

## CONJUGATED HEAT TRANSFER ANALYSIS OF GAS TURBINE VANES USING MacCORMACK'S TECHNIQUE

by

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*It is well known that turbine engine efficiency can be improved by increasing the turbine inlet gas temperature. This causes an increase of heat load to the turbine components. Current inlet temperature level in advanced gas turbine is far above the melting point of the vane material. Therefore, along with high temperature material development, sophisticated cooling scheme must be developed for continuous safe operation of gas turbine with high performance. Gas turbine blades are cooled internally and externally. Internal cooling is achieved by passing the coolant through passages inside the blade and extracting the heat from outside of the blade. This paper focuses on turbine vanes internal cooling. The effect of arrangement of rib and parabolic fin turbulator in the internal cooling channel and numerical investigation of temperature distribution along the vane material has been presented. The formulations for the internal cooling for the turbine vane have been done and these formulated equations are solved by MacCormack's technique.*

**Key words:** *internal cooling, turbine vanes, conjugate heat transfer, MacCormack's technique, parabolic fin turbulator*

### Introduction

#### ***The need for cooling***

An increase in working temperature is a suitable solution for increasing the performance of gas engine. It can be found from law of physics that an increase of the turbine entry temperature (TET) would raise the net efficiency of the gas turbine. However, and increases in temperature will increase the demands on the structures in the engine. High grade material, as are found in *e. g.*, the turbine blades or vanes, can with stand high load for infinite time at room temperature. At elevated temperature, however, the materials become weaker and begin to creep, which limits the time span for which the turbine blade can be used. There are generally two different approaches to the problem of increasing the TET, and still achieving a sufficiently long endurance of stressed parts. The materials are either changed to even higher strength materials, or cooling air is used to reduce the working temperature of the structure. While both these solutions are normally employed, only the cooling idea will be addressed here.

There are several areas in a gas turbine which will need cooling owing to the high temperature encountered. The combustor where the air and fuel is mixed and ignited will naturally have a very high temperature. This mixed air/fuel gas then flows through the turbine before ex-

isting at the back through the nozzle. These parts also work at elevated temperature of course, even if they are not as high as in the combustor.

The focus of this paper is the internal cooling of the turbine vanes and investigation of temperature distribution in the cooling channel with rib turbulator and parabolic fin with reference to the smooth cooling channel.

### The turbine-cooling technique

The turbine stage is composed of an inlet guide vane followed by a turbine blade. The stationary vane and the rotating blade are matched together and are known as a stage. A gas turbine has normally several turbine stages. Due to the high working temperatures of the turbine, it employs several different cooling techniques, both for the vane and the blade. In most cases the cooling media is high-pressure air which is drawn from the compressor through ducts in the center of the engine.

This air is then led up, through the inside of the blades and vanes in intricate ducts. There will thus be two distinct gas flows around the turbine, the main gas flow with hot gases from the combustor of the outside and the secondary air flow with cooling air from the compressor on the inside. Figure 1 shows two different cooling techniques [1]: one in which the cooling media is in contact with the hot gases on the outside of the blade and another in which the cooling

media is employed only on the inside to cool the blade material. These techniques are known as external and internal cooling. These are then subdivided into several sub-groups such as:

- internal cooling
  - convection cooling,
  - impingement cooling,
  - internally air-cooled thermal barrier, and
- external cooling
  - local film cooling,
  - full-coverage film cooling,
  - transpiration cooling.

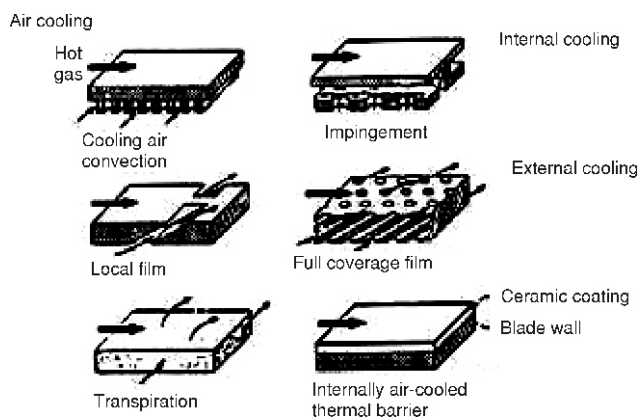


Figure 1. Blade and vane cooling technique

### Review of literature

Zhang *et al.* [1] investigated the effect of compound roughness on heat transfer and pressure drop in rectangular channel for Reynolds number between 10,000 to 50,000. Tiwari *et al.* [2] numerically investigated the flow and heat transfer in a rectangular channel with a Build-circular tube. Han *et al.* [3] has discussed about the different turbine blade internal cooling technique. Carlson *et al.* [4] in his paper, the diagonal Cartesian method is used to simulate conjugate heat transfer involving complex geometries.

Metzger *et al.* [5] investigated the heat transfer associated with flow normal to arrays of circular cylinder and developing heat transfer in rectangular ducts with staggered arrays of short pin fins.

Hann *et al.* [6] reported heat transfer enhancement in fully developed turbulent flows in annuli, in circular tubes, and between parallel plates with periodic rib rougheners. The effects of the rib height on the fully developed average heat transfer coefficient are well established.

Jang *et al.* [7], presented numerical predictions of three dimensional flow and heat transfer for a two-pass square channel with and without 60 degree angled parallel ribs.

Heat transfer enhancement in fully developed turbulent flows in annuli, in circular tubes, and between parallel plates with periodic rib rougheners has been reported in [8, 9].

Prakash *et al.* [10] presented the numerical prediction of turbulent flow and heat transfer in a 2:1 aspect ratio rectangular duct with ribs on the two shorter sides. The ribs are of square cross-section, staggered and aligned normal to the main flow direction.

### Governing equation

The governing equation is:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{1}{\alpha} \frac{\partial T}{\partial t} \quad (\text{Heat equation}) \quad (1)$$

The following assumption employed: unsteady-state, constant fluid property, and thermal conductivity of vane material is constant.

### Physics of the problem

The heat transfer in a turbine blade under operating conditions is a three-step process:

- heat transfer from the hot gas (external to the vane) to the vane by convection and conduction in the fluid,
- heat transfer within the vane material by conduction, and
- heat transfer from the vane to the coolant (inside the blade cavity).

In order to accurately predict the temperature in the vane material, particularly on the hot surface of the blade, one need to simultaneously compute all three domains, *i. e.*, the external gas flow, blade heat conduction, and internal coolant flow.

### Methodology-CFD approach

The objective of computational fluid dynamic is to calculate an entire flow field either around a turbine vane or through a rectangular channel with rib and parabolic fin turbulator. The flow is unsteady, two-dimensional, compressible, and turbulent. The equations to describe this task are the Navier-Stokes equation, the energy equation, and continuity equation.

The important steps of computational fluid dynamics (CFD) are formulating the governing equation, developing the appropriate numerical solution for these equations, developing code using the C program, and then going through all trials, finally analysis of results.

### Finite difference method

Finite difference method is one of the important methods for developing the numerical solution of the governing equation. The numerical solution enables determination of the temperature at any discrete points in the medium. The first step in any numerical analysis must therefore to be these points. This is done by subdividing the medium of interest into a number of small regions and assigning to each a reference point.

The reference point is frequently termed as nodal network, grid or mesh. The nodal points are designated by a numbering scheme. The numerical accuracy of the calculation depends strongly on the number of designated nodal point. If this number is large (a fine mesh), extreme accurate solution can be obtained.

### Finite difference solution

Once the nodal network has been established and an appropriate finite difference equation has been developed for each node, the temperature distribution may be determined. MacCormack's Technique is employed to iterate the transient equation [11]. Matrix inversion method is employed to find temperature at different node after attaining steady-state temperature.

### Assumption and boundary conditions

#### Assumption

The following assumptions are made for investigating temperature distribution in turbine vane:

- two-dimensional conduction in turbine vane cooling channel,
- unsteady state of conduction,
- constant properties for gas and vane material,
- symmetric adiabatic condition for channel flow, and
- fully developed Compressible fluid flow.

Figure 2 shows the schematic diagram of turbine vane and the section under consideration for analysis. Figure 3 shows the arrangement of rib turbulator and parabolic fin in the cooling channel.

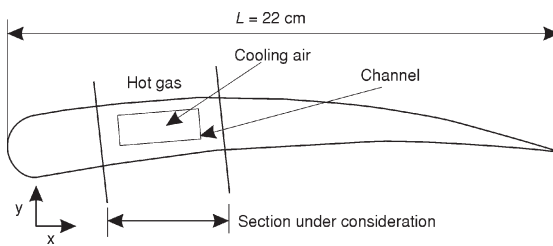


Figure 2. Schematic diagram for gas turbine vane

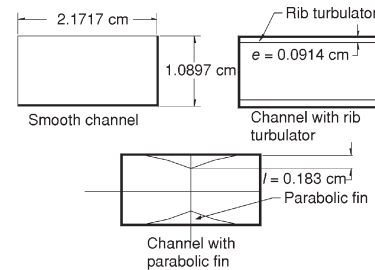


Figure 3. Dimensions of different type of channel

Main dimensions and parameters are: breadth of the channel  $b = 2.1717$  cm, width of the channel  $a = 1.0897$  cm, pitch  $p = 0.9144$  cm, thickness of rib  $e = 0.0914$  cm, cooling air temperature  $T_{a,i} = 200$  °C, hot gas temperature  $T_{a,o} = 1300$  °C, height of the vane  $H = 110$  cm, chord length  $L = 22$  cm, length of parabolic fin  $l = 0.183$  cm, Nusselt number for hot gas ( $= h_{ext}L/k$ ) = 14.0, Nusselt number for cooling air ( $= h_{int}L/k$ ) = 4.7, and thermal conductivity of the vane material (nickel and cobalt based super alloy)  $k = 17.7$  W/mK.

The general average heat transfer coefficient  $h'$  [12]

$$h = \frac{h \, dA}{A_i \, A_p} \quad (2)$$

where  $A_t$  is the true heat transfer area [ $\text{cm}^2$ ],  $A_p$  – the projected base area [ $\text{cm}^2$ ],  $h_{\text{ext}}$  – the heat transfer coefficient of hot gas [ $\text{W}/\text{m}^2\text{K}$ ], and  $h_{\text{int}}$  – the heat transfer coefficient of cooling air in smooth channel [ $\text{W}/\text{m}^2\text{K}$ ].

Area for square rib turbulator:

$$A_t = ap + 2ea \quad (3)$$

$$A_p = ap \quad (4)$$

Area for parabolic fin turbulator [12]:

$$A_t = a \left[ l \sqrt{1 - \frac{e^2}{l^2}} + \frac{l^2}{e} \ln \frac{e}{l} \right] \sqrt{1 - \frac{e^2}{l^2}} \quad (5)$$

$$A_p = ap \quad (6)$$

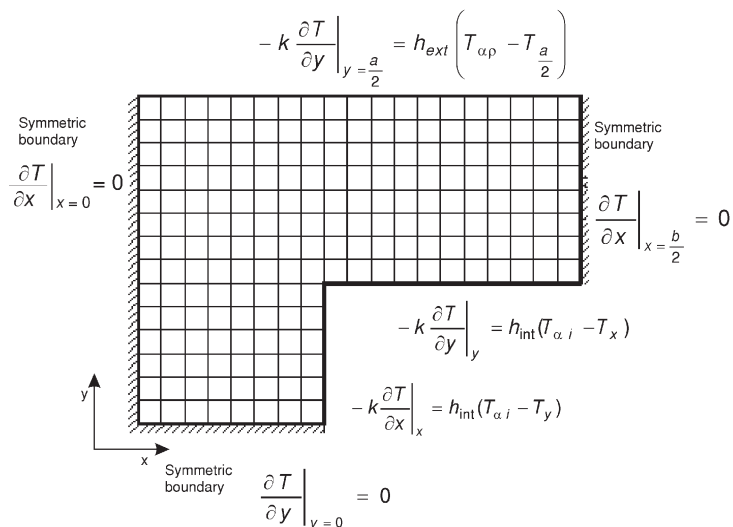
### Boundary conditions

The following boundary conditions were imposed. No inlet and exit conditions are required for a periodically fully developed analysis. Figures 4, 5, and 6 shows the boundary condition for smooth channel, channel with rib turbulator, and channel with parabolic fin.

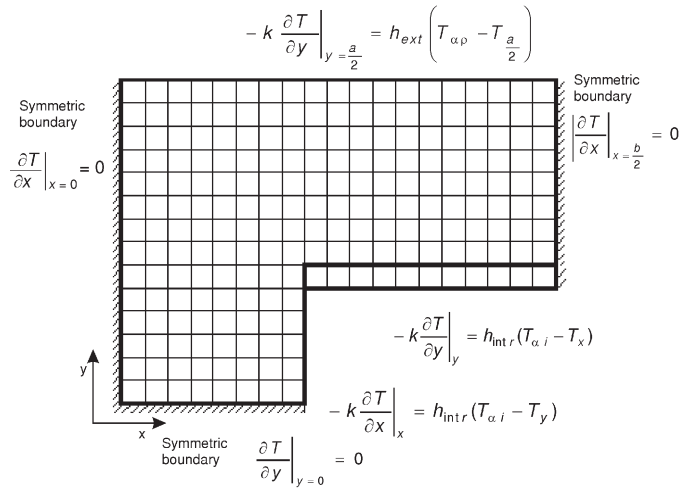
At the wall ( $x = 0, x = a/2, y = 0$ ), adiabatic boundary condition:

$$\left. \frac{\partial T}{\partial x} \right|_{x=0} = 0, \quad \left. \frac{\partial T}{\partial x} \right|_{x=\frac{a}{2}} = 0, \quad \left. \frac{\partial T}{\partial x} \right|_{y=0} = 0 \text{ respectively.}$$

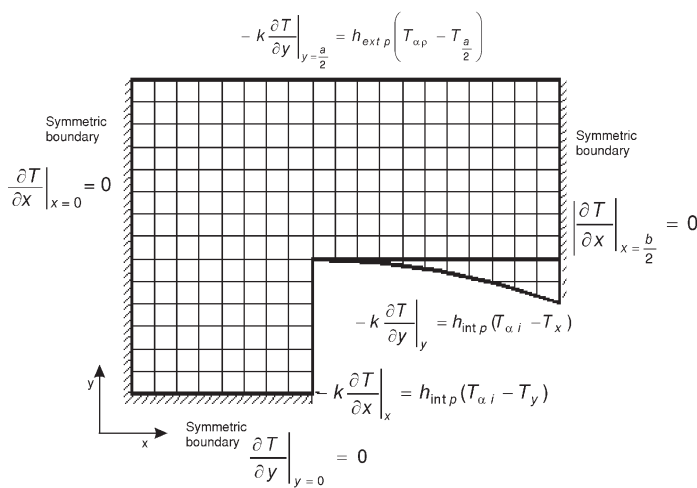
Figure 4. Smooth channel



At the outer surface of the vane and inner channel, the boundary condition of third kind corresponds to the existence of convection heating or cooling at the surface and is obtained from the surface energy balance (convection surface condition):



**Figure 5. Channel with rib  
turbulator**



**Figure 6. Channel with parabolic fin**

- at outer surface of the vane

$$k \frac{\partial T}{\partial x} \bigg|_y = \frac{a}{2} h_{\text{ext}} (T_{\alpha o} - T_{\frac{a}{2}})$$

- at inner smooth channel

$$k \frac{\partial T}{\partial y} \Big|_x \quad h_{\text{int}}(T_{\alpha i} - T_x), \quad k \frac{\partial T}{\partial x} \Big|_x \quad h_{\text{int}}(T_{\alpha i} - T_y)$$

- at inner channel with rib turbulator

$$k \frac{\partial T}{\partial y} \bigg|_x h_{\text{int}r}(T_{\alpha i} - T_x), \quad k \frac{\partial T}{\partial y} \bigg|_x h_{\text{int}}(T_{\alpha i} - T_y)$$

- at inner channel with parabolic fin

$$k \frac{\partial T}{\partial y} \bigg|_x h_{\text{int } p}(T_{\alpha i} - T_x), \quad k \frac{\partial T}{\partial x} \bigg|_x h_{\text{int}}(T_{\alpha i} - T_y)$$

where  $T$  is the temperature [ $^{\circ}\text{C}$ ],  $h_{\text{in r}}$  – the heat transfer coefficient of cooling air in channel with rib turbulator [ $\text{W}/\text{m}^2\text{K}$ ], and  $h_{\text{int p}}$  – the heat transfer coefficient of cooling air in channel with parabolic fin [ $\text{W}/\text{m}^2\text{K}$ ],

## Results and discussion

Figure 7 shows the typical results of transient temperature variation for the smooth channel, channel with rib turbulator, and channel with parabolic fins. Without cooling arrangement in vane difference between the vane surface and fluid temperature must decay exponentially to zero as time approaches infinity.

But presence of internal cooling arrangement in the vane always makes temperature difference between vane surface and the hot gases. Hence the curve will not approach zero as the time approaches infinity. It implies that the surface temperature of vane is less than the hot gas temperature even the turbine runs at infinite times. In fig. 8 it can be seen that the temperature difference between the outer surface of the vane and hot gas is more for the channel with parabolic fins when compare to that of channel with rib and smooth channel after attaining the steady-state temperature.

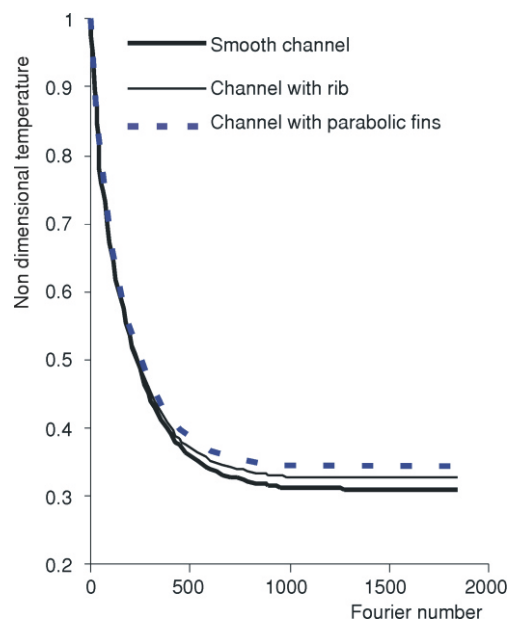


Figure 7. Fourier number vs. non-dimensional temperature

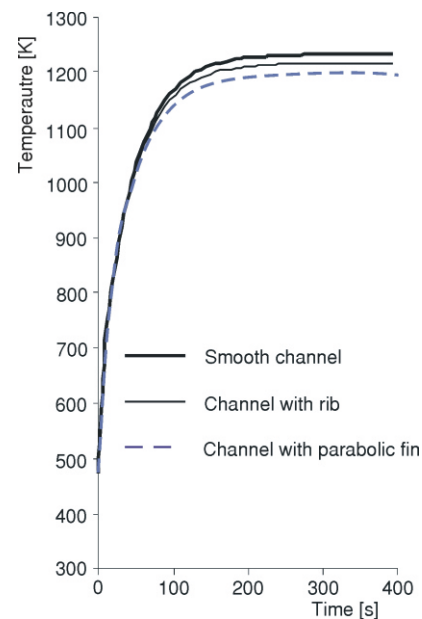
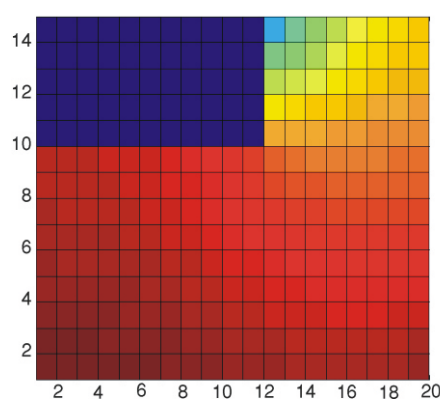


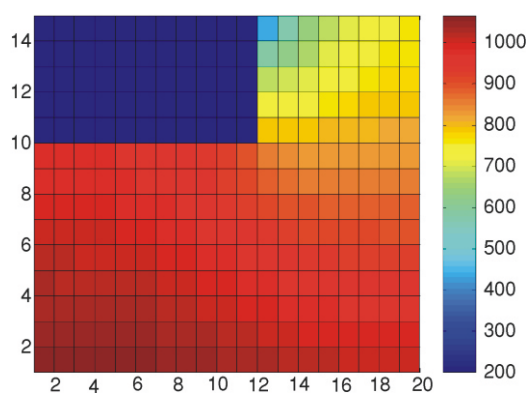
Figure 8. Variation of temperature with time

Figures 9, 10, and 11 shows the temperature field represented in the form of checker board plot (plotted in MATLAB) for smooth channel, channel with rib turbulator, and a parabolic fin, respectively. As expected the maximum temperature exists at the location far-off from the coolant, which corresponds to outer surface to the vane.

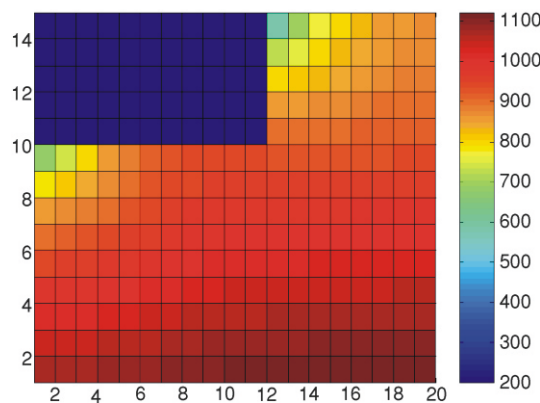
The heat transfer coefficient is more in the descending order for parabolic fin, channel with rib turbulator and smooth channel because of flow separation and flow reattachment in para-



**Figure 9. Temperature [°C] distribution for smooth channel (color image see on our web site)**



**Figure 10. Temperature [°C] distribution for channel with rib turbulator (color image see on our web site)**



**Figure 11. Temperature [°C] distribution for channel with parabolic fin turbulator (color image see on our web site)**

bolic fin and rib turbulator. Also, heat transfer coefficient is high for parabolic fin when compare to rib turbulator because of high heat transfer area.

It can be seen that higher heat transfer coefficient and best heat transfer performance can be obtained in the channel with parabolic fins.

Furthermore, from the turbine cooling design point of view, it is very important to know the detailed distribution of temperature in the blade vane for fully developed fluid flow inside the channel.

Therefore, a systematic investigation has done to find the temperature distribution.

The temperature distribution on the channel with rib turbulator and parabolic fin

has numerically investigated with reference to the smooth channel. It is concluded that the outer surface temperature for parabolic fin is less when compared to that of remaining two cases. But in the parabolic fin turbulator, fig. 11, the temperature distribution is uneven in the fin side of the channel as shown. Even temperature distributions can be achieved using the rib turbulator.

## Conclusions

In this review, the numerical method for temperature distribution in turbine vane cooling with rib and parabolic turbulator with reference to the smooth channel has been presented. Explicit finite difference technique (MacCormack's technique) has been used successfully to predict the temperature distribution. As compared to other method, internal cooling with parabolic fin turbulator is effective way to reduce the surface temperature of the turbine vane. However, use of the parabolic fin is connecting with manufacturing problems, which require special consideration.



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