI and Asad ALEBRAHIM

Original scientific paper
UDC: 66.045.1:66.011
BIBLID: 0354-9836, *12* (2008), 3, 75-84
DOI: 10.2298/TSCI0802075A

The operation of heat exchangers and other thermal equipments in the face of variable loads is usually controlled by manipulating inlet fluid temperatures or mass flow rates, where the controlled variable is usually one of the output temperatures. The aim of this work is to optimize the geometry of a tube with internal flow of water and an external cross-flow of air, based on its controllability characteristics. Controllability is a useful concept both from theoretical and practical perspective since it tells us if a particular output can be controlled by a particular input. This concept can also provide us with information about the easiest operating condition to control a particular output. A transient model of a tube in cross-flow is developed, where an implicit formulation is used for transient numerical solutions. The aspect ratio of the tube is optimized, subject to volume constraints, based on the optimum operation in terms of controllability. The reported optimized aspect ratio, water mass flow rate and controllability are studied for deferent external properties of the tube.

Key words: *geometric optimization, controllability, cross-flow heat exchanger*

Introduction

The configuration of thermal systems is usually designed based on its steady-state performance subject to global and local constraints, where its configuration is optimized for maximization of global performance or for minimization of total cost [1]. This is usually done by assuming the system configuration and then simulating its operation under various conditions to determine its best operating conditions. Recently another approach has been used, in which the optimization procedure goes into searching for the best design parameter to determine the optimal geometry that would produce the maximum performance. In this procedure the configuration of the system is simulated and the effect of the system geometric parameters on its performance is extensively studied to determine the optimal geometry for this system. The maximum performance is usually determined by the maximum output of the system. However, this does not include the performance of other parameters (pressure drop for example). That's encouraged using entropy generation as the main factor of the optimization process in which the system is optimized to produce the minimum irreversibility. This procedure has been considered in many heat exchanger optimizations [2, 3]. The minimization of irreversibility can be used for the optimization process specially in systems in which the total cost is dominated by the cost of thermodynamic irreversibility. On the other hand, thermal systems usually changing with time according to disturbances in the systems or changing in demands, therefore, a control system is always required to return the system to it is desired operating condition or to control fluid temperature at a specific location. This is often achieved by using ducts to transport fluids in theses systems according the control strategy. The effect of minimizing thermodynamic irreversibilities during the design of ducts may leads to poor controllability characteristics of these ducts. In terms of control, a good duct geometry will have a good thermal response characteristics and the ability to operate at different steady-state conditions. In designing a thermal control systems, there is a very important question that every control engineer should ask him self before designing a control for these systems. Is there away to improve the design of the system to achieve good control? Control engineers are usually not present from the beginning of the design steps. After the system is being designed to satisfy the required thermal performance, a control engineer is asked to design a control strategy for the system. This design procedure could be improved by incorporating the control requirements in design from the beginning of design steps. Therefore, another important factor that should be taken into account is the system controllability; it is the ability of a system to be controled by the available control input which could be in the form of a flow rate, an applied heat flux or an external applied temperature [4]. This can be done by investigating its state controllability; this is the ability of the complete system to be taken from any given state to any other within a prescribed time interval.

Controllability is easily tested for systems governed by a system of linear, finite-dimensional ordinary differential equations [5]. The situation is more complicated for linear infinite-dimensional systems such as those governed by partial differential equations [6, 7].

The application in thermal systems is very limited. This includes some work in industrial and chemical plants and thermal networks [8]. In the field of heat exchangers, the controllability of multi-stream heat exchanger when some operating parameters deviated from their design value has been studied by [9]. Also, the state and output controllability of a cross-flow heat exchanger has been studied by [4].

In this work we introduce the use of the dynamic controllability with the system geometry for a better system design. A trade-off between them can be finally reached for an overall optimized design. The configuration of water-to-air cross-flow heat exchanger is optimized in this work to produce the best geometry in terms of the system controllability subjected to some constraints, such as the volume and the mass of the heat exchanger. Although the geometry of cross-flow heat exchanger can be complicated, the simplest geometry that can be easily computed, *i. e.* a single tube with water flow inside and cross-flow of air outside, will be considered here. A schematic of this arrangement is shown in fig. 1. Although a straight tube is shown, it may zig-zag over the face of the heat exchanger so as to make it more compact. The mathematical model is a pair of coupled partial differential equations representing conduction in the tube wall, advection by the in-tube fluid, and lateral convection to the over-tube fluid.

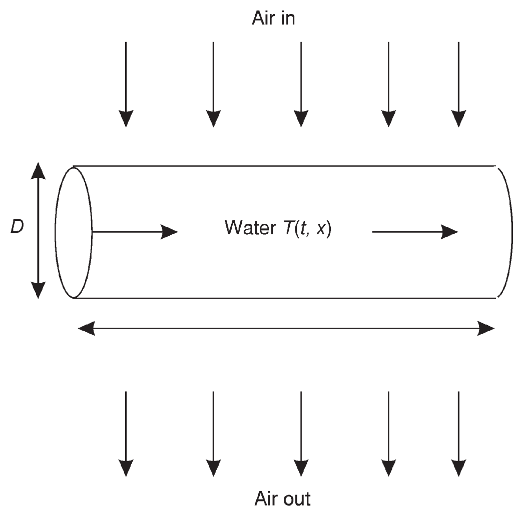

Figure 1. Schematic of a tube in cross-flow

Governing equations

To enable a one-dimensional analysis, the simplifying assumptions that the flow is hydro dynamically and thermally fully developed, and that the velocity and temperature are uniform over the cross-section of the pipe will be used. The physical properties of the fluids are also constant. There is convective heat transfer between the water and the tube wall, conduction along the tube wall, and convection between the tube wall and the surrounding air. The heat transfer coefficients are calculated using a standard correlations. The following are the governing equations for this problem with appropriate boundary conditions.

Consider a circular tube of constant cross-section as schematically shown in fig. 1.

The energy balance for the water inside the tube is:

$$\frac{\partial T_{\mathrm{w}}}{\partial t} \quad v_{\mathrm{w}} \frac{\partial T_{\mathrm{w}}}{\partial x} \quad \frac{4 h_i}{\rho_{\mathrm{w}} c_{\mathrm{w}} D_{\mathrm{i}}}(T_{\mathrm{w}} \quad T_{\mathrm{i}}) \quad 0 \tag{1}$$

where T_{w} is the water temperature, ρ_{w} – the water density, c_{w} – its specific heat, and $\dot{m}_{\mathrm{w}}$ – the water mass flow rate.

The energy balance for the tube wall is:

$$\rho_{\mathrm{t}} c_{\mathrm{t}} \pi (r_{\mathrm{o}}^2 \quad r_{\mathrm{i}}^2) \frac{\partial T_{\mathrm{t}}}{\partial t} \quad k_{\mathrm{t}} \pi (r_{\mathrm{o}}^2 \quad r_{\mathrm{i}}^2) \frac{\partial^2 T_t}{\partial x^2} \quad 2\pi r_{\mathrm{o}} h_{\mathrm{o}} (T_{\mathrm{a}} \quad T_{\mathrm{t}}) \quad 2\pi r_{\mathrm{i}} h_{\mathrm{i}} (T_{\mathrm{t}} \quad T_{\mathrm{w}}) \tag{2}$$

where ρ_{t} is the density of the tube material, c_{t} – its specific heat, k_{t} – its thermal conductivity, r_{i} – the inside radius of the tube, r_{o} – the outlet radius of the tube, h_{i} – the heat transfer coefficient in the inner surface of the tube, and T_{a} – the air temperature surrounding the tube.

On the outside of the tube, there is airflow driven by a variable-speed fan. Thus, we have convection between the tube and the air flow over it. The energy balance thus give:

$$\rho_{\mathrm{a}} v_{\mathrm{a}} A c_{\mathrm{a}} (T_{\mathrm{a}}^{\mathrm{in}} \quad T_{\mathrm{a}}^{\mathrm{out}}) \quad 2 h_{\mathrm{o}} \pi r_{\mathrm{o}} L (T_{\mathrm{a}} \quad T_{\mathrm{t}}) \tag{3}$$

where L is the length of the tube, ρ_{a} – the air density, v_{a} – the air velocity, A – the air flowing area, c_{a} – its specific heat, $T_{\mathrm{o}}^{\mathrm{in}}$ and $T_{\mathrm{o}}^{\mathrm{out}}$ are the incoming and outgoing air temperatures, and h_{o} is the heat transfer coefficient in the outer surface of the tube. For convenience, the air temperature can be assumed to be approximately:

$$T_{\mathrm{a}} \quad \frac{T_{\mathrm{a}}^{\mathrm{in}} \quad T_{\mathrm{a}}^{\mathrm{out}}}{2} \tag{4}$$

This can be substituted in eqs. (2) and (3) to eliminate the algebraic equation eq. (3).

In these equations, in general $T_{\mathrm{t}} = T_{\mathrm{t}}(x, t)$, $T_{\mathrm{w}} = T_{\mathrm{w}}(x, t)$, $T_{\mathrm{a}}^{\mathrm{out}} = T_{\mathrm{a}}^{\mathrm{out}}(t)$, and $T_{\mathrm{a}} = T_{\mathrm{a}}(t)$.

The boundary and initial conditions are $T_{\mathrm{t}}(0, t) = T_{\mathrm{w}}(0, t) = T_{\mathrm{w}}^{\mathrm{in}}$, $T_{\mathrm{t}}(L, t) = T_{\mathrm{w}}(L, t)$, and $T_{\mathrm{t}}(x, 0) = T_{\mathrm{w}}(x, 0) = 25$ °C (arbitrarily).

In the complete set of equations, the unknowns are $T_{\mathrm{a}}^{\mathrm{out}}(t)$, $T_{\mathrm{t}}(x, t)$, and $T_{\mathrm{w}}(x, t)$.

The convective heat transfer coefficients depend upon the mass flow rates of air and water, they can be evaluated from the standard dimensionless relations as in [10].

The objective is to solve for the outlet air temperature $T_{\mathrm{a}}^{\mathrm{out}}(t)$, and the outlet liquid temperature $T_{\mathrm{w}}^{\mathrm{out}}(t)$. Equations (2), (3), and (1) need to be solve together for the three unknowns $T_{\mathrm{t}}(x, t)$, $T_{\mathrm{a}}^{\mathrm{out}}(t)$, and $T_{\mathrm{w}}(x, t)$. This can be done by using finite differences schemes to approximating first- and second-order derivatives by upwind and central differences, for eqs. (1) and (2), respectively. This will enable us to put eqs. (1) and (2) after collecting the equations for all the nodes in the general form:

$$\frac{\partial T}{\partial t} \quad \mathbf{A} T(t) \quad \mathbf{B} u(t) \quad \mathbf{C} \tag{5}$$

where $T(t)$ contains all the temperatures – ($T_w(t)$ and $T_t(t)$, $u(t) = T_w^{in}$, **A**, **B**, and **C** are constant matrices. More details on the numerical solutions and the parameters used in above matrices can be found in [4]. A transformation of eq. (5) is needed to put this equation in the general linear system form to perform the controllability test as in [4].

Controllability

The control input $u(t)$ in the above equation can be any of the four inputs in the systems. The two inlet temperatures and two velocities for the liquid and the air. Different choices of the control input will leads to different problem. Using the water or air inlet temperature as a control input leads to a linear problem, and the water velocity it is non-linear since the manipulated variable v_w appears as a product with the unknown temperature $T_w(x, t)$ in eq. (1). In this case general controllability theorems are not available. For this reason the two input temperatures can be used as a control variable to the system.

In this problem T_w^{in} is used as a manipulated variable – control input $u(t)$ – as a pear in eq. (5) and T_a^{in}, v_w, and v_a are constant. It is known [5] that the state of a system of the form of eq. (5) is completely controllable if and only if the matrix:

$$\mathbf{M} = [\mathbf{B}, \mathbf{AB}, ..., \mathbf{A}^{n-1}\mathbf{B}] \quad \mathbb{R}^{n\ \ n} \tag{6}$$

is of full rank. In the present case it can be shown that:

$$\text{Rank } \mathbf{M} = n,$$

thus, matrix **M** is of full rank, indicating that the state of the system is controllable. This means that any water and air temperature within T_w^{in} and T_a^{in} (bounded inputs) can be reached along the tube. If the range of eigenvalues increases of matrix **M**, the rank becomes difficult to compute numerically. More importantly, however, this also means that the system needs large values of the control input which may not be available in practice. Therefore, the condition number of **M**, *CM*, being the ratio of the largest to smallest singular values, is thus used as an indicator of the degree to which the system may be controlled with a bounded input. Of course, **M** is singular and not of full rank if *CM* is infinite. Also, at the minimum *CM* the system is most controllable. Thus the condition number will provide us with information on the best situation for control of the tube flow when the inlet water temperature is used as a manipulated variable.

If another input is used as a manipulated variable *i. e*, T_a^{in}, the system in eq. (5) will be modified for this new control. In this case, the changes will be in matrices A and B, and the manipulated variable $u(t)$, where in this case $u(t) = T_a^{in}$. With this new control input we can also test the system controllability and compare it with the case when T_w^{in} is used by comparing the values of *CM*. This will provide us with an indication on which manipulated variable could be used to control the system more easily than the other if both of them are available. In any one of these cases, *CM* is a function of the physical parameters, and it will changed as these parameters will change, therefore, an optimization process will be used to find the minimum *CM* in terms of these parameters.

Non-linear controllability

As mentioned above, different choices of the control input will leads to different problem. In the previous results we choose the temperatures as an input, this will leads to a linear systems where their controllability is guaranteed in linear control theory using eq. (6). The problem became nonlinear when the manipulated variable $u(t)$ is v_w, in this case eq. (6) cannot be used. In this case a numerical example is used to show that our optimization results based on controllability are working in linear and non-linear situation.

As an example a simple control scheme is used to control the outlet temperature of the heat exchanger to a prescribed value using the water velocity (v_w) as a manipulated variable instead of T_w^{in} in eq. (5).

The objective of the control is to show that the non-linear system controllability is changing according to the aspect ratio values and the system is in the best controllability condition at the optimum aspect ratio corresponding to the minimum *CM* as shown in fig. 2.

Air enters to the heat exchanger at T_a^{in} = 25 °C and a target temperature was set to be T_a^{out} = 23 °C. A control scheme will be used to return the air outlet temperature to the target temperature by varying the water flow rate.

A proportional-integral control will be used for this purpose and the water flow rate will be modified using:

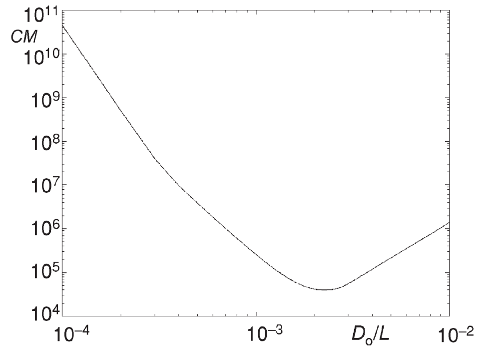

Figure 2. The variation of the controllability matrix condition number (*CM*) with respect to the tube aspect ratio

$$\frac{d\dot{m}_w}{dt} \quad K_I e \quad K_P \frac{de}{dt} \tag{7}$$

The error signal is $e = T_s - \overline{T}_a^{out}$, where T_s is the desired temperature. A detailed numerical solution of the governing equations along with the control simulation are presented in [10].

From fig. 2 the minimum *CM* which corresponds to the best controllability occurs at an aspect ratio around D_o/L = 0.0025, at this aspect ratio the system should behave easy to control than any other aspect ratio value. This fact is demonstrated by controlling the system to a prescribed value. Figures 3 and 4 shows the outlet air temperature controlled using water velocity. In each figure the above frame is for the air temperature and the lower frame is for the corresponding water velocity. In fig. 4 the system at an aspect ratio D_o/L = 0.01, it clear that the control scheme was unable to reduce the temperature to the target temperature 23 °C where the wa-

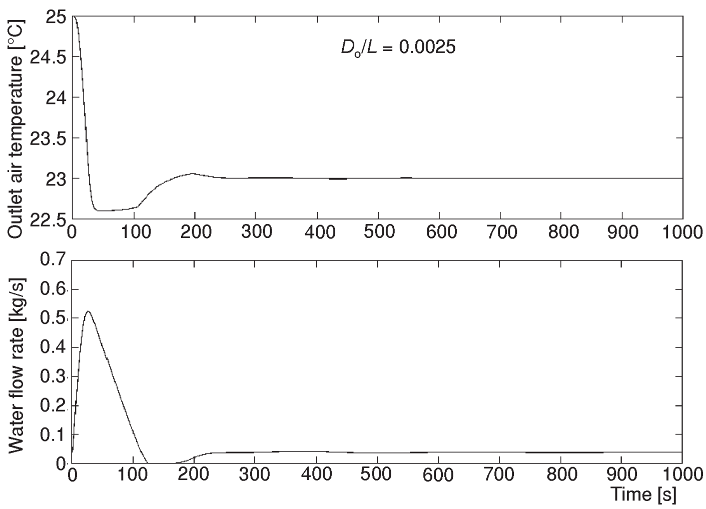

Figure 3. Control of air outlet temperature at the optimum aspect ratio D_o/L = 0.0025

ter velocity reached it is maximum. In fig. 3 the aspect ratio was set to be D_o/L = 0.0025 which correspond to the minimum *CM*. In this case the control scheme was able to control the temperature to the target with a minimum effort (water velocity).

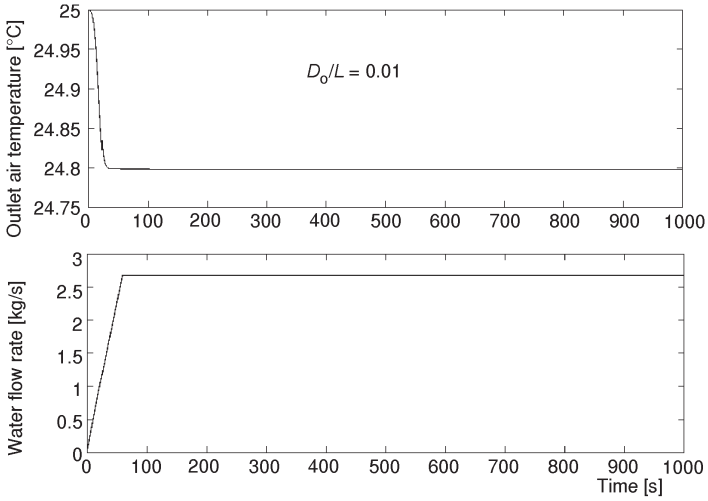

Figure 4. Control of air outlet temperature at an arbitrary aspect ratio D_o/L = 0.01

Geometric optimization

The cross-flow heat exchanger in this study is a single tube without fins. This heat exchanger tube have a three-dimensional parameters that can be varied; inner diameter D_i, outer diameters D_o, and the length L. This result in a three geometric degree of freedom. Therefore, in order to reduce this degree of freedom, two geometric constraints will be taken into account, the total volume and the mass of the heat exchanger. The total volume can be represented by the following relation:

$$V \quad \frac{\pi D_o^2}{4} L \tag{8}$$

The volume of the heat exchanger constraint can be represented by the volume occupied by the solid part of the tube:

$$V_t \quad \frac{\pi (D_o^2 \quad D_i^2)}{4} L \tag{9}$$

A non-dimensional parameter can be used now to represent the ratio of the tube volume to the total volume:

$$\phi \quad \frac{V_t}{V} \quad 1 \quad \frac{D_i^2}{D_o^2} \tag{10}$$

These two constraints will decrease the number of geometric degree of freedom to one, that will be presented by the dimensionless aspect ratio D_o/L.

The heat exchanger optimized in this work is described by the dimensionless and physical parameters shown in tab. 1. The optimization process starts by numerically simulate the cross-flow heat exchanger and minimize its controllability matrix condition number *CM* by varying its aspect ratio D_o/L while keeping all other parameters constant as shown in fig. 2. In this figure it can be seen that there is a minimum *CM* with respect to the aspect ration D_o/L. At this minimum *CM* the system has the minimum eigenvalues ratio as explained, which means that the system is best controllable at this aspect ratio. On the other hand, there are two extreme cases that can be seen in the figure. At a large aspect ratio the diameter is large while the length is small, therefore the heat transfer area between the tube and both fluids will be small. Thus, the water exit temperature T_w^{out} T_w^{in}, which means we have a very hard system to control, and that is indicated by the increase in *CM* at large aspect ration. Similarly at the other extreme, for small aspect ratio the diameter is small and the length is large. This results in a large heat transfer area and we have T_w^{out} T_a^{in}, which also leads to another hard situation to control. Therefore, at the minimum *CM* we have the optimum aspect ration in terms of the system controllability.

Table 1. Physical parameter for the heat exchanger optimized in fig. 1

$A = 1.5\, D_oL$	$V = 0.01$ m^3
$\phi = 0.1$	$\dot{m}_a = 1$ kg/s
$\dot{m}_w = 0.2$ kg/s	$T_a^{in} = 25$ °C
$T_w^{in} = 1$ °C	

The procedure illustrated in fig. 2 was repeated for different values of the water flow rate, $\dot{m}_w$. The minimum condition number in each case, labled CM_m is drawn against $\dot{m}_w$ as shown in fig. 5. It is clear that the already minimized CM_m, has a minimum value with respect to $\dot{m}_w$, which is mathematically corresponding to:

$$\frac{\partial^2}{\partial \left(\frac{D_o}{L}\right) \partial \dot{m}_w} CM \quad 0 \tag{11}$$

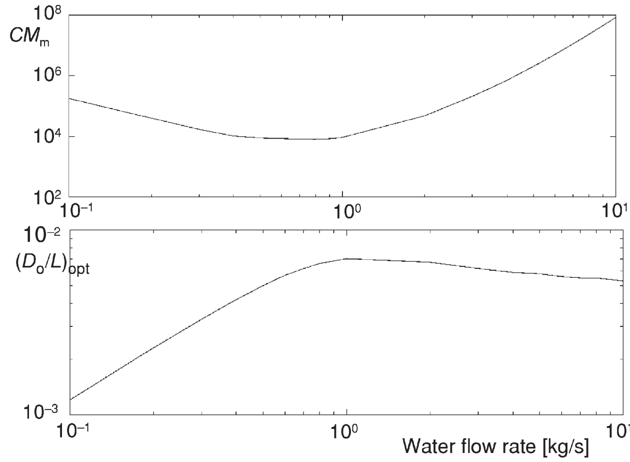

Figure 5. The variation of the minimized controllability matrix condition number (CM_m) and corresponding optimal aspect ratio with respect to the water mass flow rate

This result can be explained by discussing the extreme cases of the high and low water mass flow rate. At the extreme case of the high water mass flow rate, the heat transfer will be dominated by the cooling water, and it will be hard to control the air exit temperature, since its mass flow rate is very low compared to the water, resulting in a negligible temperature difference in the water side, therefore, the air exit temperature will always be close the water temperature. At the other extreme, low water mass flow rate, the water in the tube will be very low and it will not have the capacity to cool the air sufficiently, which will result in negligible temperature change in the airside. This, obviously, makes it harder to control the air exit temperature. The bottom frame shows the effect of the optimal aspect ratio $(D_o/L)_{opt}$. The figure shows that even though $(D_o/L)_{opt}$ changes, it stays with the same order of magnitude for the range of used in this work. This means that the effect of the $(D_o/L)_{opt}$ is not significant.

The resulting figure of merit of the second minimization of *CM* is the twice minimized controllability matrix condition number, CM_{mm}, the corresponding optimal aspect ratio, $(D_o/L)_{opt}$, and the corresponding optimal water mass flow rate, $\dot{m}_w^{opt}$. The optimization procedure illustrated in figs. 2 and 5 is repeated for different values of the total volume, V, the volume fraction, ϕ, and the air mass flow rate, $\dot{m}_a$. Figure 6 shows the effect of V on CM_{mm}, and the corresponding $(D_o/L)_{opt}$ and $\dot{m}_w^{opt}$ It can be seen from the top and middle frames that the effect of V on CM_{mm} and $(D_o/L)_{opt}$ is very small, while its effect on $\dot{m}_w^{opt}$ is noticeable such that it changes order of magnitude, as the volume increases, as shown in the bottom frame.

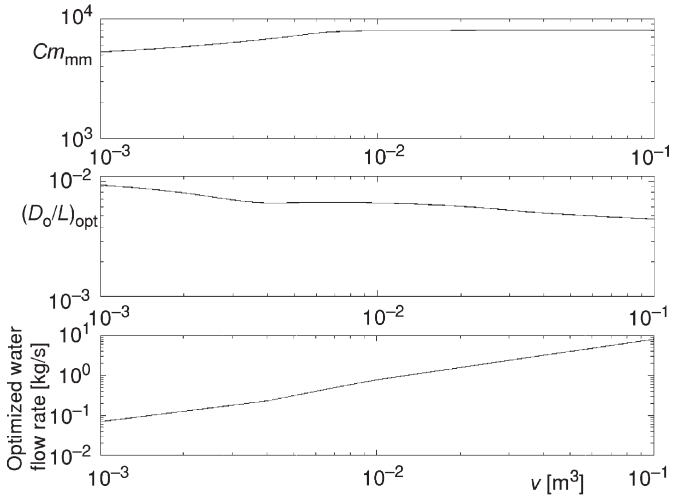

Figure 6. The variation of the twice minimized controllability matrix condition number (CM_{mm}) and corresponding optimal aspect ratio and optimal water mass flow rate with respect to the total volum

The effect of the volume fraction, ϕ, on CM_{mm}, and the corresponding $(D_o/L)_{opt}$ and $\dot{m}_w^{opt}$ is shown in fig. 7. The top figure shows that CM_{mm}, is sensitive to changes in ϕ, and the controllability decreases as the volume fraction increases. The middle frame shows that $(D_o/L)_{opt}$ de-

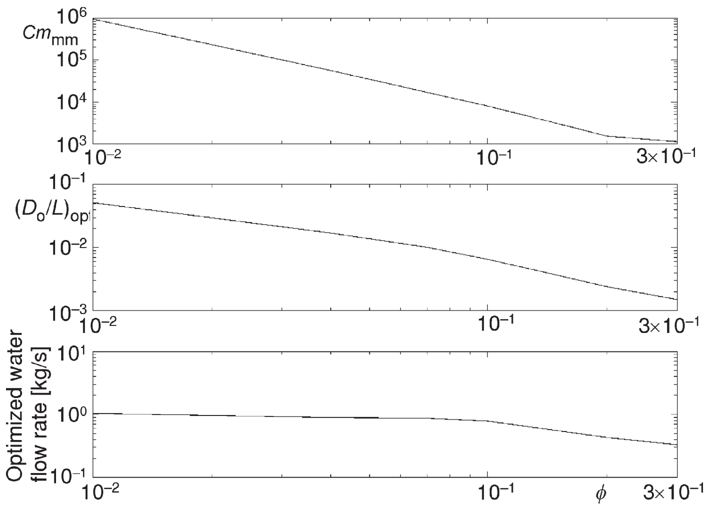

Figure 7. The variation of the twice minimized controllability matrix condition number (CM_{mm}) and corresponding optimal aspect ratio and optimal water mass flow rate with respect to the volume fraction

creases as ϕ increases, and it changes from order of 10^{-1} to order of 10^{-2} for the range studied. While the bottom frame shows that $\dot{m}_w^{opt}$ is insensitive to changes in ϕ.

The air mass flow rates $\dot{m}_a$ on the previous cases was kept constant. Figure 8 shows the effect of $\dot{m}_a$ on CM_{mm}, and the corresponding $(D_o/L)_{opt}$ and $\dot{m}_w^{opt}$. It can be seen that even though CM_{mm}, $\dot{m}_w^{opt}$, and $(D_o/L)_{opt}$ are decreases as $\dot{m}_a$ increases, the sensitivity of the change is insignificant.

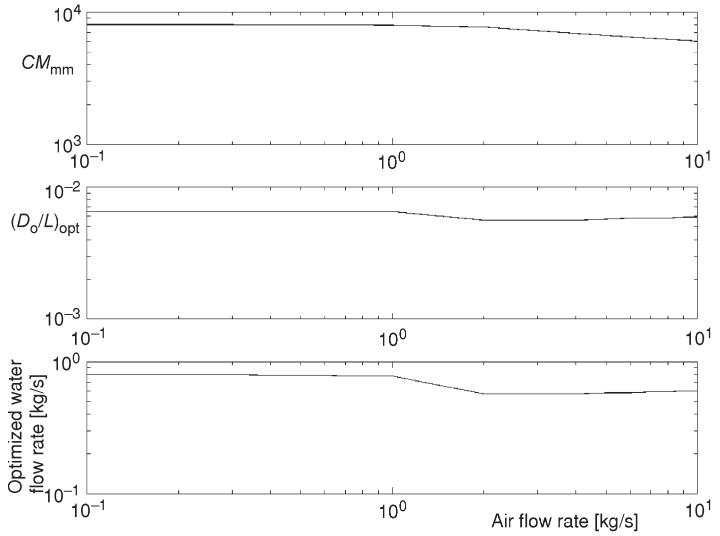

Figure 8. The variation of the twice minimized controllability matrix condition number (CM_{mm}) and corresponding optimal aspect ratio and optimal water mass flow rate with respect to the air mass flow rate

Conclusions

The aim of this work is to study the ability of optimizing the geometry of heat exchangers with respect to its dynamic controllability. A simple tube in a cross-flow was used as an example. The controllability of the system was quantified in the controllability matrix condition number, then it was minimized with respect to the tube aspect ratio and the water mass flow rate. The minimized controllability and the optimal aspect ratio were robust to changes in the volume fraction, however they were insensitive to changes in the total volume and the mass flow rate of air. The optimal water mass flow rate was sensitive to changes in the total volume, but it was not receptive to changes in the volume fraction and the air mass flow rate. This work showed that optimizing the geometry of heat exchangers with respect to its controllability is important and should be taken into account for systems where the control part is valuable. Beside testing the controllability of a certain system, it was described in this work how to choose the best manipulated variable from the available control inputs.

Even though the system used in this work is simple, it can be used as the basic for more specified and detailed heat exchangers. The insensitivity of some of the parameters on the optimized geometry showed in this work can be used to simplify future optimization of similar systems.

Nomenclature

A	– air frontal area, [m^2]	D	– diameter, [m]
CM	– controllability matrix condition number, [–]	e	– error, [K]
c	– specific heat at constant pressure, [kJkg^{-1}K^{-1}]	h	– convective heat transfer coefficient, [Wm^{-2}K^{-1}]

K_1 – integral control gain, [$kgK^{-1}s^{-2}$]
K_p – proportional control gain, [$kgK^{-1}s^{-2}$]
k – thermal conductivity, [$Wm^{-1}K^{-1}$]
L – length, [m]
M – controllability matrix, [–]
$\dot{m}$ – mass flow rate, [kgs^{-1}]
n – dimensions of matrices, [–]
r – radius, [m]
T – temperature, [°C]
t – time, [s]
u – manipulated variable, [–]
V – volume, [m^3]
v – velocity, [ms^{-1}]
x – system state, [–]

Greek symbols

ρ – fluid density, [kgm^{-3}]
ϕ – volume fraction, [–]

Subscripts and superscripts

a – air
in, out – inlet and outlet, respectively
i, o – inside and outside, respectively
m – minimum
mm – double minimized
opt – optimum
t – tube
w – water

References

[1] Bejan, A., Entropy Generation Minimization, CRC Press, Boca Raton, Fla., USA, 1996
[2] Alebrahim, A., Bejan, A., Entropy Generation Minimization in a Ram-Air Cross-Flow Heat Exchanger, *Int. J. Applied Thermodynamics, 2* (1999), 4, pp. 145-157
[3] Sekulic, D. P., Entropy Generation in a Heat Exchanger, *Heat Transfer Engineering, 7* (1986), 1-2, pp. 83-88
[4] Alotaibi, S., *et al*., Controllability of Cross-Flow Heat Exchangers, *Int. J. Heat Mass Transfer, 47* (2004), 5, pp. 913-924
[5] Antsaklis, P. J., Michel, A. N., Linear Systems, McGraw-Hill, New York, USA, 1997
[6] Butkovsky, A. G., Distributed Control System, Elsevier, Amsterdam, The Netherlands, 1969
[7] Klamka, J., Controllability of Dynamical Systems, Kluwer Academic Publishers, Dordrecht, The Netherlands, 1991
[8] Vargra, E. I., Hangos, K. M., Szigeti, F., Controllability and Observability of Heat Exchangers Networks in the Time-Varying Parameter Case, *Control Engineering Practice, 3* (1995),10, pp. 1409-1419
[9] Kequn, L., Xing, L., Niemeyer, B., Controllability Analysis of Multistream Heat Exchangers, *Proceedings,* International Symposium on Compact Heat Exchangers, Grenoble, France, 2002, pp. 163-167
[10] Alotaibi, S., *et al*., Numerical Simulation of the Thermal Control of Heat Exchangers, *Numerical Heat Transfer A, 41* (2002), 3, pp. 229-244

Authors´ address:

S. Alotaibi, A. Alebrahim
Mechanical Engineering Department,
Kuwait University
P. O. Box 5969, Safat 13060
Kuwait

Corresponding author S. Alotaibi
E-mail: sr.alotaibi@gmail.com

Paper submitted: November 11, 2007
Paper revised: March 9, 2008
Paper accepted: July 17, 2008