THERMAL AND HYDRODYNAMIC CHARACTERISTICS OF FORCED AND MIXED CONVECTION FLOW THROUGH VERTICAL RECTANGULAR CHANNELS

by

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This paper presents experimental and numerical studies for the case of turbulent forced and mixed convection flow of water through narrow vertical rectangular channel. The channel is composed of two parallel plates which are heated at a uniform heat flux, whereas, the other two sides of the channel are thermally insulated. The plates are of 64 mm in width, 800 mm in height, and separated from each other at a narrow gap of 2.7 mm. The Nusselt number distribution along the flow direction normalized by the Nusselt number for the case of turbulent forced convection flow is obtained experimentally with a comparison with the numerical results obtained from a commercial computer code. The quantitative determination of the normalized Nusselt number with respect to the dimension-less number $Z = (Gr/Re^{21/8}Pr^{0.5})$ *is presented with a comparison with previous experimental results. Qualitative results are presented for the normalized temperature and velocity profiles in the transverse direction with a comparison between the forced and mixed convection flow for both the cases of upward and downward flow directions. The effect of the axial locations and the parameter Gr/Re on the variation of the normalized temperature profiles in the transverse direction for both the regions of forced and mixed convection and for both of the upward and downward flow directions are obtained. The normalized velocity profiles in the transverse directions are also determined at different inlet velocity and heat fluxes for the previous cases. It is found that the normalized Nusselt number is greater than one in the mixed convection region for both the cases of upward and downward flow and correlated well with the dimension-less parameter Z for both of the forced and mixed convection regions. The temperature profiles increase with increasing the axial location along the flow direction or the parameter Gr/Re for both of the forced and mixed convection regions, but this increase is more pronounced in the case of the mixed convection flow. For the forced convection region, the velocity profile depends only on Re with no difference between the upward and downward flow directions. Whereas, for the case of mixed convection flow, the velocity profile depends on the parameter Gr/Re with a main difference between upward and downward flow. These results are of great importance for any research reactor using plate type fuel elements or for any engineering application in which mixed convection flow through rectangular channel is encountered.*

Key words: *mixed convection, rectangular channels, turbulent flow*

Introduction

In mixed or combined convection both natural and forced convection participate in the heat transfer process, in other words in the mixed convection buoyant forces are comparable to pumping forces. Mixed or combined convection in vertical rectangular channels is frequently encountered in many engineering applications such as cooling of nuclear reactors, large scale heat exchangers, solar energy collectors. A large number of research nuclear reactors in the world use plate – type fuel elements which have thin rectangular coolant channels with channel gap in the range between 2 to 5 mm [1]. A famous situation for the occurrence of mixed convection is when failure occurs for the primary cooling pump due to loss of electrical power supply of the reactor which is considered as an anticipated operational occurrence. For the determination of the mixed convection region, many dimension-less numbers were proposed. All of these numbers are combination of Gr, which represents the effect of buoyancy force, and Re which represents the effect of inertia force. Some of these numbers contain another dimension-less number, which is Pr as it eliminates the effect of fluid type. Perhaps the most suitable one was $Z = \mathrm{Gr}/\mathrm{Re}^{21/8}\mathrm{Pr}^{1/2}$. Previous studies showed that the mixed convection will prevail for $Z > 1.2\cdot10^{-4}$ [2]. Several studies on mixed convection problems for a Newtonian fluid in a vertical channel have been presented in the literature. Interested reader may refer to Gebhart *et al*. [3] to get a comprehensive list of research publications to that date.

Sandip Dutta *et al*. [4] carried out an experimental heat transfer measurements and analysis for mixed convection of water flow in a vertical square channel with two sides heated and two sides insulated for both the cases of assisting or opposing flow. They reported that:

– for high Re, the Nusselt number ratios is nearly one at the developed flow region ($X/D > 10$). For low Reynolds number flows, Nusselet number ratio first decreases as the flow develops downstream of the inlet and then the flow gets affected by buoyancy in the region $5 < X/D < 10$ as the thermal boundary layer grows significantly and Nusselt number ratio increase significantly; visual observations confirmed large-scale motions of the flow by buoyancy effects, and
– for the case of opposed flow: even for high Re the Nusselt number ratio is greater than one and at other Re a gradual increase in the Nusselt number ratio is observed; they attributed this to the deceleration of the flow near the heated plate and hence the growth of the thermal boundary layer will be faster than the case of assisted flow, and hence the effect of buoyancy is felt almost immediate downstream to the inlet enhancing the heat transfer.

Nobuhide Kasagi and Mitsugu Nishimura [5] numerically investigated the fully developed turbulent combined convection between two vertical parallel plates kept at different temperatures. The direction of the mean flow was upward, while the buoyancy forces acts upward (aiding flow), and downward (opposing flow) near the high and low temperature walls respectively. The Reynolds number was assumed to be 150 while the Grashof number varied from 0 to $1.0\cdot10^6$. Their results can be summarized as follow.

– the friction coefficient is increased in the aiding flow (on the heated wall), while decreased in the opposing flow (on the cooled wall), with increasing $\mathrm{Gr}/\mathrm{Re}^2$; however, the Nusselt number exhibited an inverse trend *i. e.* it is decreased and increased in the aiding and opposing flows, respectively, as the buoyancy force is increased, and
– the velocity profile became more asymmetric as Gr is increased.

Aung and Worku [6] studied numerically the combined convection for upward flow in a parallel plate vertical channel with asymmetric wall heating at uniform wall heat flux. The effect of buoyancy and asymmetric heating on the hydrodynamic and thermal parameters is presented. As for the results they reported that: when symmetric heating occurs ($r_\mathrm{H} = 1$), the value of the centerline velocity is 1.5 of the entrance velocity in the fully developed flow region and full development is accomplished at a small value of the distance from the entrance of the channel x as compared with the uniform wall temperature case for Gr/Re = 0. For Gr/Re > 0 the development length is increased and this is consistent with the uniform wall temperature case, but the centerline velocity value fails to develop to the universal value of 1.5 of the entrance velocity value unlike the uniform wall temperature case. Instead the center line velocity value becomes progressively smaller as Gr/Re increases.

The laminar fully developed mixed convection in a parallel plate channel is analyzed for the case of two-dimensional buoyancy opposing flow by Hamadah and Wirtz [7]. Complete, closed form solutions for the velocity and wall heat transfer are obtained for different cases of boundary conditions. As for their results they reported that: for the case that both walls are at uniform and constant heat flux (*i. e.* $r_H = 1$), infliction occurs adjacent to both walls and the velocity profiles pops in the center of the channel as Gr/Re increases.

Measurements of velocity and temperature distributions were reported for the case of laminar mixed convection flow of air adjacent to an inclined flat plate (45°) with uniform heat flux for range of buoyancy parameter $Gr_x/Re_x^{5/2}$ from 0 to 2.91 for buoyancy assisting condition by Abu-Mulaweh [8]. All reported velocity and temperature measurements were taken along the mid-plane (z = 0). As for the results, it was found that:

– as the buoyancy forces (buoyancy parameter) increases, both the velocity and temperature gradients at the wall increase, causing an increase in both the local Nusselt number and the local friction coefficient; similar results were reported by Abu-Mulaweh, *et al.* [9] for the case of mixed convection flow adjacent to a vertical surface with uniform heat flux, and
– the flow field is found to be more sensitive to changes in the buoyancy force than the thermal field.

Experimental technique

The test loop

The experimental test loop is a semi-opened circulation loop which consists of a test section, a circulating pump, a storage tank, an orifice flow meter and control valves. A schematic layout of the experimental setup is shown in fig.1. Water which is used as the coolant is forced to flow through the loop by means of a circulating pump to the test section. The pump suction is connected to a storage tank. Either upward or downward flow through the test section

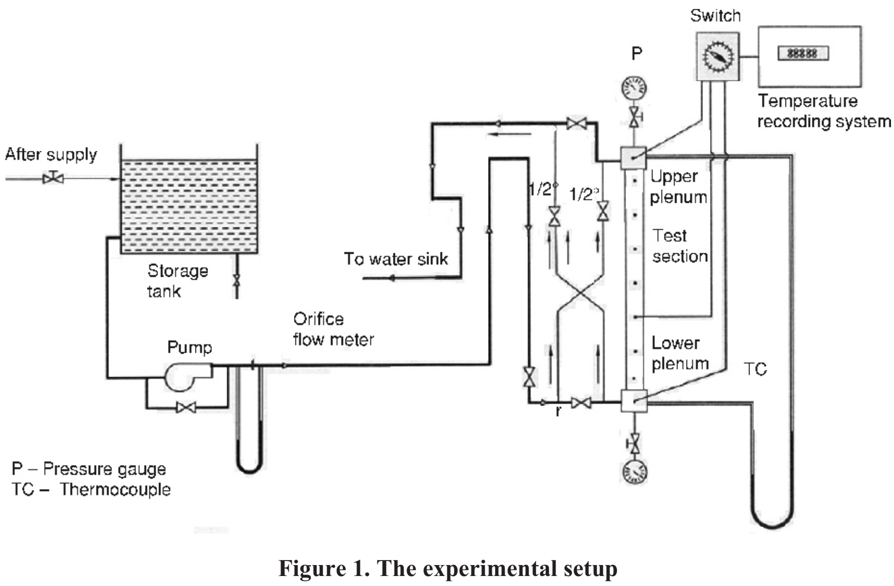

Figure 1. The experimental setup

can be chosen by means of the valves arrangement. Water flows to the storage tank after passing through the test section either in upward or downward direction, where it is cooled down to the desired temperature. Inlet temperature to the test section is maintained at a constant values throughout all of the runs by discharging the hot water leaving the test section to water sink and supply row water in the intake tank by adjusting the supply valve to the tank so as to give approximately the same flow rate to the tank as the discharge flow rate to the water sink. A schematic layout of the experimental setup is shown in fig. 1. The flow rate through the test section is measured by using an orifice plate. The pressure drop across the orifice is measured by means of a suitable differential pressure U-tube manometer. Water is used as the working fluid to allow the accurate measurements of low values of flow rate.

Test section

The main functional parts constructing the test section are the two heating plates and the upper and lower plenums. The heating plates are apart from each other by 2.7 ± 0.1 mm, and made of stainless steel because of its relatively high resistivity with respect to the other metals to limit the passing current as well as its very low temperature coefficient of resistance, that is to say the variation of the resistance due to the variation of the temperature along the plate surface, can be neglected and hence uniform heat flux can be obtained. The plate has 2.6 mm thickness, 64 mm width, and 800 mm length. The configuration of the flow channel is rectangular with 64 mm width, 2.7 mm gap, and 800 mm length. This channel simulates the dimensions of a sub-channel in the Egyptian second research reactor. Figure 2 shows an elevation, a sectional side view and a sectional plan of the test section. Seventy thermocouples, type K (chromel-alumel), are used to measure the wall temperature of the heating plates, thirty four for each one, along the longitudinal direction to allow the measurement of the wall temperature at different positions along the flow channel, and two thermocouples at the inlet and outlet water plenums. Each heating plate is mounted in a bakelite block of low thermal conductivity (1.59 J/kgK) [10], to reduce heat losses by conduction. On one of the heating plates, thermocouples are mounted along the back of the heating plate, that is to say, on the thermal insulator side. On the other heating plate, thermocouples are mounted along the surface of the heating plate on the water side. Each thermocouple on the water side corresponds to a thermocouple on the thermal insulator side at the same vertical location. Hence, the thermocouples on the thermal insulator side serve for two functions. First is to verify the wall temperature value measured by the corresponding thermocouple on the water side and second, their

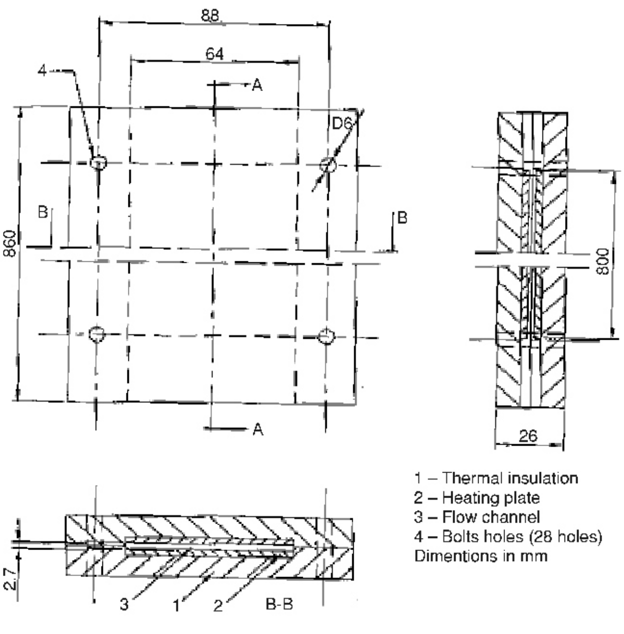

Figure 2. Sectional view of the test section

readings are being used in the calculation of the heat loss through the thermal insulator. The heating plates are uniformly heated, that is, at a uniform surface heat flux by direct current from a DC power supply. The two plenums are made of stainless-steel; each has a hole in which a pipe is welded for the inlet and/or outlet of the circulating water. Also each plenum is equipped with a hole to allow the measurement of the pressure difference between the inlet and outlet by means of a manometer in which mercury is used as the working fluid, and a thermocouple for measuring the temperature of the water at the inlet and/or the outlet of the test section. Water flows in the test section under approximately atmospheric pressure.

Experimental results and discussion

Normalized Nusslet number distribution

Previous studies have proved that a very suitable number for the determination of the buoyancy effects relative to inertia, and the determination of the mixed convection region and includes the effect of Pr is $Z = \mathrm{Gr}/\mathrm{Re}^{21/8}\mathrm{Pr}^{1/2}$, which was first proposed by Jackson and Fewester [11]. Experimental results are obtained for Re from1600 to 30000, Gr from 200 to 147000 (covering both the forced and mixed convection regions), Pr from 1.5 to 9, heat flux from 20510 to 90000 W/m^2 and flow rates from 1000 to 17000 m^3/h. Figures 3 and 4 show the Nusselt number distribution along the flow channel for the cases of upward and downward flows, respectively. The vertical axis shows the experimentally measured local Nusselt number (Nu) normalized by calculated Nusselt number ($\mathrm{Nu_{ft}}$) obtained from a correlation applied for the case of the fully developed turbulent forced convection flow. Dittus-Boulter correlation [12] $\mathrm{Nu_{ft}} = 0.023\ \mathrm{Re}^{0.8}\mathrm{Pr}^{0.4}$, is used to obtain $\mathrm{Nu_{ft}}$ as it gives satisfactory results for almost all types of geometry including the rectangular channels of the present study. The hydraulic diameter of the rectangular channel is

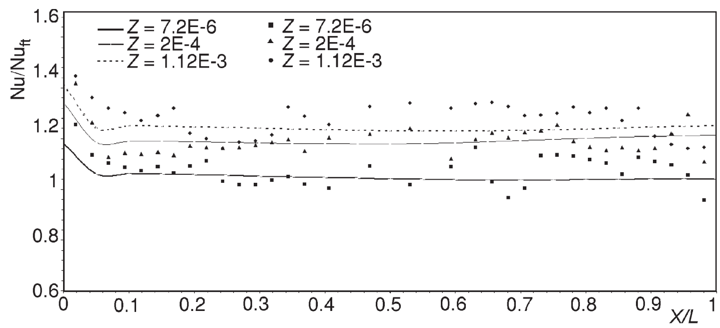

Figure 3. Comparison between normalized experimental and numerical Nusselt number distribution for the case of upward flow

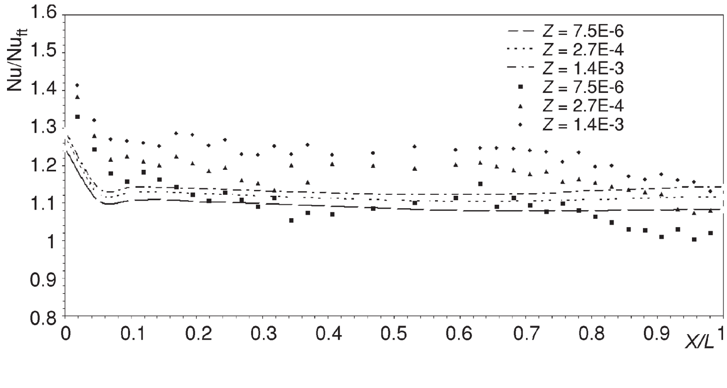

Figure 4. Comparison between normalized experimental and numerical Nusselt number distribution for the case of downward flow

used as the characteristic length in the all of the dimensionless numbers. On these figures, the reliability of the computer code (ANSYS) is checked by means of the comparison between the experimental results and those from the computer code. As shown in these figures, the numerical results from the computer code are in a fairly good agreement with the experimental ones. From these figures it can be seen that, in the channel entrance region, there is a sharp drop in the local Nusselt number as the flow is considered to be developing in this region. For low values of the buoyancy parameter Z, the Nusselt number ratios are nearly one in the developed flow region ($X/L > 0.09$), so it is clear that these conditions represent the case of pure forced convection. As buoyancy forces becomes significant (mixed convection region), *i. e.* by increasing the buoyancy parameter Z, the heat transfer parameter represented by Nu/Nu_{ft} increases, and becomes significantly grater than one. This indicates that, in the mixed convection region the heat transfer rate is greater than that in the forced convection region and this is true for both the cases of upward and downward flow. In the mixed convection regions (high values of Z), there is no difference between Nu ratio for buoyancy assisted flow, and that of buoyancy opposed flow.

If the previous local Nuselt number results are rearranged, and Nu/Nu_{ft} is plotted against Z as a variable on the abscissa, these results are shown in figs.5 and 6 for the case of upward and that of downward flow, respectively. For the purpose of comparison, the results of Usui and Kaminaga [13] in their experimental research on a vertical rectangular channel of 2.5

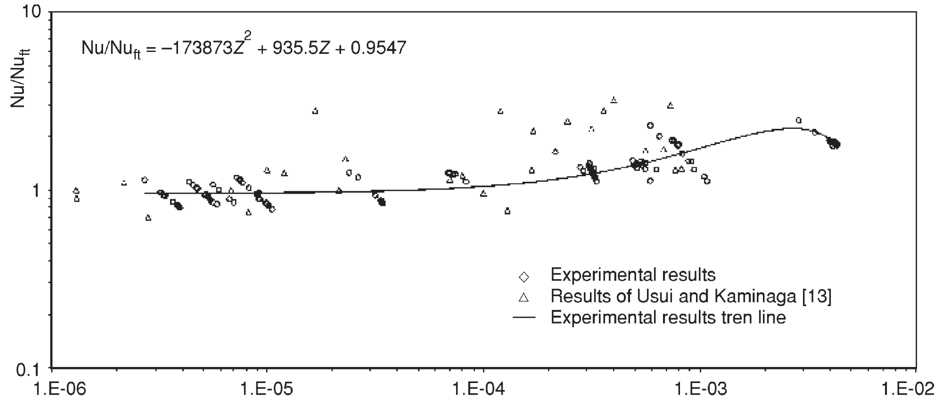

Figure 5. Normalized Nu as a function of *Z* for the case of upward flow

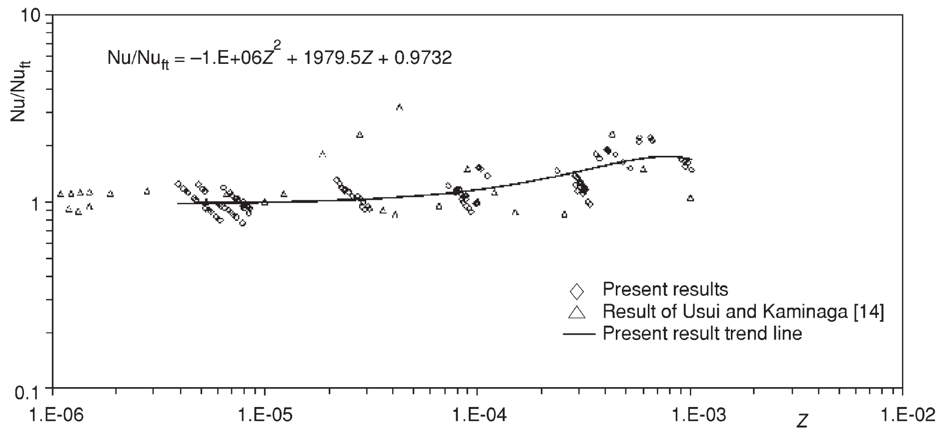

Figure 6. Normalized Nu as a function of *Z* for the case of downward flow

mm in gap are also shown in these figures. From these figures it can be shown that for low values of the buoyancy parameter Z, the present experimental results trend line is approximately a horizontal line of a value of approximately one. So, this region is considered to be the forced convection region as measured Nu is approximately equal Nu_{ft}. By increasing the buoyancy parameter Z, and at a certain value for each of the cases shown in these figures (figs. 5 and 6) the heat transfer parameter represented by the Nusselt number ratio starts to increase significantly above one, and this value of Z is the one which has previously mentioned above for the onset of mixed convection region. The equation of the best fit curve is shown on each figure and it is necessary to emphasize that these equations can predict the Nusselt number ratio as a function of Z only in the range of Z shown on theses figures.

Numerical results and discussion

In the present work, the thermal and hydrodynamic characteristics which can not be determined by experimental measurements, such as the transverse temperature and velocity profiles, are determined by considering the main laws of flow such as the law of conservation of mass, momentum, and energy. These laws are expressed in forms of differential equations. Solving these equations analytically is considered to be so complicated, so these equations are discretized by a finite element base technique. The commercial computer code ANSYS is used for this purpose.

A computer code (ANSYS) is used for two purposes:
- verification of the experimental work, and
- determination of the parameters which can not be determined from the experimental measurements.

Transverse temperature profiles

Figures 7 and 8 depict the typical temperature profiles in the transverse direction (Y) ,obtained from the computer code, for the case of upward flow through the channel heated from both sides for two different fixed values of the buoyancy parameter (Z) corresponding to the case of forced convection and that of mixed convection, respectively. Since the same trend is obtained for the case of downward flow, the corresponding results for downward flow are omitted here. These profiles are obtained at different axial locations along the flow direction. The temperature values are normalized in the form of the dimension-less number $(T_r - T_{in})/(T_w - T_{in})$ and are shown on the vertical axis, where T_r is the temperature at a certain position (Y) in the transverse direction, T_w is the corresponding wall temperature, and T_{in} is the inlet water temperature.

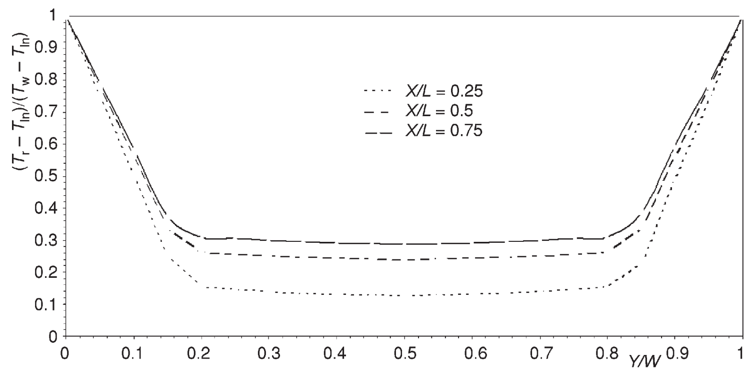

Figure 7. Normalized transverse temperature distribution at different axial locations for the case of upward forced convection flow

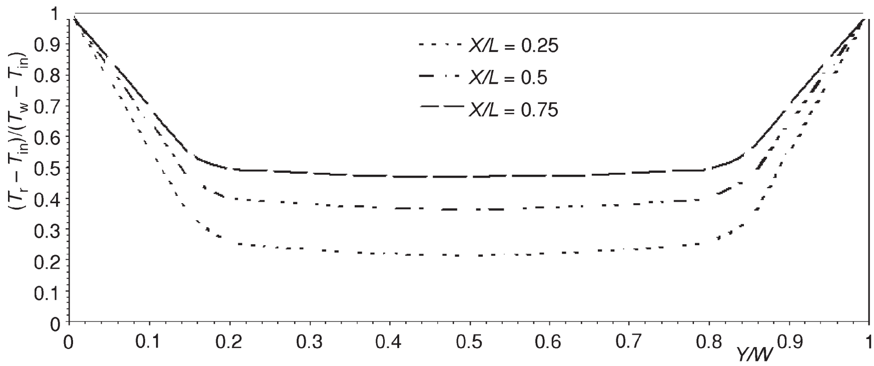

Figure 8. Normalized transverse temperature distribution at different axial locations for the case of upward mixed convection flow

This normalization eliminates the effect of the change in the inlet temperature and serves to generalize the results. The horizontal axis represents the transverse position (Y) normalized by the channel width (W). As shown from these figures, the thermal profiles have a similar concave shape and they are approximately parallel to one another suggesting constant axial temperature gradient due to the constant wall heat flux. These characteristics are the same for both the cases of forced and mixed convection, with the only difference which lies in the fact that, for the case of mixed convection, the ratio of the temperature values at a certain axial position (X/L), and that at another axial position is greater than the corresponding ratio for the case of forced convection flow at the same positions.

To show the effect of the buoyancy parameter (Z) on the transverse temperature profiles, the normalized transverse temperature at arbitrary fixed axial location ($X/L = 0.5$) is plotted at different values of the buoyancy parameter. These results are shown in figs. 9 and, 10 for the cases of forced and mixed convection flows, respectively. As show in these figures, the buoyancy parameter has a slight effect on the transverse temperature distribution for the case of

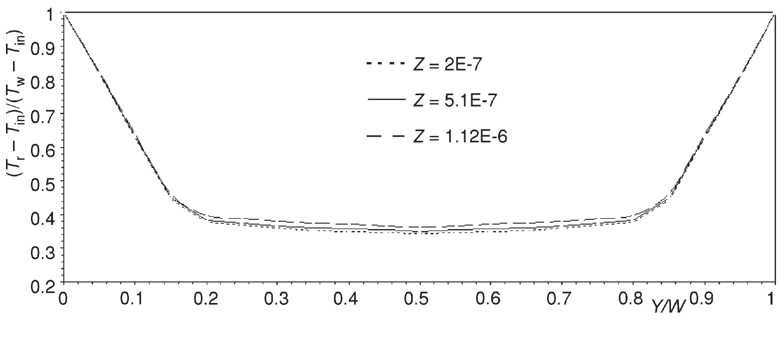

Figure 9. Normalized transverse temperature distribution for the case of upward forced convection flow for different values of *Z*

forced convection flow and this effect is to increase the temperature values. For the case of mixed convection flow this is not true, *i. e.* in the case of mixed convection flow, the increase in the temperature values with the increase in the buoyancy parameter is more pronounced than the case of forced convection flow, and this characteristic is true for both the case of upward and that of downward flow, so the downward flow results are omitted here again.

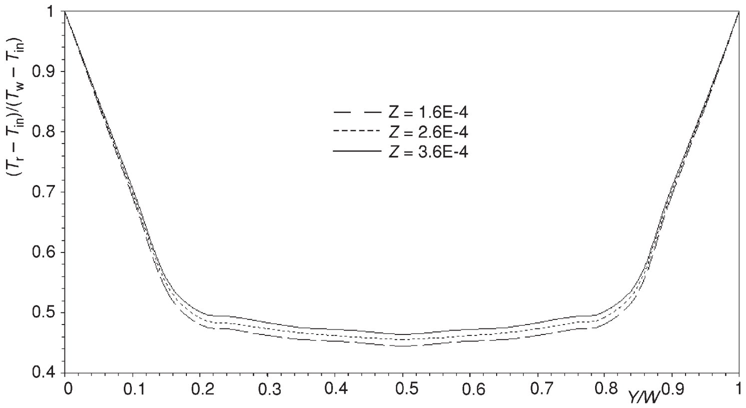

Figure 10. Normalized transverse temperature distribution for the case of upward mixed convection flow for different values of *Z*

Hydrodynamic characteristics

In this section the local streamwise velocity distribution in the transverse direction is studied for both the cases of forced convection and that of mixed convection. Both the cases of upward and downward flows are studied. The local velocity values in the transverse direction is normalized by the average velocity value at the cross-section under study, whereas the transverse distance (Y) is normalized by the channel width (W). In all of the cases, the velocity profiles are obtained for the fully developed flow case, at the cross section in the middle of the channel (X/L = 0.5). Figure 11 shows the effect of the variation of inlet velocity on the velocity profiles at the considered cross-section (X/L = 0.5) at constant heat flux of 70040 W/m^2 for the case of upward forced convection flow. The velocity profiles are obtained for inlet velocities of 0.55, 0.7, and 0.9 m/s. Figure 12 shows the effect of the variation of heat flux on the considered velocity profiles at constant inlet velocity of 0.55 m/s for the case of upward forced convection flow. The velocity profiles are obtained for heat fluxes of 20508, 35020, and 70040 W/m^2. Since the case of downward forced convection flow gives the same trend as that of upward forced convection flow the corresponding results for the case of downward forced convection flow are omitted here. From these figures it can be deduced that, by increasing the heat flux at fixed inlet velocity or increasing the inlet velocity at fixed heat flux the local velocity near the wall is increased on account of the decrease in the velocity values near the center region as shown from the focuses on the wall and center region. This can be attributed the increase in Re. The increase in Re is due to increasing the inlet velocity at fixed heat flux in the first case, and the decrease in

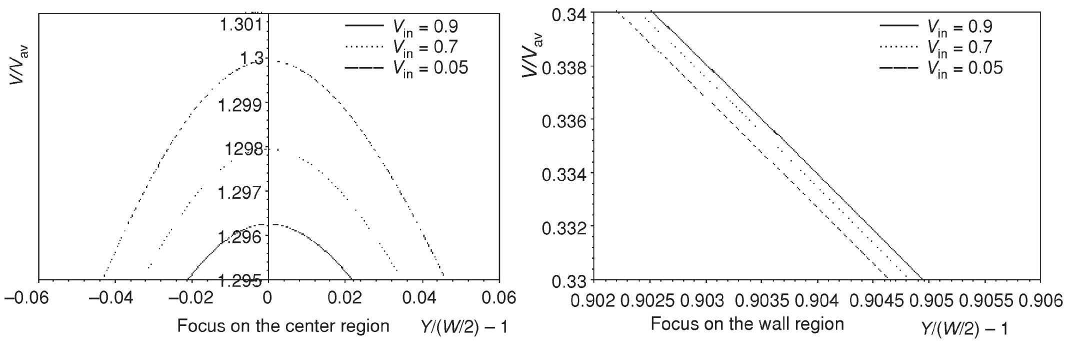

Figure 11. Velocity profiles at different values of inlet velocity and constant heat flux for the case of upward forced convection flow

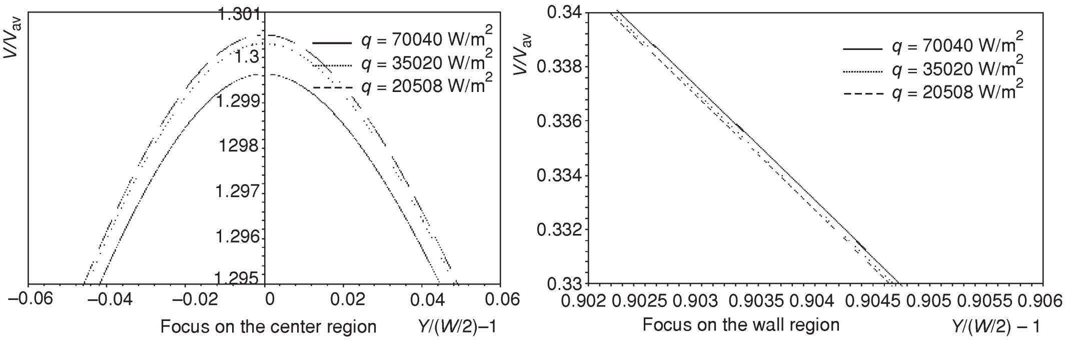

Figure 12. Velocity profiles at different values of heat flux and constant inlet velocity for the case of upward forced convection flow

the kinematic viscosity due to increasing the heat flux at fixed inlet velocity in the second case. By increasing the Re, the hydrodynamic boundary layer thickness at the same axial location (X) decrease as deduced from the correlations for the hydrodynamic boundary layer thickness on flat plate [14]:

$\delta/X = 5/\mathrm{Re}_x^{1/2}$ for laminar forced convection flow, and

$\delta/X = 0.381\mathrm{Re}_x^{-1/5}$ for turbulent forced convection flow.

Thus, by increasing Re, the local velocity values near the wall increase on account of the decrease in the local velocity values near the center, *i. e.* the velocity profile suffers a center depression or concavity by increasing Re. In case of decreasing Re, the reverse is true, *i. e.*, by decreasing Re the boundary layer thickness at the same longitudinal distance (X) increases, hence the local velocity values near the wall decrease, and the local velocity values near the center of the channel increase to substitute this reduction in the velocity values near the wall.

Figure 13 shows the effect of the variation in the inlet velocity at constant heat flux on the normalized velocity profiles for the case of upward mixed convection flow. As shown in this figure, increasing the inlet velocity at constant heat flux causes the local velocity near the wall to reduce, and the local velocities near the center increase accordingly. The effect of the variation in the heat flux at constant inlet velocity on the velocity profiles for the case of upward mixed convection flow is shown in fig. 14. As shown in this figure, increasing the heat flux causes the local velocities near the wall to increase on account of the decrease in the local velocities near the center. In the two previous cases, this can be attributed to the increase in the buoyancy forces

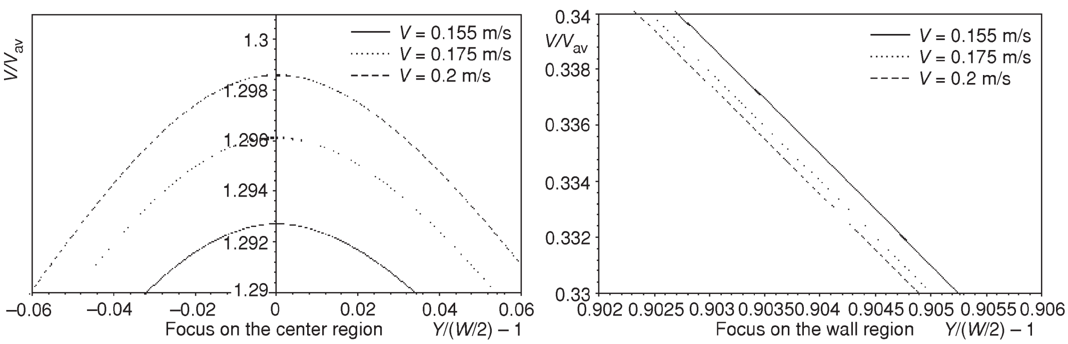

Figure 13. Velocity profiles at different values of inlet velocity and constant heat flux for the case of upward mixed convection flow

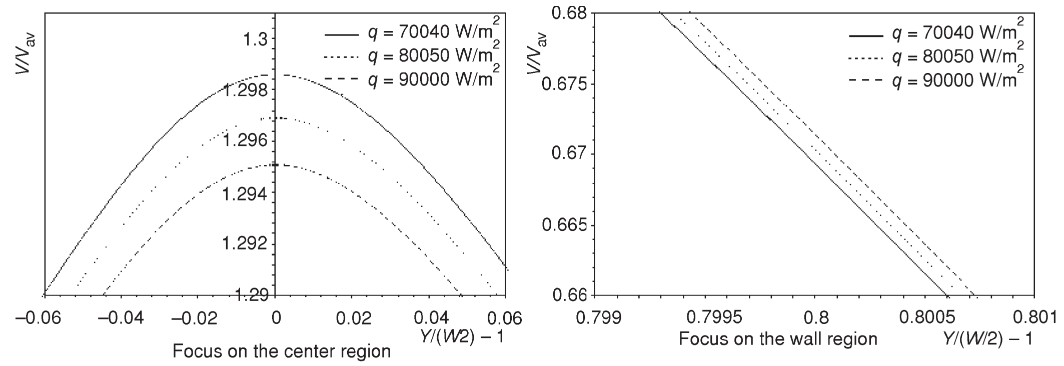

Figure 14. Velocity profiles at different values of heat flux and constant inlet velocity for the case of upward mixed convection flow

effect either by reducing the inlet velocity keeping the heat flux constant or by increasing the heat flux at constant inlet velocity. The increase in the buoyancy force effect accelerates the fluid near the wall for the case of upward flow as the buoyancy force in this case acts in the same direction of the bulk fluid flow and this is done on account of the velocities in the center.

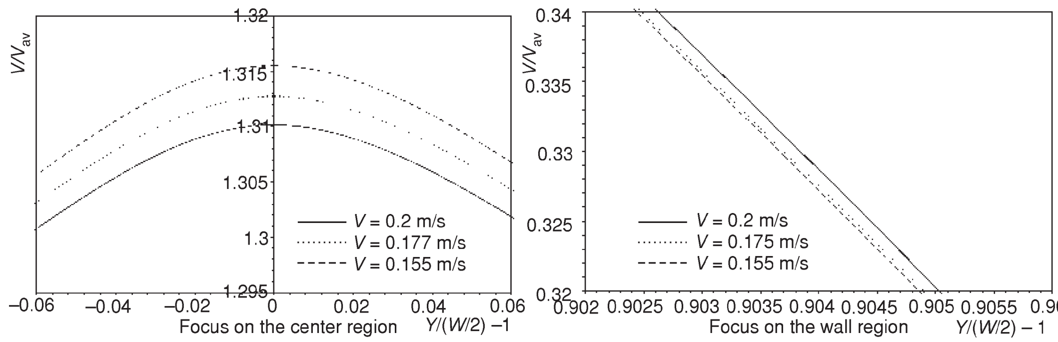

Figure 15. Velocity profiles at different values of inlet velocity and constant heat flux for the case of downward mixed convection flow

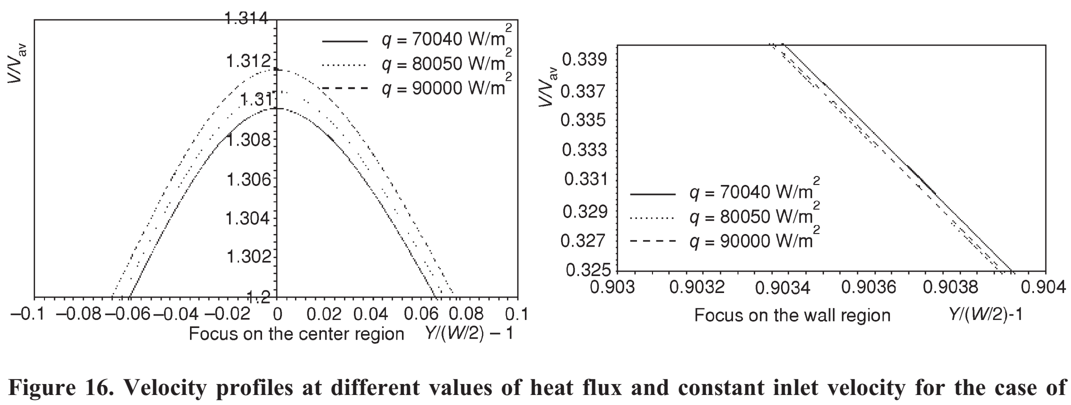

Figure 16. Velocity profiles at different values of heat flux and constant inlet velocity for the case of downward mixed convection flow

The previous argument is checked for the case of downward mixed convection flow through the channel heated from both sides and the results are shown in figs. 15 and 16. Figure 15 shows the effect of the variation in the inlet velocity at constant heat flux on the velocity profiles. This figure shows that, increasing the inlet velocity causes the local velocities near the wall to increase on account of the decrease in the velocities near the center. Figure 16 shows the effect of the variation of the heat flux keeping the inlet velocity constant on the velocity profiles. This figure depicts that, by increasing the heat flux at constant inlet velocity the fluid near the wall decelerates and the velocities near the center increase. The results of the two cases can be attributed to the increase in the buoyancy force effect with the increase in the heat flux at constant inlet velocity, or with the decrease in the inlet velocity at constant heat flux. Note that in the case of downward flow through the heated channels, buoyancy forces oppose the bulk fluid flow, thus by increasing the buoyancy forces effect, which acts near the heated walls, this causes the fluid near the wall to be decelerated, and thus the fluid near the center is accelerated.

From the previous discussion, it can be deduced that, in the region of mixed convection flow, the velocity profiles do not depend on the value of Re as in the case of forced convection but depend on a parameter which represents the ratio of the effect of buoyancy forces in terms of the inertia forces. Thus the velocity profilers are re-plotted again taking Gr/Re as a parameter. The results are shown in fig. 17 for the case of upward flow (aiding flow) and fig. 18 for the case of downward flow (opposing flow). Figure 17 shows that, in the case of upward mixed convection flow, where buoyancy forces aid the main flow, by increasing the parameter Gr/Re the velocity values near the wall increase by the action of the buoyancy forces which aides the flow near the heated walls. This increase in the velocity values near the heated walls occurs in

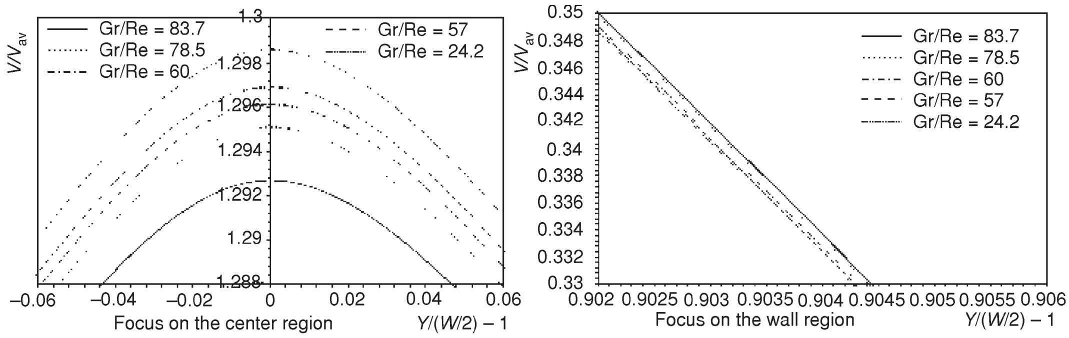

Figure 17. Velocity profiles at different values of Gr/Re for the case of upward mixed convection flow

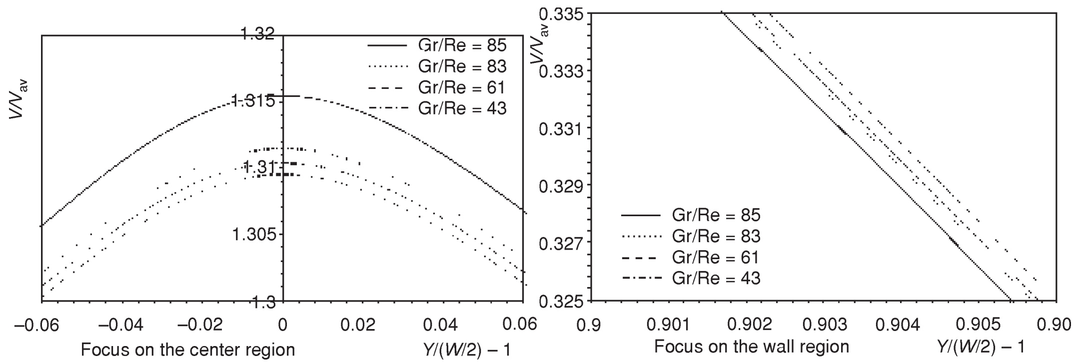

Figure 18. Velocity profiles at different values of Gr/Re for the case of downward mixed convection flow

account of the decrease in the velocity values near the center as all of the values are normalized by the average velocity in this cross-section. Thus, in case of upward mixed convection flow, the velocity profiles suffer a center depression or concavity that increase with the increase in Gr/Re.

Figure 18 shows the velocity profiles for the case of downward mixed convection flow taking Gr/Re as a parameter. The effect of the buoyancy forces which oppose the main flow in this case is opposite to the previous case of upward flow, *i. e.* by the increase in the buoyancy force effect represented by the increase in Gr/Re, inflection occurs adjacent to both walls and he velocity profile pops in the center of the channel.

Conclusions

The conclusions reached in this work may be summarized as follows.

Thermal characteristics

At the entrance of the flow channel, the heat transfer parameter represented by the Nusselt number ratio between the measured local Nusselt number and that is predicted for the case of pure forced convection is very high as the thermal boundary layer has not been established yet. Then the Nusselt number ratio decreases when the fluid move downstream from the channel entrance and then it assumes a nearly constant value in the fully developed region. This trend is approximately the same for all values of the buoyancy parameter Z. So, it can be said that the local Nusselt number suggests a typical distribution along the flow channel for both of the forced and mixed convection regions.

At low values of the buoyancy parameter Z the Nusselt number ratio takes approximately a constant value of one so this region is considered to be the forced convection region. By further increase of the buoyancy parameter to the predefined values corresponding to the mixed convection region, the effect of buoyancy forces are to increase the heat transfer rate significantly above that of the forced convection region for both the case of assisted and opposed flow. The results of the Nusselt number ratio as a function of Z can be correlated by correlations corresponding to the results best fit curves. These correlations can be applicable for the rough estimation of the Nusselt number ratio as a function of Z in the studied range of Z.

For both the case of forced and mixed convection, the temperature profiles in the transverse direction at different axial locations along the flow channel (X/L = 0.25, 0.5, 0.75) have a similar concave shape and approximately parallel to one another suggesting equal axial temperature gradient which is a characteristics of the thermally developed flow. These profiles have their minimum values at half of the channel width ($W/2$). The temperature profile shifts upward with the increase in the axial location (X/L) at constant value of the buoyancy parameter (Z). The temperature profile shifts upward also with the increase in the buoyancy parameter at fixed axial location. The difference between the case of forced convection and that of mixed convection flow lies in the fact that, the shift in the temperature profiles either by the increase in the axial location at the same buoyancy parameter value or the increase in the buoyancy parameter value at fixed axial location increases in the case of mixed convection than that of force convection for both the cases of upward and that of downward flow. But this increase is more significant in case of maintaining the buoyancy parameter constant and increasing the axial location than varying the buoyancy parameter at the same axial location.

Hydrodynamic characteristics

In the case of forced convection flow, the velocity profiles depend on Re, and with the increase in Re either by increasing the inlet velocity to the channel or the applied heat flux, the velocity values near the wall increase as a result of the decrease in the hydrodynamic boundary layer thickness at the same position (X), whereas the velocity values near the center decrease to account for the increase in the velocity values near the wall, *i. e.* the profile suffers a center depression or concavity. By increasing Re either by decreasing the inlet velocity to the channel or the applied heat flux, the velocity values near the wall decrease as a result of the increase in the hydrodynamic boundary layer thickness at the same axial location (X), and the velocity values near the center pop to account for the decrease in the velocity values near the wall. No difference is observed in the velocity profiles between the case of upward flow and that of downward flow provided that the flow lies in the forced convection region.

In the mixed convection region, the velocity profiles depend on Gr/Re or Gr/Re^2 which represents the ratio between the buoyancy and inertia forces. For the case of mixed convection flow, there is difference between the case of upward and downward flow which lies in the fact that in the case of upward flow the buoyancy forces cause the velocity values near the heated walls to increase as it acts in the same direction as the bulk fluid direction and its effect is concentrated near the hot walls. This is done on account of the decrease in the velocity near the center. Thus in the case of upward mixed convection flow the velocity profiles suffer a depression or concavity in the center which increase with Gr/Re. In the case of downward mixed convection flow the opposite is true, *i. e.* with the increase in Gr/Re inflection occurs in the velocity profile near the hot wall as the effect of buoyancy forces which increase with the increase in Gr/Re is to oppose the main flow. Thus the velocity profile near the center pops with the increase in Gr/Re.

Nomenclature

C – specific heat, [Jkg^{-1}K^{-1}]
Gr – Grashof number (= $g\beta D^3(T_w - T_b)/u^2$), [–]
D – equivalent hydraulic diameter, [m]
h – heat transfer coefficient, [Wm^{-2}K^{-1}]
K – thermal conductivity, [Wm^{-1}°C^{-1}]
L – length of the flow channel, and heating plate, [m]
Nu – Nusselt number, [hL/k], [–]
Pr – Prandtl number ($\mu C/k$), [–]
q – heat flux, [Wm^{-2}]
Re – Reynolds number ($\rho VD/\mu$), [–]
r_H – ratio of the heat flux on the two plates, [–]
r_T – ratio of the uniform temperature on the two plates, [–]
T – temperature, [°C]
V – fluid velocity in the streamwise direction, [ms^{-1}]
W – plate width, [m]
X – distance from the inlet of the flow channel, [m]
Y – distance from the wall, [m]
Z – Gr/(Re$^{21/8}$Pr$^{1/2}$), [–]

Greek letters

β – coefficient of volumetric expansion, [K^{-1}]
ν – kinematic viscosity, [m^{2}s^{-1}]
μ – dinamic viscosity, [kgm^{-1}K^{-1}]

Subscripts

av – average condition
ft – turbulent forced convection
in – condition at the inlet of the flow channel
r – transverse direction
w – wall condition of heating plates on flow – channel side
x – distance from the inlet of the flow channel

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