EMPIRICAL CORRELATION FOR PREDICTION OF THE ELUTRIATION RATE CONSTANT

by

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In vessels containing fluidized solids, the gas leaving carries some suspended particles. This flux of solids is called entrainment, E, or carryover and the bulk density of solids on this leaving gas stream is called the holdup. For design we need to know the rate of this entrainment and the size distribution of these entrained particles R_{im} in relation to the size distribution in the bed, R_{ib}, as well as the variation of both these quantities with gas and solids properties, gas flow rate, bed geometry and location of the leaving gas stream.

Steady-state elutriation experiments have been done in a fluidised bed 0,2 m diameter by 2,94 m high freeboard with superficial gas velocities up to 1 m/s using solids ranging in mean size from 0,15 to 0,58 mm and with particle density 2660 kg/m³. When the fine and coarse particles were mixed, the total entrainment flux above the freeboard was increased. None of the published correlations for estimating the elutriation rate constant were useful. A new simple equation, which is developed on the base of experimental results and theory of dimensional analyses, is presented.

Introduction

A fluidization vessel usually consists of two zones, a dense bubbling phase having a more or less distinct upper surface separating it from a lean or dispersed phase. The section of the vessel between the surface of the dense phase and the exiting gas stream from the vessel is called freeboard and its height is called freeboard height, H. The purpose of the freeboard is to allow the solids to separate from the gas stream, and as its height is increased entrainment lessens. A freeboard height is reached above which entrainment becomes nearly constant. This is called the transport disengaging height, TDH. Unfortunately there are two related but fundamentally different definitions in current use:

(1) The TDH is the height above the surface of the bed required for the coarses, flung up by the bubbles, to disengage and fall back into the bed. Above the TDH only fines are found;

(2) The TDH is the height above which the elutriation rate remains constant or declines only slightly.

For a freeboard height less than the TDH, the size distribution of solids in the freeboard changes with position. When the gas stream exits are above the TDH then both the size distribution and entrainment rate become nearly constant. Elutriation refers to the selective removal of the fines from mixture and this may occur either below or above the TDH.

Particles are splashed into the freeboard when bubbles burst at the bed surface. The solids trown up into the freeboard contains the whole spectrum particles sizes present in the bed. Although bubbles clearly play an important role in entrainment, there is still no general agreement concerning the mechanisms by which solids are ejected into and transported throughout the freeboard. The basic ejection mechanisms emphasize the role of the bubble nose (roof) particles, the particles in the bubble wake or both nose and wake particles. The picture is further complicated by the freeboard effects, in which particles are transported upward as dispersed particles or as agglomerates and downwards near the wall of the column as a flowing suspension or in the center as large agglomerates. The term fines is reserved for size fraction of particles which have terminal velocity smaller than the superficial gas velocity.

Definition of elutriation rate coefficient

Several models and correlations for estimating total entrainment flux are reported in the literature. The total entrainment flux can be expressed by the following equation:

$$E = E_\infty + (E_0 - E_\infty) e^{-ah} \qquad (1)$$

where E is the entrainment flux at a point h above the bed surface, and E_∞ is the elutriation flux (or the entrainment flux above TDH).

E_0 is the entrainment flux of particles at the bed surface and it can be obtained by extrapolation of the entrainment flux data in the freeboard to the bed surface. Such a correlation equation is given by Wen and Chen [5]:

$$\frac{E_0}{AD_b} = 3.07 \cdot 10^{-9} \frac{\rho_f^{3.5} g^{0.5}}{\mu^{2.5}} (U_f - U_{mf})^{2.5} \qquad [kg/m^5 s] \qquad (2)$$

where A is the cross-section area of column. D_b is the bubble diameter at the bed surface and can be calculated using an appropriate equation at different flow regimes. U_{mf} is the minimum fluidization velocity.

The constant a is independent of the bed's composition and can be evaluated from experimental data for E as a function of h. This constant representing the characteristics of the fluidized bed entrainment system. Values of a are ranged from 3.5 to 6.4 m^{-1}, and a

value of 4 m^{-1} is recommended for a system in which no information on entrainment rate is available.

The total flux of particles elutriated can be calculated by the summation of the flux of each particle elutriated (a component elutriation flux). Thus,

$$E_{\infty} = \sum_i E_{i\infty} \tag{3}$$

Experimental evidence shows that only a small particles whose terminal velocity is less than the gas velocity will be carried out of the freeboard if the freeboard height is tall enough and if no internals are present in the freeboard. The proportionality between the elutriation flux of a component and its concentration in the bed is well established over a very wide range of concentration and sizes. The flux of the particles elutriated, $E_{i\infty}$, is determinate by both the elutriation rate constant, $K_{i\infty}$, and a weight fraction of the fine particles present in the bed, X_i, as:

$$E_{i\infty} = K_{i\infty} X_i \tag{4}$$

The elutriation rate constant can be regarded as the amount of particles blown away from a bed composed of one-sized particles. $K_{i\infty}$ is defined mathematically as:

$$\frac{1}{A}\frac{dW_i}{dt} = K_{i\infty} X_i \qquad K_{i\infty} = \frac{W_i}{W} \tag{5}$$

For a continuous feed of solids to a bed at equilibrium, X_i and W are constant and then elutriation rate constant can be experimentally finding from relation

$$K_{i\infty} = \frac{x_i W_e}{X_i A} = \frac{x_i}{X_i} E_{\infty} \tag{6}$$

where x_i is weight fraction of fine particles with size d_i concerned into the entrained flux.

Many correlations published (see table on the next page) for determining the elutriation rate constant are limited to the experimental conditions employed by each of the investigators. Extrapolation of these correlations to different operating conditions often leads to very strange results. The purpose of this study is to provide a new perspective based on the existing entrainment and elutriation data and to propose a new correlation which is more reliable than existing correlations for the simulation and modeling of freeboard phenomena in fluidized bed.

Dimensional analysis

Following three equations may be considered as the basic equations for elutriation:

Empirical correlations for elutriation rate constant $K_{i\infty}$

Author	Equation
Yagi-Aochi (1955)	$\dfrac{K_{i\infty}\,g d_{pi}^2}{\mu_f (U_f - U_{ti})^2} = 0.0015\,\text{Re}_{ti}^{0.6} + 0.01\,\text{Re}_{ti}^{1.2}$
Zenz-Weil (1958)	$\dfrac{K_{i\infty}}{\rho_f U_f} = 1.26 \times 10^7 \left(\dfrac{U_f^2}{g d_{pi}\rho_p^2}\right)^{1.88}\;;\;\dfrac{U_f^2}{g d_{pi}\rho_p^2} < 3 \times 10^{-4}$ $\dfrac{K_{i\infty}}{\rho_f U_f} = 4.31 \times 10^4 \left(\dfrac{U_f^2}{g d_{pi}\rho_p^2}\right)^{1.15}\;;\;\dfrac{U_f^2}{g d_{pi}\rho_p^2} > 3 \times 10^{-4}$
Wen-Hashinger (1960)	$\dfrac{K_{i\infty}}{\rho_f (U_f - U_{ti})} = 1.7 \times 10^{-5} \left[\dfrac{(U_f - U_{ti})^2}{g d_{pi}}\right]^{0.5} \text{Re}_{ti}^{0.725} \left(\dfrac{\rho_p - \rho_f}{\rho_f}\right)^{1.15}$
Tanaka et al. (1972)	$\dfrac{K_{i\infty}}{\rho_f (U_f - U_{ti})} = 4.6 \times 10^{-2} \left[\dfrac{(U_f - U_{ti})^2}{g d_{pi}}\right]^{0.5} \text{Re}_{ti}^{0.3} \left(\dfrac{\rho_p - \rho_f}{\rho_f}\right)^{1.15}$
Merrick-Highley (1974)	$\dfrac{K_{i\infty}}{\rho_f U_f} = A + 130 \exp\left[-10.4 \left(\dfrac{U_{ti}}{U_f}\right)^{0.5} \left(\dfrac{U_{mf}}{U_f - U_{mf}}\right)^{0.25}\right]$
Geldart (1979)	$\dfrac{K_{i\infty}}{\rho_f U_f} = 23.7 \exp\left(-5.4 \dfrac{U_{ti}}{U_f}\right)$
Colakyan (1979)	$K_{i\infty} = 33\left(1 - \dfrac{U_{ti}}{U_f}\right)^2$
Bachovchin (1979)	$K_{i\infty} = 3.35 \times 10^5 \left[\dfrac{U_f}{(d_{pm}g)^{0.5}}\right]^{4.67} \left(\dfrac{\rho_f}{\rho_p}\right)^{1.62} \left(\dfrac{\mu_f}{d_{pm}}\right) \left(\dfrac{D_s x_{i0}^{0.5}}{d_{pm}}\right)^{1.15}$
Lin et al. (1980)	$\dfrac{K_{i\infty}}{\rho_f U_f} = 9.43 \times 10^{-4} \left(\dfrac{U_f^2}{g d_{pi}}\right)^{1.65}$
Wen-Chen (1982)	$K_{i\infty} = \rho_p (1 - \varepsilon) U_{si}$ $\varepsilon_i = 1 - \left[\dfrac{\lambda (U_f - U_{ti})^2}{2 g D_s}\right]^{1/4.7}$ $\dfrac{\lambda \rho_p}{d_{pi}^2}\left(\dfrac{\mu_f}{\rho_f}\right)^{2.5} = 5.17 \left[\dfrac{\rho_f (U_f - U_{ti})d_{pi}}{\mu_f}\right]^{1.5} D_s^2$ for $\dfrac{\rho_f (U_f - U_{ti})d_{pi}}{\mu_f} < \dfrac{2.38}{D_s}$ $= 12.3 \left[\dfrac{\rho_f (U_f - U_{ti})d_{pi}}{\mu_f}\right]^{2.5} D_s$ for $\dfrac{\rho_f (U_f - U_{ti})d_{pi}}{\mu_f} > \dfrac{2.38}{D_s}$
Colakyan-Levenspiel (1984)	$K_{i\infty} = 0.011 \rho_p \left(1 - \dfrac{U_{ti}}{U_f}\right)^2$ for $U_{ti} < U_f$

– for the momentum balance of single particles

$$\rho_p \frac{d_{pi}^3 \pi}{6} \frac{\partial U_p}{\partial t} \quad C_d \frac{\pi d_{pi}^2}{4} \frac{(U_p \quad U_f)^2 \rho_f}{2} \quad \frac{\pi d_{pi}^3}{6}(\rho_p \quad \rho_f)g \tag{7}$$

– for the momentum balance of fluid in column

$$\rho_f \frac{\partial U_f}{\partial t} \quad \frac{\partial p}{\partial z} \quad \mu \frac{1}{r} \frac{\partial}{\partial r} \quad r \frac{\partial U_f}{\partial r} \quad \rho_f g \tag{8}$$

– for elutriation

$$\frac{1}{A_s} \frac{d(X_i M_b)}{dt} \quad K_{i\infty} X_i$$

Using the theory of dimensional analysis, the following equation is obtained:

$$\frac{K_{i\infty}}{\rho_f(U_f \quad U_t)} \quad f \frac{\rho_p \quad \rho_f}{\rho_f}, \frac{(U_f \quad U_t)^2}{g d_{pi}}, \frac{d_{pi}(U_f \quad U_t)\rho_f}{\mu}, \frac{D_s}{d_{pi}} \tag{9}$$

where the second and third term on the right side, denotes Froudes' and Reynolds' number, respectively.

Experimental

The research of particles entrainment was carried out in fluidized beds with different constant bed composition. The steady-state fluidized bed was obtained by a continuos feeding and discharging of the material from the bed. The cylindrical column with a diameter D_s = 200 mm was made of glass and PVC parts. An outline of the experimental apparatus is shown in fig. 1. A gas off-take was constructed like a conical section with a dimension 200/50/150 and it was equipped at the top of the column. The amount of entrainment material was collected in a filter cloth. Air was used as fluidizing gas. The air flow rate was measured by a orifice inserted in 36 mm diameter pipe-line.
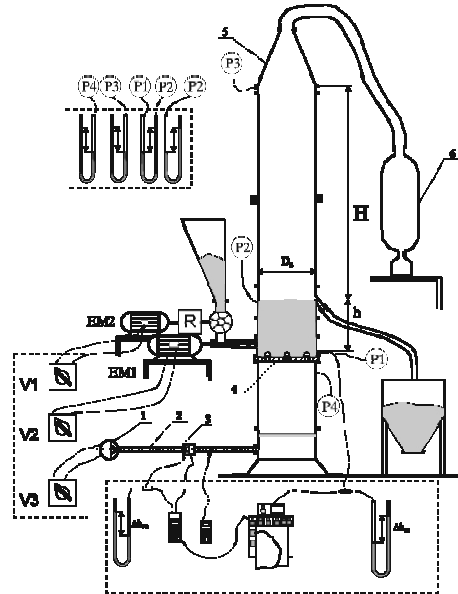

Figure 1. Experimental apparatus
1 – fan, 2 – pipe-line, 3 – orifice, 4 – distribution plate, 5 – conical section, 6 – filter cloth, P1-P2 – pressure taps, V1-V3 – variation of voltage, R – rear reductor, EM1, EM2 – electrical motor, H – height of freeboard, h – bed height

The bed material was silica sand, which particle density was ρ_p = 2660 kg/m³. Two narrow fractions of a material were used in the experiments: a fraction with equivalent diameter of particles d_{pm}^F 0.1487 mm (approximately fine – F) and a fraction with d_{pm}^P 0.5798 mm (approximately coarse – P), as well as their mixtures: P20/F80 (20% coarse and 80% fine material), P40/F60, P60/F40, and P80/F20. The main physical properties of the bed mixtures are given in tab. 1.

Table 1. The main physical properties of bed mixtures [7]

Material	F	P20/F80	P40/F60	P60/F40	P80/F20	P
d_{pm} mm	0.1487	0.1760	0.2202	0.2769	0.2788	0.5798
ρ_o kg/m³	1280	1345	1395	1406	1410	1420
ε_o –	0.5188	0.4944	0.4756	0.4714	0.4699	0.4662
$U_{mf\text{-}P}$ m/s	0.0359	0.0403	0.0542	0.1078	0.1328	0.302
$U_{mf\text{-}T}$ m/s	0.091	0.109	0.13	0.16	0.234	0.302

The experiments presented in this paper were carried out for:
– the superficial gas velocity was ranged from U_f = 0.3-0.9 m/s,
– the freeboard height was H = 2940 mm,
– the fluidized bed height was h = 320 mm, and
– the fluidization air temperature ranged from 20 to 35 °C.
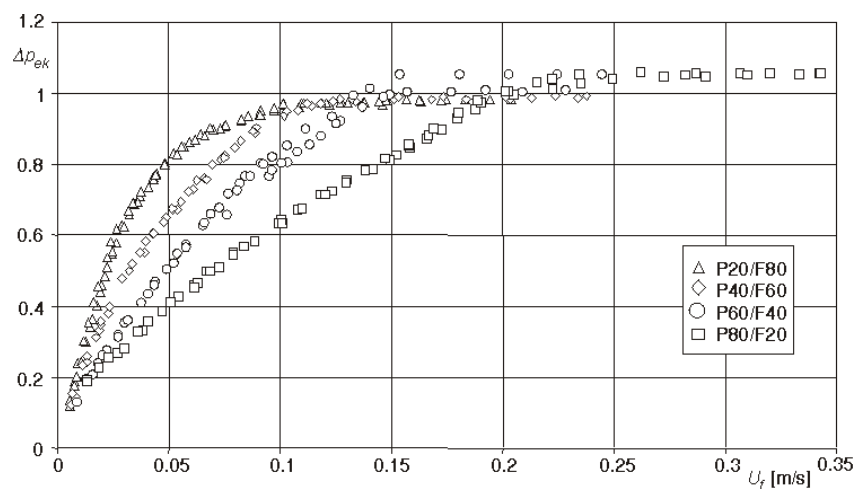

Figure 2. Experimentally obtained curves of fluidization for different mixtures

Experimentally determination on minimum fluidization velocity

The fluidisation curve of polidisperzive materials has a transition area between the filtration and fluidisation state of the bed. The apparent, U_{mf-P} and the total U_{mf-T} minimum fluidisation velocity can be determinate. The experimentally obtained fluidisation curves for different mixtures are presented on fig. 2. The values for apparent and total minimum fluidisation velocity are given in tab. 1. The relation U_f/U_{mf} is termed as a fluidisation number.

Experimental results and discussion

Effect of superficial gas velocity on total entrainment flux

The total entrainment flux is the sum of all component fluxes and the variation of E with U_f depends therefore on the bed composition as well as on particle size. If the superficial gas velocity is initially below the terminal velocities of most of the powder and is increased to a value above that of most particles, a larger and larger proportion of the powder becomes available for carry-over. The experimental results presented in fig. 3 conforms that the superficial gas velocity had a significant effect on the total flux of particles entrained above the TDH. Since the freeboard height was always well above the TDH, the increase in the flux of particles entrained above the TDH can be attribute to an increase of bursting bubbles intensity at the bed surface and reasonly to an increase in the flux of particles ejected from the bed surface.

The experimental data of the total elutriation flux obtained for different bed mixtures are grouped when its present in depence of total number of fluidisation. This
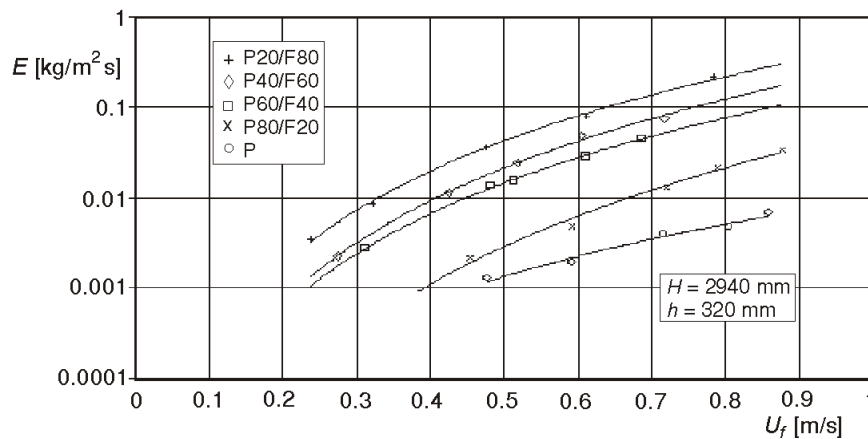

Figure 3. Total entrainment flux as a function of the superficial gas velocity for various bed materials, bed height, and freeboard height
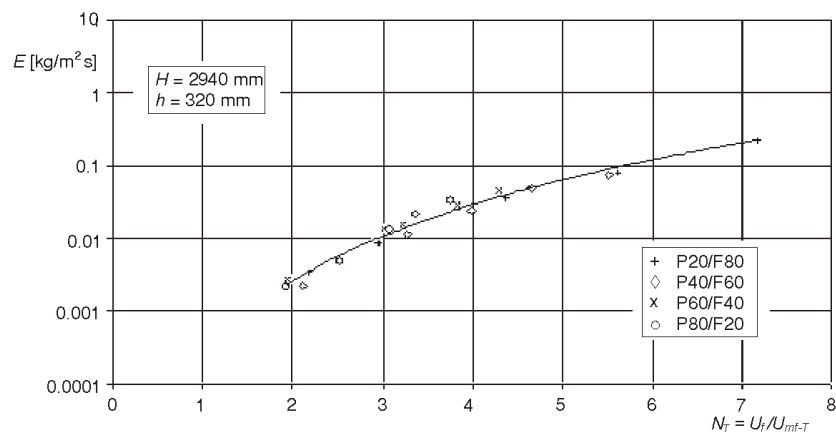
Figure 4. Elutriation flux in function of total number of fluidization

result is presented on fig. 4. This result can be explained by the conditions of the fluidised bed at the total minimum fluidisation velocity. The total minimum fluidization velocity is occurred when the all particles in the bed are in fluidised condition. In that case the segregation in the bed is exceeded, the origin of bursting bubbles is appears and the intensive mixing of the particles in bed is occurred. These hydrodynamic conditions are similar for the beds with various compositions at the point of total minimum fluidisation velocity.

Effect of fines in bed on total entrainment flux

The size distribution of bed material had an influence on the hydrodynamic of the bed and the freeboard. The segregation within the bed had the influence on the total entrainment flux. If the size distribution is very wide elutriation rates may increase due to segregation. As the concentration of fines in the bed is increased, the height required by the coarses for disengagement increases (fig. 3).

Effect of fluidisation velocity on size distribution of elutriated particles

The sieve analysis on entrained material is done. The cumulative curves for size distribution of particles, for a freeboard height $H = 2940$ mm and various superficial gas velocity are given on fig. 5. The superficial gas velocity has a dominant influence of the size distribution of elutriated particles because the increasing of the velocity enables to reach the terminal velocity of the wide particles fraction. On the other side, the limited freeboard height and ensuring the TDH depends of the superficial gas velocity intensity.
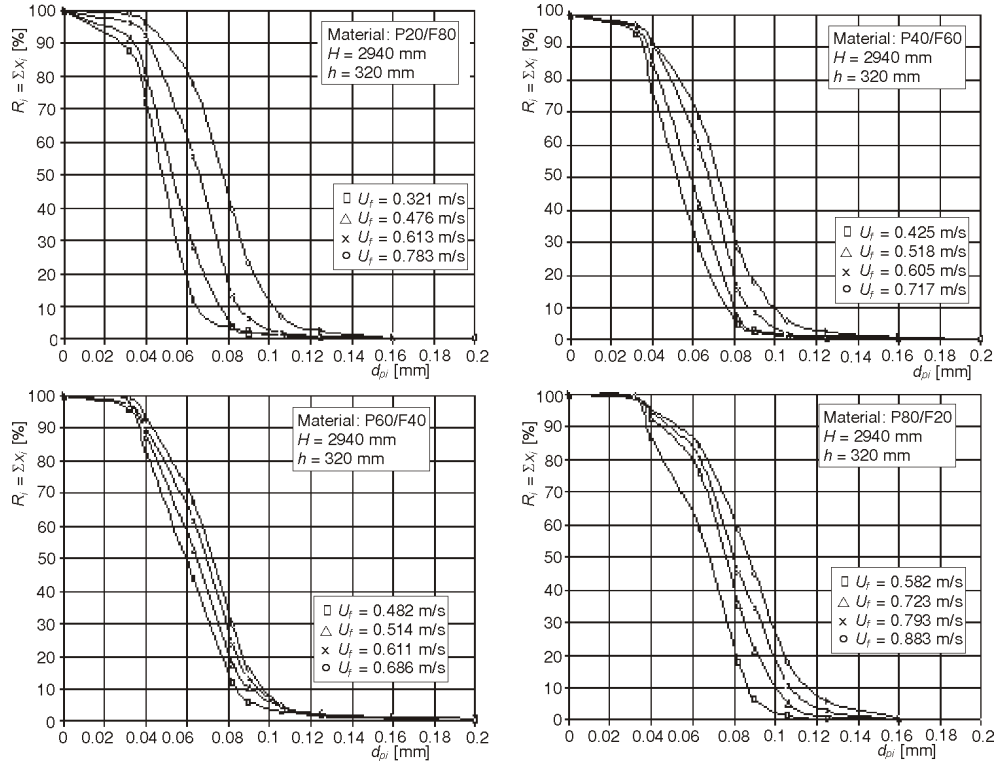
Figure 5. Cumulative curves for size distribution of particles comprised in the elutriated material at freeboard height _H_

The experimental results shows that the rate of the coarse particles into the entrained material is less than 3%, what means that the experimental conditions (flow and geometrical) refer for freeboard height nearest to TDH. The presence of coarse particles in the elutriated material is greater for the higher superficial gas velocity.

Determination of the elutriation rate constant

The elutration rate constant for experimentally obtained results was determined by using the eq. (6). The teminal velocity for each fraction of particles was obtained with Todes' equation:

$$Re_t = \frac{Ar}{18 + 0.61\sqrt{Ar}} \, 0.843 \log \frac{\phi_s}{0.065}$$

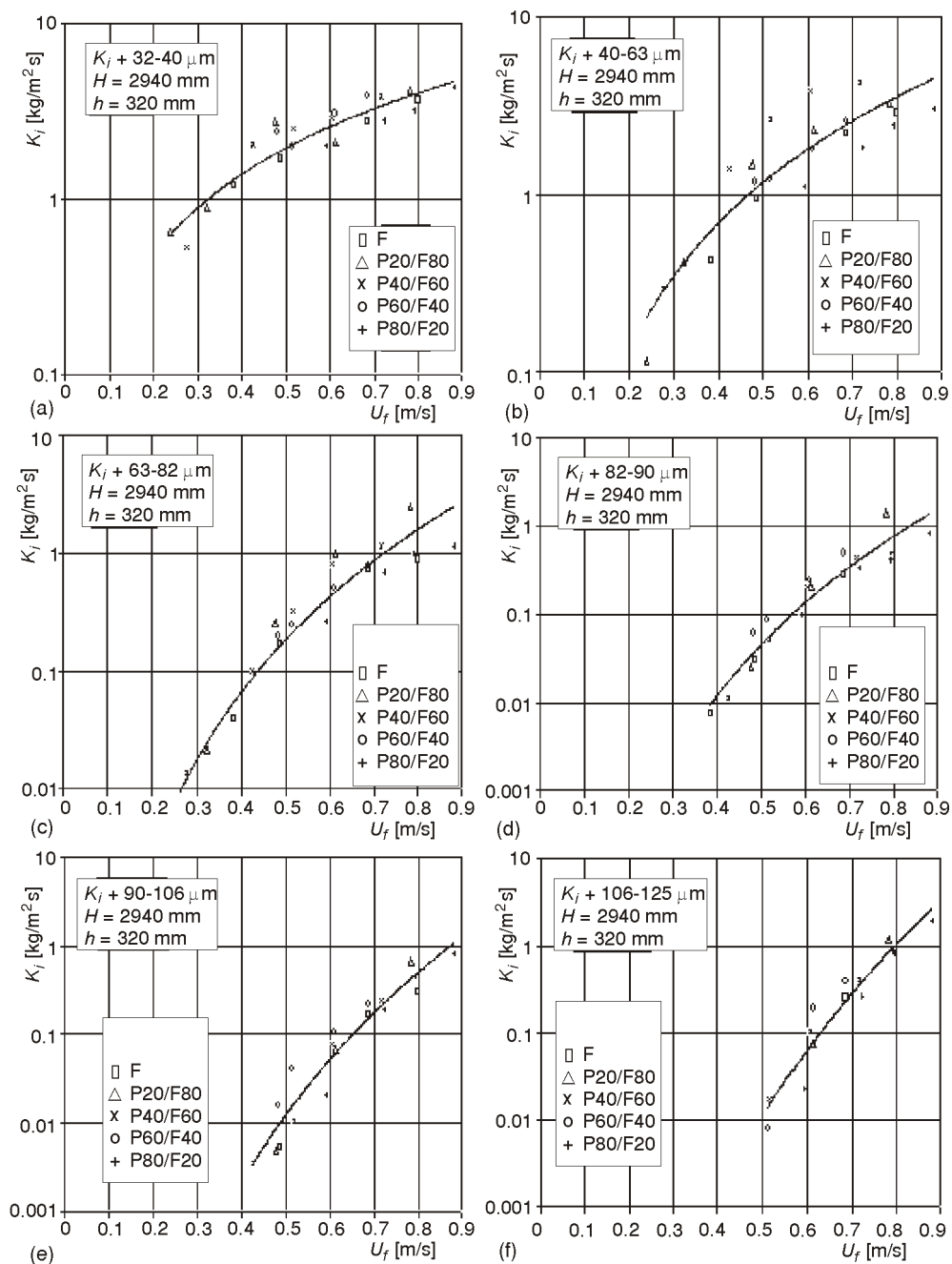The value of particles shape factor $\phi_s = 0.75$ was experimentally determinated [6].

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Figure 6. Elutriation rate constant in dependence of the superficial gas velocity and the fine particles content in the bed

Effect of fluidisation velocity on the elutriation rate constant

The influence of superficial velocity on the elutriation rate constant for the different bed composition of fine particles is given on fig. 6.

The increase of superficial gas velocity U_f increased the value of elutriation rate constant and had a strong influence on its determination. On the other side, the increase of the presence of fine particles in the bed mixture does not have effects of change to the value of the elutriation rate constant. This experimental result confirms the physical meaning of the elutriation rate constant as a coefficient of proportion between the ratio of the fine particles concentration at freeboard height and the total weight of fine particles remaining in the bed.

Comparison with the published equations in the literature

The experimentally determined elutriation rate constants, by using eq. (6), are compared with equations published in the literature. The comparative diagrams are presented on fig. 7. Remarkable differences between experimentally obtained results and prediction of elutriation rate constant by using the published correlation occurred. Concerning the number of variables and the complexity (and inter-relation of the mechanisms occurring in, and above the bed), it is hardly surprising that the numerous published correlations predict widely different rates of carry-over (when applied to systems other than those which they are based on). It is not certain at all that the gas off-take was above the TDH in the used columns. Another problem that complicates the prediction of carry-over rates is that fines may be generated in the bed. The best fit to the experimental results are obtained by using the Geldarts' empirical correlation.

New empirical correlation for the elutriation rate constant

Taking into account the dominant dimensionless number derivate by using the theory of dimension analyses, eq. (9), a new form has been found to fit the experimentally data the best:

$$\frac{K_{i\infty}\left(1-\dfrac{U_{ti}}{U_f}\right)^{1.25}}{\rho_f U_f}=K_1\left(\frac{\mathrm{Fr}_i^{1.125}}{\mathrm{Re}_i}\right)^{K_2} \tag{10}$$

where K_1 and K_2 are constants, and Froudes' and Reynolds' number is defined with the

relations:

$$\mathrm{Fr}_i=\frac{(U_f-U_{ti})^2}{g d_{pi}}; \quad \mathrm{Re}_i=\frac{(U_f-U_{ti})d_{pi}\rho_f}{\mu}$$
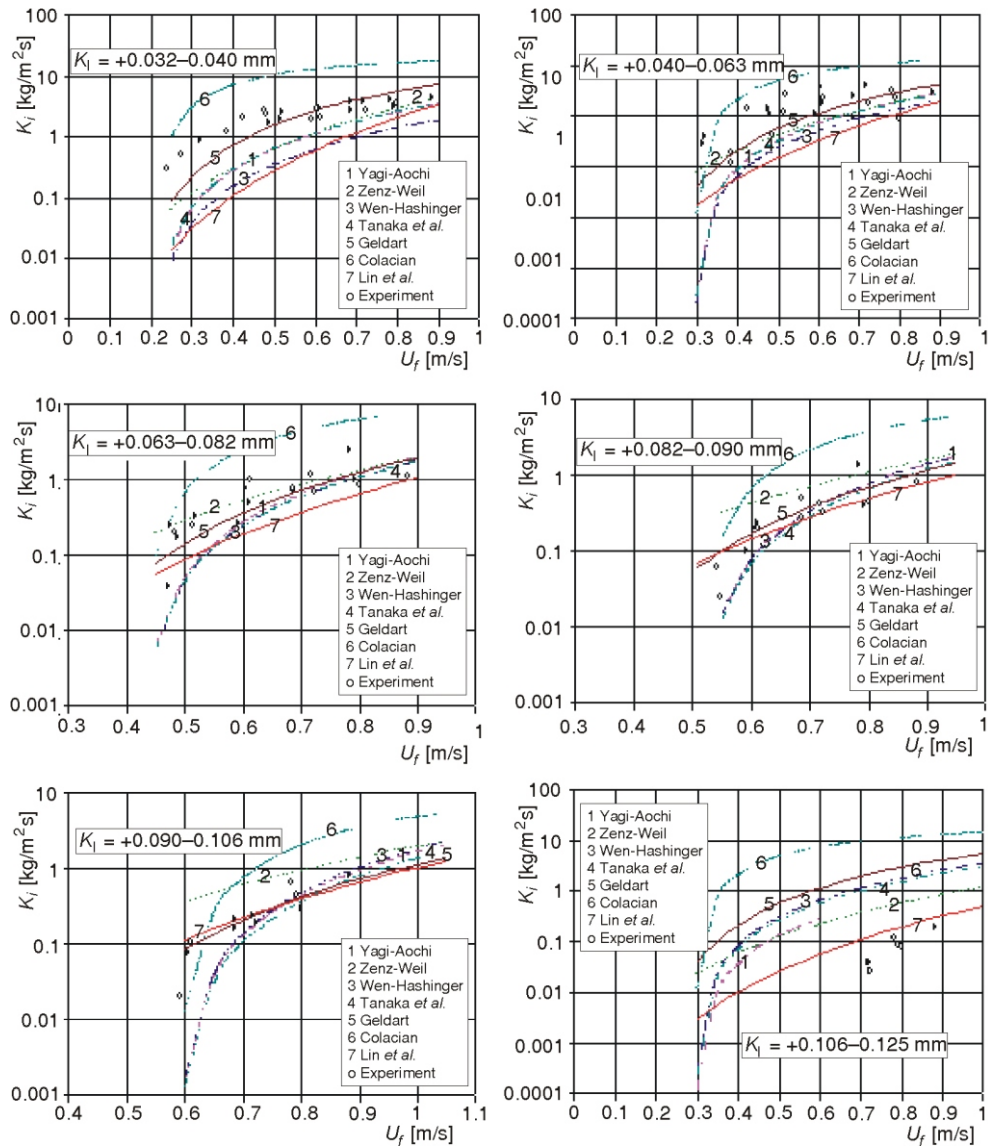
Figure 7. Comparison of the experimentally obtained elutriation rate constant and the equations published in the literature

The values of the constants K_1 and K_2 are changed in dependence of the freeboard heights. On a base of the realized analyses for the lower freeboard heights [7], the change of constant values with the freeboard height is given on the diagram on fig. 8.
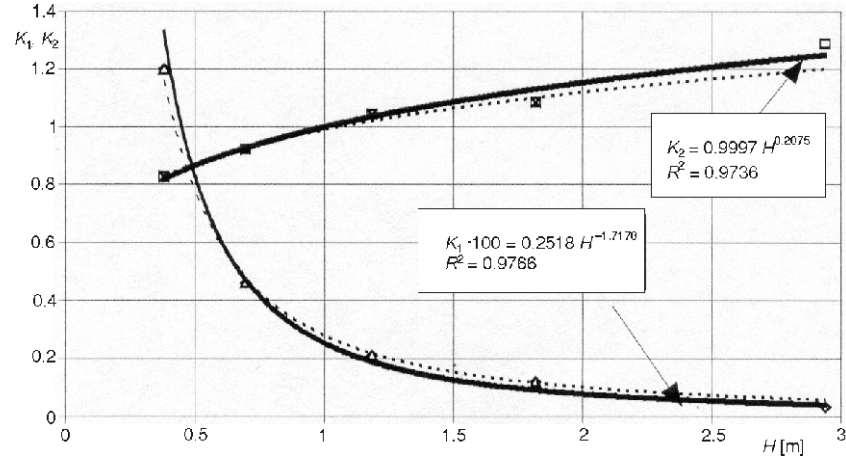
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Figure 8. Values of constant K_1 and K_2 in dependence of a freeboard height

The fit of the experimentally obtained results with the new proposed empirical correlation is given on fig. 9. The accuracy of the experimental data interpolation with the proposed correlation is $R^2 = 0.95$.

The new proposed correlation is also used for presentation of the experimental results published by Colakyan [6]. The diagram presented on fig. 10(a) shows the
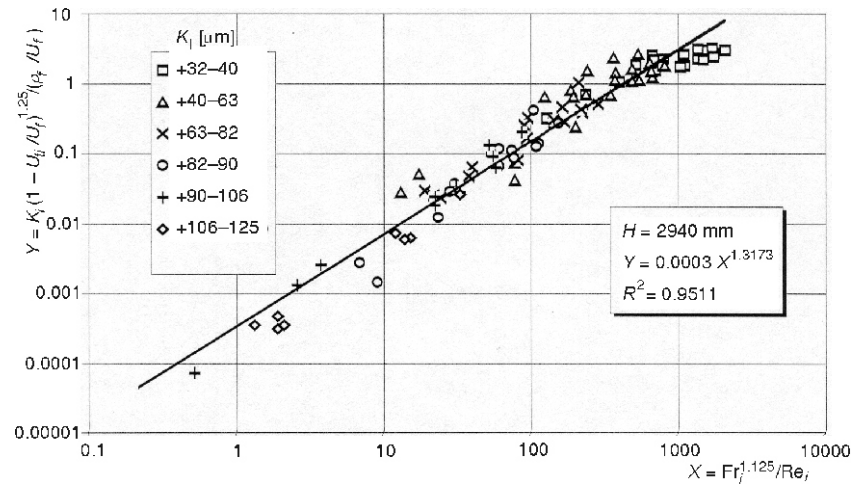

Figure 9. Comparison between experimental results and proposed correlation
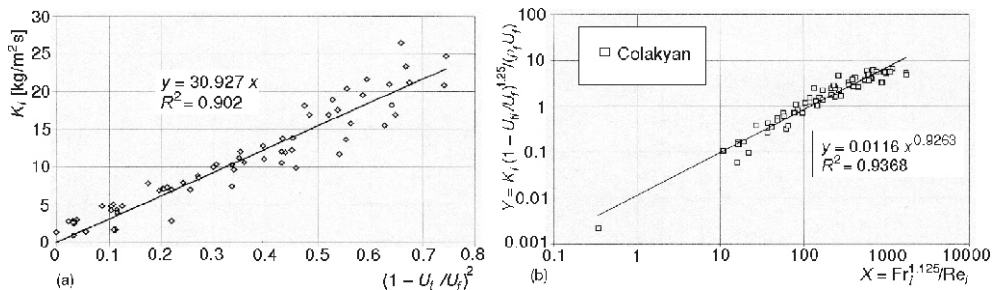
Figure 10. Using the new correlation for fitting the experimental results from Colakyan [6]

Colakyan's approach for the fitting of oven experimental results, and on fig. 10(b) the same experimental results are fitted by using the new proposed correlation for estimation the elutriation rate constant. As it can be seen, the new proposed correlation enable to fit the same experimental results with higher accuracy ($R^2 = 0,94$).

The applying of the new proposed empirical equation for fitting the experimental results from Colakyan shows that the values for constant K_1 and K_2 are much higher than that given on fig. 8. The comparison of the experimental results with the published equation, fig. 7, shows that the proposed Colakyan equation predicts the greatest values for the elutriation rate constant than all another equations. However, the proposed empirical correlation gives a better grouping of the experimental results and higher accuracy for fitting function. The definition and determination of the values for the constants K_1 and K_2 are work for further analyses.N

Conclusions

Using a steady-state fluidized bed with a sufficiently high freeboard, this experiment was carried out in order to investigate the effect of the freeboard height, superficial gas velocity and bed composition on the elutriation rate constant. Within the operating conditions, the following conclusions were drown:

- A physical meaning of the elutriation rate constant $K_{i\infty}$ as a constant of proportional to the ratio of the fine particles concentration at the freeboard height and the total weight of fine particles remaining in the bed is experimentally obtained.
- The effect of superficial gas velocity on $K_{i\infty}$ was rarely investigated. The value of $K_{i\infty}$ increases with the increase U_f.
- None of the published equations predicts all the experimental results; though some show the correct trends, other show deviations of several orders of magnitude.
- The elutriation rate constant is affected by Froudes' and Reynolds' numbers, determined with the theory of dimensional analysis. The empirical correlation for elutriation rate constant from a fluidised bed is well correlated with the developed empirical equation.

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Nomenclature

A_s – cross-sectional area of bed at surface, m²
C_d – drag coefficient, –
D_s – inner diameter of column, m
d_{pi} – aritmetical mean of adjacent sieve apertures, m
d_{pm} – particle mean size of powder, m
E – total entrainment flux, kg/m²s
F_i – component entrainment flux, kg/m²s
Fr_i – particle Froude number, –
g   – acceleration of gravity, m/s²
H – freeboard height, m
h – bed height, m
$K_{i\infty}$ – elutriation rate constant, kg/m²s
K_1, K_2 – coeficients in eq. (10), –
K_I – upper and down dimension of particles fraction, μm , mm
M_b – mass of solid in bed, kg
R_i – component entrainment rate, kg/s
Re_i – particle Reynolds number, –
t – time, s
TDH   – transport disengaging height
U_f – superficial gas velocity, m/s
U_{mf} – minimum fluidization velocity, m/s
U_p – velocity of particles, m/s
U_t – terminal velocity, m/s
X_{bi} – equilibrium concentration of size i in bed, –
x_i – mass fraction of size i in elutriated material, –

Greek letters

μ – gas viscosity, kg/ms
ρ_f – gas density, kg/m³
ρ_p – particle density, kg/m³

References

1   Zenz, P. A., Weil, N. A., A Theoretical-Empirical Approach to the Mechanism of Particle Entrainment from Fluidized Bed, *AIChE J., 4* (1958), pp. 472-479
2   Wen, C. Y., Hashinger, R. F., Elutriation of Solid Particles from a Dense Phase Fluidized Bed, *AIChE J., 6* (1960), pp. 220-226
3   Tanaka, I., Shinohara, H., Tanaka, Y., Elutriation of Fines from Fluidized Bed, *J.Chem.Eng.Jpn., 5* (1972), pp. 57-62
4   Lin, L., Sears, J. T., Wen, C. Y., Elutriation and Attrition of Char from a Large Fluidized Bed, *Powder Technology, 27* (1980), pp. 105-115
5   Wen, C. Y., Chen, L. H., Fluidized Bed Freeboard Phenomena: Entrainment and Elutriation, *AIChE J., 28* (1982) 1, pp. 116-129
6   Colakyan, M., Levenspiel, O., Elutriation from Fluidized Beds, *Powder Technology, 38* (1984), pp. 223-232
7   Stojkovski, V., Entrainment and Elutriation of Particle from Fluidized Bed, M. Sc Thesis, University St. Kiril and Metodij, Faculty of Mechanical Engineering, Skopje, 1995

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