DETERMINATION OF THERMAL CONDUCTIVITY OF ROCKS SAMPLES USING FABRICATED EQUIPMENT

by

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The aim of the paper is to describe how inexpensive/simple physics equipment was fabricated and used in the determination of thermal conductivity of rock samples. We used the experimental techniques known as transient method of measuring thermal properties of rock samples at ambient temperature.
We investigated samples found in five locations/region (Ewekoro, Ile-Ife, Igara, Ago-Iwoye, Abeokuta) in South western Nigeria. Those samples are limestone, dolerite, marble, gneiss, and granite. Although the samples are multi-mineral as revealed by photomicrograph, the thermal conductivity results obtained 1.40, 1.50, 1.57, 1.75, and 2.94 W/m°C, respectively, are found to be consistent with the ones in literature where highly expensive and sophisticated (not easily affordable in developing nation) equipment are used.

Key words: *thermal conductivity, thermal diffusivity, specific heat, density, rocks*

Introduction

Nigeria lies between latitudes 50 and 140N and 30 and 140E longitudes. Crystalline basement rocks of Precambrian age underlie about 50% of the country. These are unconformably overlain by sedimentary rocks of Cretaceous to recent age (fig. 1). Samples used in this study were collected from south western Nigeria comprising rocks of the Precambrian basement (granite, gneiss, dolerite, and marble) and limestone from the Ewekoro formation generally considered to be of Paleocene age [1]. This region of the country is highly involved in geological exploration activities especially borehole construction and well-logging.

It is a known fact that the heat form of transformation and transfer of the Earth's inner energy determines such fundamental parameter as the temperature of depths, which in turn influences the physical properties of depth, their phase conditions, metamorphic processes and other fundamental properties of the Earth [2].

The local variations of the terrestrial heat flow are very small within the same geological region and consequently the differences between geothermal gradients within areas of the same tectonic character are mainly due to difference between the thermal conductivities of the rocks [3].

Investigations of thermal properties of rocks using various methods such as steady-state divided bar technique, needle-probe method, transient methods, *etc*., had been carried out [4-7].

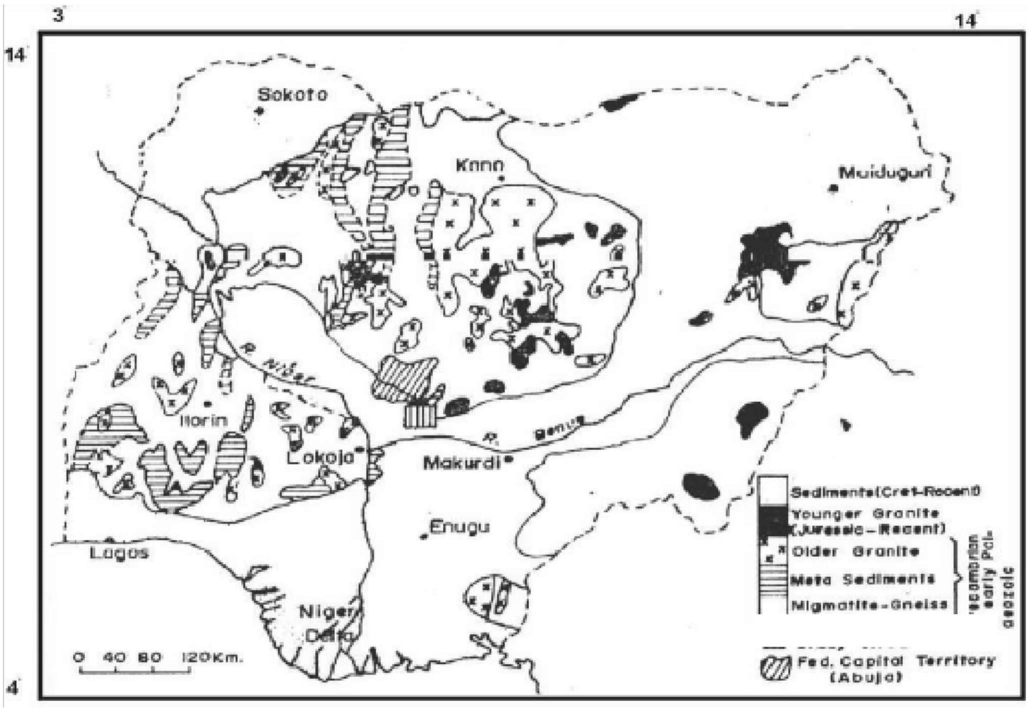


Figure 1. Generalized geological map of Nigeria [8]

Transient method based on one-dimensional heat flow inside the sample was adopted in this study. It directly determines thermal diffusivity that is not important in its own right but offers a convenient, economical and accurate method for determining the thermal conductivity (the most important thermal properties of rocks).

This study therefore is aimed at determining: (1) the thermal parameters for each rock samples, (2) the physical characteristics of the samples by visual inspection, (3) relative ratio of mineral composition by thin section analysis, and (4) compile thermal conductivity data for the rock samples used from thermal diffusivity measurements.

Materials and method

Theory of method

The theory of thermal conduction describing the 1-D single-slab is given by Middleton [7] and Carslaw and Jaeger [9]. They consider a slab that is initially at zero temperature, that is insulated at the surface $x = 0$, and has a constant heat flux introduced at the surface $x = a$ at time $t = 0$.

They showed that the temperature T at a distance x within the slab, and at time t (after the introduction of the constant heat at $x = a$) (fig. 2) is given by:

$$T \quad \frac{F_0 t}{\rho c a} \quad \frac{Fa}{k}\frac{3x^2 \quad a^2}{6a^2} \quad \frac{2}{\Pi^2}\sum_{n\ 1}^{\infty}\frac{(\ 1)^n}{n^2}\mathrm{e}^{\ \alpha n^2\Pi^2 t/a^2}\cos\frac{n\Pi x}{a} \tag{1}$$

$$T(a,t) \quad \frac{F\alpha t}{ak} \quad \frac{Fa}{k}\frac{3x^2 \quad a^2}{6a^2} \quad \frac{2}{\Pi^2}\sum_{n\ 1}^{\infty}\frac{(\ 1)^n}{n^2}\mathrm{e}^{\ \alpha n^2\Pi^2 t/a^2}\cos\frac{n\Pi x}{a} \tag{2}$$

Also:

$$T(x,t) = \frac{F\alpha t}{ak} + \frac{Fa}{k}\,\frac{3x^2 - a^2}{6a^2} - \frac{2}{\Pi^2}\sum_{n=1}^{a}\frac{(-1)^n}{n^2}\,\mathrm{e}^{-\alpha n^2\Pi^2 t/a^2}\cos\frac{n\Pi x}{a} \tag{2b}$$

where α is the thermal diffusivity, k – the thermal conductivity, and a is the thickness of the slab.

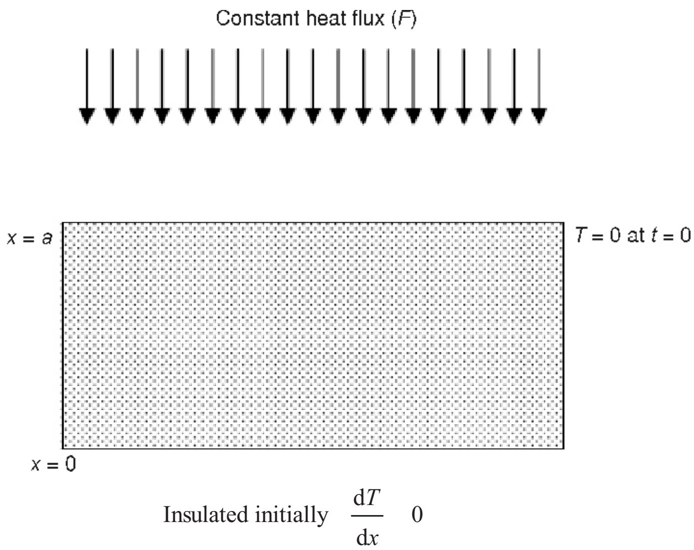


Figure 2. Diagram showing constant heat flux *F* at the top surface of one slab

If the measurement is made at the base of the slab ($x = 0$), the expression for temperature becomes:

$$T(a,t) = \frac{F\alpha t}{ak} + \frac{Fa}{6k} + \text{transient terms} \tag{3}$$

For times large relative to $\alpha\Pi^2/a^2$, the transient terms are negligible, and the temperature *vs*. time behaviour becomes linear; the intercept a_i on the $T = 0$ axis is:

$$t_i = \frac{a^2}{6\alpha^2} \tag{4}$$

where a is the thickness of the sample.

Heating device

A nickel-chromium 22swg (0.6426 mm) wire is used to design the heat source (coiled wire element) at the Department of Physics, Olabisi Onabanjo University, Ogun, Nigeria, to supply a constant heat flux when placed at about 1 cm above the exposed surface of the sample. The coiled wire (heat source) placed at about 1 cm above the top surface of the sample amount to an "oven effect" [9] and as such the problems of thermal contact resistance are avoided at the top surface, hence the necessity to provide a fairly smooth surface.

The constant heat flux was maintained with the help of a transformer which steps the 220 V down to 12-15 V through which a constant current flows. Once a constant current is flow-

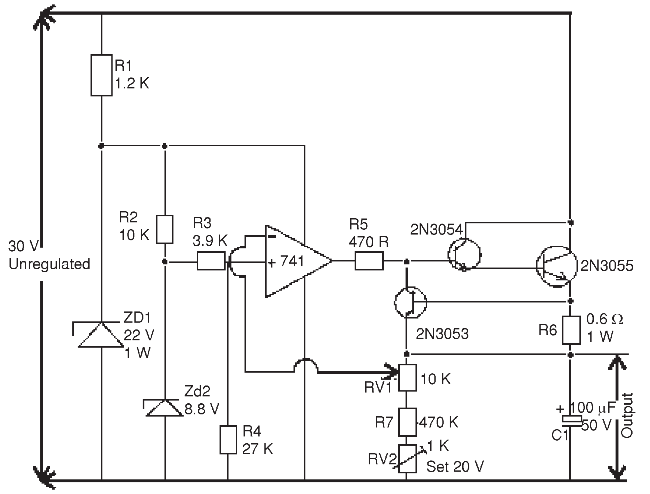


Figure 3. The circuit diagram of a stabilized low voltage and steady current power supply unit

ing, the wire will radiate near constant heat flux [9]. A stabilized low voltage and steady current power supply unit powered the heater. The circuit diagram of the supply unit is as shown in fig. 3

The circuit is made-up of operational amplifier used as a variable voltage reference by wiring it as a voltage follower and applying a suitable reference to its input. The op-amp has a very high input impedance when used in the "follower" mode and thus draws near-zero current from the impedance and can supply several milliamps, output loading cause little change in the output voltage value. The circuit then was made to act as a high-current regulated voltage (power) supply by wiring Current-boosting transistor networks into its output.

In this circuit the available current is boosted by the Darlington-connected $Q_1$and Q_2 pair of transistors, the circuit gain is fully variable from unity to 10 via RV_1 and the ZD_1 pre-regulator network enhances the stability of the 3 V reference input to the op-amp.

The circuit also incorporates an automatic overload protection, here R6 senses the magnitude of the output current and when it exceeds the maximum current the resulting volt drop starts to bias Q3, thereby shunting the base drive current of Q1 and automatically limiting the circuit's output current.

The output of the supply was set at 15V by RV1, so there was a constant 15 stabilized voltage at the output.

Estimation of thermal conductivity

Thermal conductivity, a basic physical property of rocks varies with changes in rock composition. It is inversely proportional to thermal gradient and a temperature (gradient) log in a well in thermal equillibrum shows the actual variation of geothermal gradient (*i. e*. thermal conductivity) with lithology [10].

However, the expression in eq. (4) was used in this study to find the thermal diffusivity directly from a series of temperature *vs*. time measurements.

In practice, temperature is plotted against time (fig. 4a-e), the intercept t_i is read from the resulting graph and the thermal diffusivity is calculated using eq. (4)

The relationship between thermal conductivity k and diffusivity α is given as:

$$k = \rho C \alpha \tag{5}$$

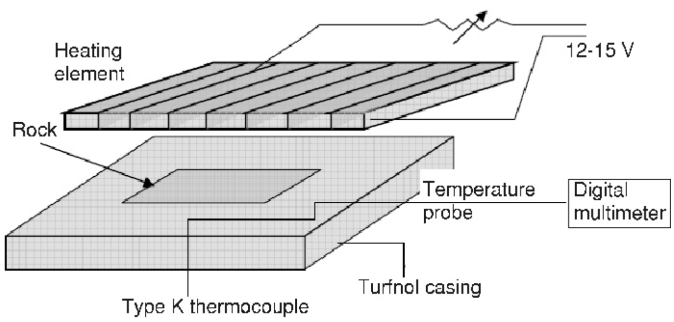


Figure 4. Diagram showing the experimental configuration for a single sample rock (one-slab solution)

where ρ is the density, and C – the specific heat. Equation (5) forms the basis for measuring thermal properties using this technique.

The specific heat and density are measured while the thermal conductivity is then calculated using eq. (5).

Density and specific heat measurements

The measurements of the specific heat capacity of the samples were made by simple calorimetry experiments in the laboratory while the density measurements were determined from the mass-volume measurements of the samples.

Sample preparation and temperature measurement

A block of approximately 0.4 m 0.4 m 0.1 m is cut and lightly polished to achieve flat surfaces. Surfaces other than the top are thermally insulated with very low thermal conductivity material (glass wool).

The temperature is measured with a K-type thermocouple probe centred at the base of the sample rock for 150 s in 10 s interval. The K-type thermocouple plug is connected to a digital multi metre (AVD890G). The K-type thermocouple probe has a temperature range of –50-400 °C, an accuracy of ±0.75% of rgd ±3 °C, and a resolution of 1 °C.

A thin leaf of aluminum foil of 0.00002 and 0.00003 m thickness is placed above the insulation layer beneath the rock sample to improve the thermal contact and to produce a uniform basal temperature since the major sources of error is the base contact between the rock sample, the K-type thermocouple probe and the basal insulation.

In effect, the aluminum foil, which is a much better thermal conductor than the rock sample or insulation, will distribute the basal temperature around the temperature probe "almost instantaneously". However, the error due to using the aluminum foil at the base of the sample slab is less than 1% and as such can be neglected. The experimental set up is as shown in fig. 4.

Thermocouple is capable of detecting 0.05 degree change above the ambient temperatures. When using this sensor several precautions are required such as the sensor must be focussed at the centre of the base of the sample and the sample must be well insulated.

The measured temperatures are reduced by subtracting the initial ambient temperature effectively, making the measurements relative to zero initial temperature. This reduced temperature is plotted against time and the linear segment and intercept time t_i are then identified to determine the thermal diffusivity.

In order to obtain a representative average value for a large volume of rock it is always necessary to measure a number of conductivities from different samples of the same rock type.

Hence, all the thermal properties were determined for 4 slabs per each rock samples and the mean values obtained except dolerite (where we had only 3 slabs available) in this way more reliable data of mean conductivities were obtained.

Results and discussion

Rocks analysed

Limestone:

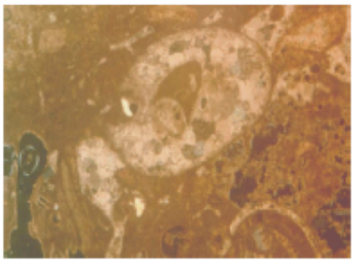
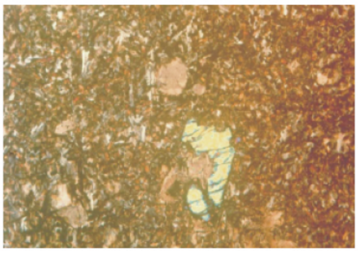


Plate 1: Photomicrograph showing the typical texture of the limestone sample. Crossed polars (XP). Long dimension of the photograph represents 4.55 mm

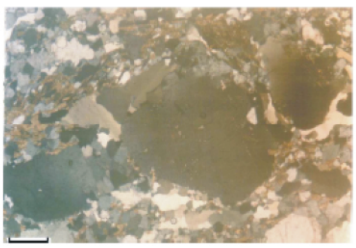


Dolerite:

Plate 2: Photomicrograph showing the fine-grained, porphyritic texture of the dolerite.

Marble:

Plate 3: Photomicrograph showing the coarse-grained, granoblastic texture of marble sample. Crossed polars (XP). Long dimension of the photograph represents 4.55 mm.

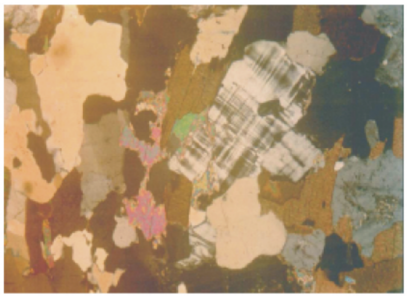


Gneiss:

Plate 4: Photomicrograph showing the coarse-grained and foliated texture of the gneiss. Crossed polars (XP). Long dimension of the photograph represents 4.55 mm.

Granite:

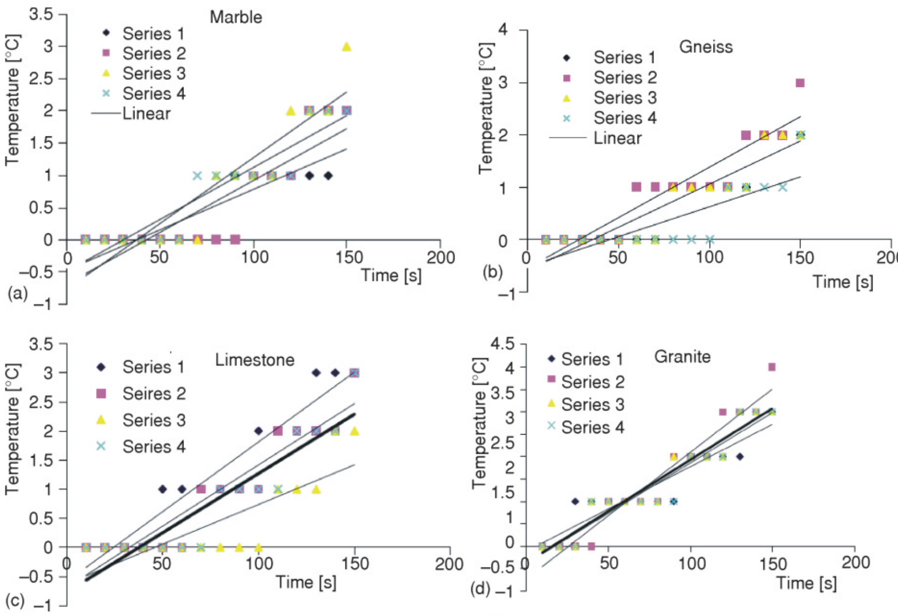
Plate 5: Photomicrograph showing the coarse-grained, granular texture of granite sample. Crossed polars (XP). Long dimension of the photograph represents 4.55 mm.



The reliability/accuracy of the method was ascertained by the comparison of the results obtained from this study with those obtained by [3].

The linear relationship of the temperature-time plot (fig. 5a-e) to give the intercept on the time axis from which the thermal diffusivity is determined for the samples agrees well with Middleton, [7].

Graphs of temperature aginst time for the samples



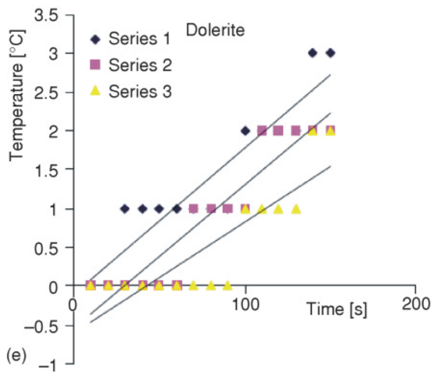


Figure 5. Graphs of temperature-time
(a) marble, (b) gneiss, (c) limestone, (d) granite, (e) dolerite

The physical characteristics by visual examination, mineral content estimation from thin section analyses (Plates 1-5) are summarized on tab. 1. While density, thermal diffusivity, specific heat capacity, and thermal conductivity of the five samples are summarized in tab. 2.

All of the samples were multimineral, with density ranging from (2.04-2.29E+03), (2.2-2.33E+03), (2.39-2.56E+03), (2.30-2.73E+03) and (2.04-2.24E+03) kg/m^3 for limestone, dolerite, marble, gneiss, and granite, respectively.

Table 1. Summary of the description of materials used

Samples	Colour	Grain size	Fabric	Mineral contents
Limestone	Light grey	Fine	Isotropic	Calcite (95%), quartz (3%)
Dolerite	Dark grey	Fine	Foliated	Olivine and pyroxene (45%), Plagioclase (40%), Opaque ore and calcite (15%)
Marble	Light grey	Coarse	Isotropic	Calcite (99%), palagioclase and opaque ore (<1%)
Gneiss	Dark grey	Coarse	Foliated	Quartz (40%), plagioclase (30%), opaque ore and sphene (5%)
Granite	Light grey	Coarse	Isotropic	Quartz (30%), microcline (35%), Plagioclase (30%)

Table 2. Thermal properties of samples obtained and comparison results Kappelmayer and Haenel

Samples	Density [kg/m^2]	Thermal diffusivity [m^2/s]	Specific heat capacity [J/kg°C]	Thermal conductivity [W/m°C]	
				This study	Kappelmayer and Haenel
Limestone	2375.7	6.59E-07	978.5	1.40	1.69
Dolerite	2496.0	9.06E-07	973.6	1.50	1.60
Marble	2454.7	9.60E-07	664.6	1.57	2.65
Gneiss	2553.7	9.32E-07	735.4	1.75	2.08
Granite	2114.0	1.78E-07	781.0	2.94	2.95

The thermal conductivity values of the samples compare well with the published data [3], with a little variations which could arise from the fact that the samples were cut perpendicular to the fabric which is said to give much lower values of conductivity than samples that were cut parallel to the fabric [11].

Conclusions

It can be seen that these obtained values are consistent with the mineral composition and their relative abundances. Rocks are, as a rule, poor conductors of heat and have a comparatively narrow range of values of thermal conductivity (0.1-7 W/m°C), which agree favourably with this study.

The differences observed in the present results and published work could be traced to a number of factors: (1) differences in the relative abundances of the mineral compositions and (2) fabrics of the rocks. The accuracy of the thermal diffusivity in this study is strongly related to strict adherence of the above-described experimental procedures. The errors of the temperature and thickness measurement on the samples is approximated to be from 1-2% and that of thermal conductivity, specific heat and density about +15%.

However, the thermal conductivity values obtained will help in the study of the original thermal condition in the surface of the study area, and the increase in temperature with depth can also be determined by the terrestrial heat flow and the thermal conductivity of rocks.

Moreover if the mean thermal conductivity cannot be accurately predicted, even the most sophisticated and appropriate modelling techniques for analysing thermal histories and maturation levels may fail when applied to real basins.

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