# A NUMERICAL STUDY OF FLOW AND TEMPERATURE FIELDS IN CIRCULAR TUBE HEAT EXCHANGER WITH ELLIPTIC VORTEX GENERATORS

## by

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The two-dimensional fluid flow and heat transfer in a circular tube heat exchanger with two elliptic obstacles at the back is studied numerically. The computational domain consists of a circular tube and two elliptic obstacles that are situated after the tube, such that the angle between their centerlines and the direction of free coming flow is 45 degrees. The numerical solution is achieved by numerical integration of full Navier-Stokes and energy equations over the computational domain, using finite volume method. The fluid flow is assumed to be laminar, incompressible and steady-state with constant thermo-physical characteristics. In this study major thermo-fluid parameters such as temperature, pressure and velocity fields as well as Nusselt number and friction factor variations are computed and some results are presented in the graphs. It is shown that using of elliptic obstacles leads to an increase in the average Nusselt number and also pressure.

Key words: numerical computations, fluid flow, heat transfer, elliptic vortex generator, heat exchanger

## Introduction

Motion of bluff and non-aerodynamic objects through fluids is encountered widely in the engineering practice; submarines, ships, passenger aircrafts, automobiles, and missiles are examples where the object is in motion through a stationary fluid medium. High buildings, cooling towers, chimneys, and tube banks in heat exchangers are examples where the fluid is in motion, while the object is stationary. In each class, the wake of the bluff object predominantly determines device performance, whether it is forces, vibration or heat transfer rates. Flow over several bluff body geometries including circular and rectangular cylinders have been experimentally and numerically investigated in the past. Recently, the flow past over heat exchangers have been studied numerically, especially the flow past tube in a tube bundle, which were done by Sparrow and Liu [1], Kundu et al. [2], and Buyruk et al. [3]. Alessio and Dennis [4] have studied using of elliptical tubes in the heat exchangers, in forced convection, for low Reynolds numbers. Kashevarov [5] reported the exact solution of forced convection problem in an elliptical cylinder. He considered a dominant potential flow around cylinder and solved the two dimensional energy equations analytically. One of the most important researches about the forced convection on a straight elliptical tube has been done by Badr [6]. He has studied two dimensional, laminar, forced convection, heat transfer from a constant temperature elliptic tube, across the uniform flow. In his studies, the using of vortex generator obstacles in the heat exchanger has been investigated for triangular and rectangular obstacles [7-9]. The most important observation from these studies is that the large eddies are responsible for much of the transport of momentum, heat and mass. The flow patterns in the base region and near the wake of a circular cylinder show a high level of mixing but have not been studied in detail yet. In the above studies, via numerical solution of governing equations in fluid mechanics and heat transfer that are kind of partial differential equations (PDE), system of linear differential equations has solved in the domain of interest. It is very important how to convert continuous domain of interest to discrete domain, as it plays an important role in solving of the governing equations. Furthermore, applying boundary fitted grids is an effective method for grid generation [10]. In this system, the physical spatial domain can be converted to the Cartesian uniform spatial domain without any concern about its geometry.

## **Problem statement**

#### Geometry



Most of complex the heat exchangers use circular section tubes, through which first fluid passes while the second fluid passes over the outside surface of the tubes. In this study, a heat exchanger with circular tube is considered. There are also two elliptic obstacles that are situated after the tube, such that the angle between their centerlines and the direction of free coming fluid flow is 45 degrees. The tube and two elliptic obstacles are shown schematically in fig. 1. The preceding and following edges of the obstacles are considered aerodynamic to achieve better numerical results.

Figure 1. A schematic of the tube with two obstacles and relative dimensions

#### Governing equations

The governing equations are the mass, momentum, and energy equations which were simplified in accordance with the assumptions of two dimensional, incompressible, steady-state flow, constant properties and Newtonian fluid. Considering mentioned assumptions, the governing equations would be as follows:

$$\frac{\partial u_i}{\partial x_i} \quad 0.0 \quad (i \quad 1, 2) \tag{1}$$

$$\frac{\partial (u_{i},\phi)}{\partial x_{i}} \quad \frac{\partial}{\partial x_{i}} \quad \Gamma_{\phi} \frac{\partial \phi}{\partial x_{i}} \qquad s_{\phi} \tag{2}$$

In the above equations,  $\phi$  is a general dependant variable,  $\Gamma_{\phi}$  is diffusion coefficient, and  $s_{\phi}$  is the source term. The two latter terms are variables depend on  $\phi$ . The  $\phi$ , general dependant variable, is used instead of velocity and temperature in the second equation. Above equations can be rewritten into non-dimensional forms as:

$$\frac{\partial u}{\partial x} \quad \frac{\partial v}{\partial y} \quad 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} \quad v\frac{\partial u}{\partial y} \quad \frac{\partial p}{\partial x} \quad \frac{1}{\text{Re}} \quad \frac{\partial^2 u}{\partial x^2} \quad \frac{\partial^2 u}{\partial y^2} \tag{4}$$

$$u\frac{\partial v}{\partial x} \quad v\frac{\partial v}{\partial y} \quad \frac{\partial p}{\partial y} \quad \frac{1}{\text{Re}} \quad \frac{\partial^2 v}{\partial x^2} \quad \frac{\partial^2 v}{\partial y^2}$$

$$\frac{\partial (uT)}{\partial x} \quad \frac{\partial (vT)}{\partial x} \quad 1 \quad \frac{\partial^2 T}{\partial x^2} \quad \frac{\partial^2 T}{\partial y^2}$$
(5)

$$\frac{\partial (uT)}{\partial x} \quad \frac{\partial (vT)}{\partial y} \quad \frac{1}{\text{Re Pr}} \quad \frac{\partial^2 T}{\partial x^2} \quad \frac{\partial^2 T}{\partial y^2} \tag{6}$$

Following relations have been used for non-dimensionalization:

$$x^* \quad \frac{x}{XL} \tag{7}$$

$$y^* = \frac{y}{XL} \tag{8}$$

$$v^* \quad \frac{v}{u_{\infty}} \tag{9}$$

$$u^* \quad \frac{u}{u_{\infty}} \tag{10}$$

$$p^* \quad \frac{p}{p_{\infty}u_{\infty}^2} \tag{11}$$

where L is specified length,  $\rho_{\infty}$  is density of free fluid,  $u_{\infty}$  is velocity of free fuid,  $T_{\infty}$  is base temperature, and  $T_{w}$  is wall temperature. Star symbols have been omitted in eqs. (3) to (6). Dimensionless numbers are also considered as follows:

$$\operatorname{Re} \quad \frac{\rho_{\infty} u_{\infty} L}{\mu_{\infty}} \tag{12}$$

$$\Pr \quad \frac{\mu c}{k} \tag{13}$$

The governing equations turn to discrete equations and resulted equations are solved in the computational domain. For calculating the Nusselt number, which shows the superiority of convection or conduction heat transfer, we have used the grade temperature gradient as follows:

$$Nu(i, j) = \frac{T(i, j = 1) - T(i, j)}{y(i, j = 1) - y(i, j)}$$
(14)

To study the effect of elliptic obstacles on the pressure losses after the heat exchanger's tubes, the pressure coefficient has used. The pressure coefficient relates to the local pressure as:

$$C_{\rm p} = \frac{\Delta p}{\frac{\rho u^2}{2}} \tag{15}$$

## Boundary conditions

## (a) Inlet (X=0)

Profiles of velocity, pressure, and temperature are known in the inlet. At the inlet, a constant stream-wise velocity has been used with other components of velocity being set to zero and the temperature distribution is also constant.

$$u = u_{\rm in} \tag{16}$$

$$v = 0$$
 (17)

$$p = p_{\rm in} \tag{18}$$

$$T = T_{\rm in} \tag{19}$$

(b) Outlet (X = L)

At outlet, the gradient of the variables are set to zero, so a Neumann boundary conditions are used for all variables.

$$\frac{\partial u_{i}}{\partial x} \quad 0.0 \tag{20}$$

$$\frac{\partial p}{\partial x} = 0.0$$
 (21)

$$\frac{\partial T}{\partial x} = 0.0$$
 (22)

(c) Lateral boundaries (y = -H, y = +H)

At lateral boundaries, we apply the symmetric boundary conditions. In this boundary, the net flow passes through the symmetric line is set to zero. The velocity component vertical to the boundary is also set to zero and the gradient of other variables are considered to be zero.

$$\frac{\partial u}{\partial y} = 0.0$$
 (23)

$$\frac{\partial p}{\partial y} = 0.0$$
 (24)

$$\frac{\partial T}{\partial y} = 0.0$$
 (25)

$$V = 0.0$$
 (26)

(d) Solid wall

The no-slip boundary conditions has been used for solid walls, so all velocity components are set to zero and also temperatures distribution on the wall is considered constant.

$$u = 0 \tag{27}$$

$$v = 0 \tag{28}$$

$$T_{\rm tube} = T_{\rm wall} = \text{const.}$$
(29)

#### **Results and discussions**

From now on, we just use non-dimensional variables in the discussions as well as the plots. Figures 2 and 3 show the variations of the vertical and horizontal velocity components around the tube of heat exchanger for the case of the Reynolds number equals to 120.



Figure 2. Variations of vertical velocity component around the tube of heat exchanger when Re = 120

The pressure variation around the tube of heat exchanger when Re = 120 is shown in fig. 4. The figure shows that the pressure is gradually increases as the flow reaches the tube of heat exchanger and then the pressure reaches to its maximum amount at the front stagnation point. Also pressure increases again gradually from the rear stagnation point to the end of the computational domain but the pressure difference on the tube of heat exchanger is quite high. In this region the pressure decreases until the flow reaches to the highest point of heat exchanger tube (lateral boundary of the tube) and then there is an adverse pressure gradient and then the pressure increases. Velocity variations of fluid flow near the tube of heat exchanger are varying inversely by pressure variations. According to the figs. 2 and 3, velocity components on the surface of tube decrease and then increase, respectively.



Figure 3. Variations of horizontal velocity component around the tube of heat exchanger when Re = 120



Figure 4. Variation of pressure around the tube of heat exchanger when Re = 120

The variation of the pressure coefficient with and without elliptic obstacle for the case of Re = 300 are shown in figs. 5 and 6. According to the figs. 5 and 6, it is clear that existence of elliptic obstacles leads to higher pressure coefficient.

The local Nusselt number distributed around the tube is shown in figs. 7 and 8, for both cases of with and without elliptic obstacles for Reynolds numbers of 120 and 300. It can be seen



Figure 5. Pressure coefficient when Re = 300 with elliptic obstacle



Figure 7. Comparison of local Nusselt number distributed around the tube with and without elliptic obstacle for Re = 300



Figure 6. Pressure coefficient when Re = 300 without elliptic obstacle



Figure 8. Comparison of local Nusselt number distributed around the tube with and without elliptic obstacle for Re = 120

that in case of existence elliptic obstacle, the values of Nusselt number for the tube are mostly higher than those of the tube without obstacle.

The variation of average Nusselt number *vs*. Reynolds number for tube surface for both cases of with and without elliptic obstacle are shown in figs. 9 and 10, it can be seen that the average Nusselt number increases with the presence of elliptic obstacle. As an example, for Re of 300 the average Nusselt number is about 12% higher than the case of without obstacle with a penalty increase in pressure of about 22%. In addition the very small size of the elliptic obstacle in comparison with the size of tubes of heat exchanger is noticeable, hence the increase of pressure losses are moderate since the small size of elliptic obstacle.





Figure 9. Variation of average Nusselt number vs. Reynolds number with elliptic obstacle

Figure 10. Variation of average Nusselt number vs. Reynolds number without elliptic ostacle

## Conclusions

A study on the effect of elliptic vortex generators on the local Nusselt number for flow around a tube heat exchanger is reported. The computational domain consists of a circular tube and two elliptic obstacles that are situated after the tube, such that the angle between their centerlines and the direction of free coming flow is 45 degrees. The momentum and energy balance equations have been written in terms of dimensionless quantities. A numerical solution based on the finite volume method was applied to solve the Navier-Stokes and energy equations over the computational domain. The fluid flow is assumed to be laminar, incompressible and steady-state with constant thermo-physical characteristics. The local Nusselt numbers in entrance region are numerically calculated and showed in some graphs as a function of dimensionless length. It has shown that the average Nusselt number increases with presence of elliptic obstacle although it leads to an increase in pressure coefficient too.

In the higher Reynolds numbers, especially when the flow is turbulent, the effects of turbulence, turbulent wake, and eddies behind the tube and around the obstacles might play a major role in flow and thermal fields; so, further study of the subjected problem for higher Reynolds numbers could be considered as a future study and as an extension of the current work.

## Nomenclature

$C_{\rm p}$	<ul> <li>pressure coefficient, [-]</li> </ul>	$T_{\rm w}$	- wall temperature, [K]
c	- heat capacity, [Jkg <sup>-1</sup> K <sup>-1</sup> ]	u	<ul> <li>horizontal Cartesian velocity component,</li> </ul>
D	– tube diameter, [m]		[ms <sup>-1</sup> ]
Η	– channel height, [m]	v	- vertical Cartesian velocity component,
k	- thermal conductivity coefficient, $[Jm^{-1}K^{-1}]$		$[ms^{-1}]$
L	– length, [m]	XL	- lenght, [m] (fig. 1)
Nu	- Nusselt number (= $hL/k$ ), [-]	<i>x</i> , <i>v</i>	<ul> <li>Cartesian coordinates</li> </ul>
Р	– pressure, [Pa]		
P	- pressure loss, [Pa]	Greek	e letters
Pr	- Prandtl number $(=\mu c/k)$		
Re	- Reynolds number $(=\rho_{\infty}\mu_{\infty}L/u_{\infty}), [-]$	$\Gamma_{\phi}$	<ul> <li>diffusion coefficient, [-]</li> </ul>
Т	- temperature, [K]	$\mu^{r}$	<ul> <li>dynamic viscosity of the fluid, [Pa·s]</li> </ul>

$egin{array}{c}  ho \ \phi \ S_{\phi} \end{array}$	<ul> <li>density, [kgm<sup>-3</sup>]</li> <li>general variable, [-]</li> <li>source term, [-]</li> </ul>	<i>tube</i> wall ∞	<ul> <li>characteristic of tube</li> <li>characteristics of wall</li> <li>free stream</li> </ul>
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#### **Subscripts**

Superscript

i – covariant vector in – inlet characteristic of fluid \* – non dimensional

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