QUASI-EQUILIBRIUM CHANNEL MODEL OF AN CONSTANT CURRENT ARC

by

Alexander V. Gerasimov, Alexander P. Kirpichnikov

Original scientific paper
UDC: 533.95:66.088
BIBLID: 354-9836, 7 (2003), 1, 101-108

The rather simple method of calculation of electronic and gas temperature in the channel of arc of plasma generator is offered. This method is based on self-consistent two-temperature channel model of an electric arc. The method proposed enables to obtain radial allocation of gas and electronic temperatures in a non-conducting zone of an constant current arc, for prescribed parameters of discharge (current intensity and power of the discharge), with enough good precision. The received results can be used in model and engineering calculations to estimate gas and electronic temperatures in the channel of an arc plasma generator.

Key words: Low-temperature plasma, constant current plasma generators, two-temperature model, engineering approach

It is known, that even at atmospheric pressure in an electric arc there can be a considerable difference between temperatures of atom-ionic gas and electronic temperature. Their difference can reach several thousand degrees 1, 2 .

Investigation of the exchange of energy between electronic and atom-ionic gas has scientific as well as, considerable self-maintained interest for optimization of efficiency of a heating of gaseous mediums in various types of plasma installations which use constant current arc.

At the moment, for theoretical study of heat exchange in multicomponent plasma mediums is based on mathematical modelling of the relevant processes.

Numerical treatment of a problem impede understanding of a physical nature and mutual influence of several processes, which proceed in an electrical arc. These methods are difficult in simulation, and in particular engineering calculations.

Thereof, it is seems useful to develop a simple physical analogy to consider the basic analytical relationships of an energy exchange in a positive column of an constant current arc.

In the present paper this problem is solved within the framework of a known Steenbeck's channel model, which is convenient for the analysis of a wide class of

problems about maintenance of plasma of a discharge through gases in electrical and magnetic fields 3, 4 .

Let's assume, according to approach of channel model, a value of electronic temperature on the border of the current-conducting channel $T_{e0} \equiv T_e(r_c) \approx T_e(0) \equiv T_{ec}$.

Where r_c radius of the channel. In the proposed concept "channel", "border of the channel" have the same sense, as in paper [3].

Therefore the relation between temperature on an axis of the channel T_{ec} and specific power W (power per unit of length of a column) can be expressed as follows:

$$T_{ec} \quad \sqrt{\frac{IW}{8\pi\lambda_e k} \quad \frac{T_c \quad T_0}{2k} \frac{\lambda_{ai}}{\lambda_e} I} \tag{1}$$

Where $T_c \equiv T(0)$ and $T_0 T(r_c)$ are the gas temperatures on an axis and on boundary of the current-conducting channel, respectively; $\lambda_e = \lambda_e(T_c)$ and $\lambda_{ai} = \lambda_{ai}(T_c)$ are the thermal conductivities of electronic and atom-ionic gas; I is the potential of ionization of plasma forming gas; k is a Boltzman's constant.

The conductance in the discharge s $\sim n_e \approx$ const·exp($-I/2kT_{ec}$) is proportional to electron concentration and is defined by the formula Saha 2 . The arc is placing in a cylindrical tube of radius R. The walls of a tube are maintained at fixed, reasonably low temperature $T(R)$ = const. The problem implies that: (1) to express at first T_c and T_0 as functions, and (2) then to determine T_{ec} and r_c at given W.

Let's write down combined equations expressing balance of energy of an unconsumed arc (radiation we shall not take into account). Inside the current-conducting channel:

$$T_e \text{const} = T_{ec} \text{ and}$$

$$\frac{1}{r}\frac{\partial}{\partial r} \quad r\lambda_{ai}\frac{\partial T}{\partial r} \quad \frac{3}{2}k\delta n_e \nu(T_{ec} \quad T) \quad 0 \tag{2}$$

In a non-conducting zone (in zone of heat removal)

$$\frac{1}{r}\frac{\partial}{\partial r} \quad r\lambda_e\frac{\partial T_e}{\partial r} \quad \frac{3}{2}k\delta n_e \nu(T_e \quad T) \quad 0 \tag{3}$$

$$\frac{1}{r}\frac{\partial}{\partial r} \quad r\lambda_{ai}\frac{\partial T_e}{\partial r} \quad \frac{3}{2}k\delta n_e \nu(T_e \quad T) \quad 0 \tag{4}$$

Here n_e is the density of electrons in the discharge; δ is the share of energy, which is lost by electrons at collisions with heavy particles; and ν is the frequency of these collisions.

The eq. (3) expresses energy balance for electronic gas, eq. (2) and (4) the same for atoms and ions. For further calculations in these and subsequent formulas,

coefficients $\lambda_e, \lambda_{ai}\delta, \nu, n_e$ shall be considered equal to the average and stationary values in the cross sections of the discharge.

It is believed, that such assumption does not impose essential qualitative contortions to a pattern of the discharge and allows to obtain the most evident representation of its basic relationships.

The given simplification is sole in the given (model) definition of a problem 3 .

Boundary conditions for eqs. (2-4), are:

$$T(0) < \infty;\ T(R) = \text{const};\ T(r_c - 0) = T(r_c + 0);$$

$$T_e(r_c \quad 0) \quad T_e(r_c \quad 0);\ \lambda_e \left.\frac{\partial T_e}{\partial r}\right|_{r \quad r_c} \quad \lambda_{ai} \left.\frac{\partial T}{\partial r}\right|_{r \quad r_c} \quad \frac{W}{\pi r_c} \tag{5}$$

$$\left.\frac{\partial T_e}{\partial r}\right|_{r \quad R} \quad \varepsilon(\varepsilon \quad 0)$$

The first four boundary conditions are physically obvious. The fifth represents a requirement of balance of power, put in the discharge.

The last requirement in (5) is written in the form used frequently in the majority of numerical models describing processes of heat exchange in nonequilibrium arcs (a so-called Neuman's condition) 5- 9 .

The quantity ε, in the right part of a relation (5), depends on parameters of the discharge and for each particular case has to be chosen, depending on the accepted physical model of interaction of electronic gas with walls of a cylindrical pipe.

The case $\varepsilon = 0$ thus corresponds to the assumption of adiabatic conditions for electronic gas.

Speaking in other words, the conditions (5) define heat flux at the wall surrounding electronic gas. Let's note, that other type possible types of boundary conditions (for example, 10), are not applicable, however, for channel model of the discharge.

For simplicity of calculations we shall consider case when temperatures of electrons and of heavy particles in central area are not too different from each other. Then, it is possible to consider in this area plasma as practically in equilibrium.

This case is present in practice in the majority of high-current arcs at atmospheric pressure with a current intensity in the discharge $I \geq 50$ A 4 .

Let's make therefore next the following basic guess: let's consider, that the mentioned equality of temperatures takes place in all the current-conducting channel, or, that at $r < r_c$ is valid $T_c = T_{ec}$, $T_0 = T_{e0}$.

In this case from equation (1) follows 3 :

$$T_c \quad \sqrt{\frac{I}{8\pi k\lambda} W} \tag{6}$$

Inside the channel $T = Tc \approx Tc =$ const, and in a non-conducting band the energy balance is defined (determined) by the equations (3) and (4) with boundary conditions similar to (5).

By adding and subtracting equations (3) and (4), it is possible to obtain the following equations for unknown variables $\lambda T + + \lambda_e T_e \;\; Te - T$:

$$\frac{1}{r}\frac{\partial}{\partial r}\; r\frac{\partial(\lambda_e T_e \quad \lambda_{ai} T}{\partial r} \quad 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}\; r\frac{\partial(\lambda_e T_e \quad \lambda_{ai} T}{\partial r} \quad \frac{3}{2}k\delta v n_e \;\; \frac{1}{\lambda_{ai}} \;\; \frac{1}{\lambda_e}\;(T_e \quad T) \quad 0$$

By integrating above written equations, electronic and gas temperatures in a non-conducting band of an electric arc in function of coordinate r can be obtained in the following form:

$$T_e(r) \;\; T(R) \quad \frac{W}{2\pi\lambda}\ln\frac{R}{r} \quad \frac{W}{2\pi\lambda} \;\; \varepsilon R \;\; \frac{\lambda_e \Psi(R) \quad \lambda_{ai}\Psi(r)}{\lambda_{ai} bR\Phi(r_c)} \tag{7}$$

$$T(r) \;\; T(R) \quad \frac{W}{2\pi\lambda}\ln\frac{R}{r} \quad \frac{W}{2\pi\lambda} \;\; \varepsilon R \;\; \frac{\lambda_e[\Psi(R) \quad \Psi(r)]}{\lambda_{ai} bR\Phi(r_c)} \tag{8}$$

where

$$\Phi(r_c) = K_0(br_c)\cdot I_1(bR) + I_0(br_c)\cdot K_1(bR)$$

$$\Psi(r) = K_0(br_c)\cdot I_0(br) - I_0\,(br_c)\cdot K_0(br), \quad rc \le r \le R$$

It is obvious that $\Psi(r_c) = 0$.

The expressions (7) and (8) have clear physical meaning and are convenient for examinations and estimations.

For comparison we shall note, that in traditional one-temperature channel model of Steenbek the similar formula for temperature in zone of heat removal $T_g(r)$ looks like:

$$T_g(r) \;\; T_g(0) \quad \frac{W}{2\pi\lambda}\ln\frac{R}{r}$$

For $r = r_c$, from formulas (7) and (8) the following expression for calculation of temperature of the channel can be derived:

$$T_c(r) \quad T(R) \quad \frac{W}{2\pi\lambda}\ln\frac{R}{r_c} \quad \frac{W}{2\pi\lambda}\varepsilon R \quad \frac{\lambda_e[K_0(br_c)I_0(bR) \quad I_0(br_c)K_0(bR)]}{\lambda_{ai}bRF(r_c)} \tag{9}$$

According to the properties of modified Bessel functions 11 in any case the following inequality is true:

$$\frac{K_0(br_c)I_0(bR) \quad I_0(br_c)K_0(bR)}{\Phi(r_c)} \quad 0$$

From this inequality physically obvious result follows, that at one and the same fixed axial temperature T_c (which in given definition of the problem is equivalent to the power put in the discharge) radius of current-conducting area is greater than calculated using the formulae of classical equilibrium model of the cooled unconsumed arc.

From the point of view of physics it means, that the mechanism of a dissipation of energy in two-temperature model appears to be less effective, than for thermally equilibrium discharge.

Thereof, to formulate balance of energy (representing conditions of existence of a static electric arc) it is necessary, that heat flux from a central zone where heat is generated (at $r < r_c$) is $J \sim W/2\pi r_c$ for a nonequilibrium case smaller than calculated using formulas of one-temperature model.

From these relations the following inequality is valid:

$$r_c^{T \quad T_e} \quad r_c^{T \quad T_e}$$

The formula (9) thus is the transcendental equation for definition of radius of the channel r_c for given temperature T_c, determined by formula (6).

At the end we shall consider one of the most interesting and most critical question in the theory and practice of a two-temperature electric arc – determination of the sharp increase of the electronic temperature $T_e(R) - T(R)$ at the boundary of the basic zone of heat removal (at $r = R$).

The importance of this problem comes from the fact that present methods for direct measurement of temperature of atoms and ions are satisfactorily developed, while the measurement of electronic temperature or temperatures close to them, using methods of spectrum analysis are widely used.

Up to now wide experimental evidence about allocation of T_e in arc plasma generators is accumulated, including value of T_e in functions of r on the surface (actually, certainly, near to the surface) of cylindrical pipe stabilizing the discharge and playing a role of calorstat for atom-ionic gas.

Let's look at the formula (7) for $r = R$, in order to obtain the following basic relation based on the theory of a two-temperature electric arc:

$$T_e(r) \quad T(R) \quad \frac{W}{2\pi\lambda} \quad \varepsilon R \; \frac{\lambda[K_0(br_c)I_0(bR) \quad I_0(br_c)K_0(bR)]}{\lambda_{ai} bR\Phi(r_c)} \tag{10}$$

The expression (10) can be simplified.

Let's assume that parameter $bR \gg 1$ (the case, when radius of a tube R is great enough). By simple calculations, using asymptotic representations of functions $K_n(x)$ and $I_n(x)$ for large values of argument, for various values of tube radius - R_1, R_2 , it is possible to get the following simple relations:

$$\frac{T_e(R_1) \quad T(R_1)}{T_e(R_2) \quad T(R_2)} \quad \frac{R_2}{R_1} \quad \text{at} \quad \frac{W}{2\pi\lambda} \quad \varepsilon R_1, \varepsilon R_2$$

$$\frac{T_e(R_1) \quad T(R_1)}{T_e(R_2) \quad T(R_2)} \quad \frac{R_2}{R_1} \quad \text{at} \quad \frac{W}{2\pi\lambda} \quad \varepsilon R_1, \varepsilon R_2$$

It can be shown, that the first approximation can be obtained by development of the formula (10) in series according the small order parameter $1/R$, the "jump" of the electronic temperature near the surface of a cylindrical pipe restricting an arc, for the same power W, is inversely proportional to radius of this pipe, $T_e(R) - T(R) \approx 1/R$.

Though, certainly, the fact, that for small increment of R the discrepancy of electronic and gas temperatures on a wall should decrease, and the following behavior should be noticed:

$$\lim T_e(R) \to T(R) \quad \text{at} \quad R \to \infty,$$

which is physically justified.

In figs. 1 and 2 the results of calculations of electronic temperature T_e and gas temperature T in a non-conducting zone (in zone of heat removal) for values $r > r_c$ are presented. Also, the results of calculation of a temperature change according to Steenbek's formula is shown.. The input data are taken from paper 12 . The calculations are performed for argon plasma at atmospheric pressure for currents $I = 78$ A and $I = 200$ A.

Algorithm of calculations is the following: (a) at given temperature of the channel T_c, power W was calculated from expression (6); (b) by solving the transcendental equation (9), radius of the channel can be determined; (c) finally electronic and gas temperature were calculated.

From figs. 1 and 2 it is obvious, that the proposed formulas for T_e and T with a sufficient degree of accuracy describe allocation of electronic and gas temperatures in a column of an arc.

The profiles T_e and T qualitatively approximate Steenbeck's profile. In wall vicinity of an arc column, as it was expected, the greatest discrepancy of temperatures of electronic and atom-ionic gases is observed. This result is in agreement with the results presented in paper 12 . It is also obvious from the figures, that radius of the channel calculated using Steenbeck' formula, is smaller, than obtained using proposed procedure. With the increase of current intensity the difference between $r_c^{T\ T_e}$ and $r_c^{T\ T_e}$ decreases.

The results obtained in the paper, can be used for simple model estimations of complex physical processes and for engineering calculations.

Nomenclature

b_r	– parameter of the Bessel function
I	– intensity of electric current, A
I_ω, I_0, I_1	– Bassel functions
J	– heat flux, W/m^2
K_ω, K_0, K_1	– Bassel functions
k	– Boltzman constant
n_e	– density of electrons in the discharge
R	– radius of a cylindrical tube, m
R	– radial coordinate, m
R_c	– radius of the channel, m
T	– gas temperatura, K
T_e	– electronic temperature, K
T_g	– gas temperature in one- temperature model, K
W	– power puf per unit length of a columu, W/m

Greec letter

δ	– share of energy fost by electron collisions
ε	– parameter of electron-wall interaction
λ	– thermal conductivity of gas, W/mK
λ_{ai}	– thermal conductivity of atom-ionic gas, W/m
λ_e	– thermal conductivity of electronic gas, W/mK

Subscripts

c	– on the channel wall
0	– on the axia

References

[1] Asinovskii, E. I., Pakhomov, E. P., Yartsev Y. M. Investigation of the Characteristics of Laminar Flow of an Argon Plasma Torch (in Russian), in Chemical Reactions In Low-Temperature Plasma, Moscow: INKHS (Institute of Petrochemical Synthesis). 1977. P.83-103

[2] Asinovskii, E. I., Pakhomov, E. P., Yartsev, Y. M. Determination of the viscosity of argon plasma using stabilized electric arc (in Russian), Teplofizika Vysokikh Temperatur (High Temperature) vol. *16* (1978), 1. pp. 28-36
[3] Raizer, Yu. P. Fundamentals of a Modern Physics of Gas-Discharge Processes (in Russian), Moscow: Nauka. 1980. 416 p.
[4] Raizer Yu. P. Physics of Gas Discharge (in Russian), Moscow: Nauka. 1987. 592p.
[5] CIàrk Ê. J., IncãÎðåãà F. Ð. Thermochemical nonequilibrium in an argon constricted arc plasma // AIAA Paper 1971. ¹ 71-593. 15 ð.
[6] Nazarenko Y. P., Panevin Y. G. Calculation of the channel arc assuming heat radiation and nonequlibrium plasma state (in Russian), in The theory of an electrical arc in conditions of stimulated heat exchange) Novosibirsk: Nauka. 1977. P. 61-87.
[7] Mathematical modeling of an electrical arc (in Russian), Engel'sht V. S. Ed. Frunze: Ilim. 1983. 363 p.
[8] Lelevkin V. M., Pakhomov E. P., Semenov V. F., Engel'sht V. S. Calculation of the characteristics of electric arc in inlet part of the channel based on two-temperature model of plasma (in Russian), Teplofizika Vysokikh Temperatur (High Temperature) 1986. V. 24. ¹ 3. P. 587-593.
[9] The theory of thermal electrical arc plasma (in Russian), Gukov M. F., Koroteev Ed.. Novosibirsk: Nauka. 1987. Part. 1. 287 p.
[10] Physics and technique of low-temperature plasma (in Russian), Dresvin S. V. Ed. Moscow: Atomizdat. 1972. 352 p.
[11] Dvait G. B. Integrals and other mathematical formulas (in Russian), Moscow: Nauka. 1983. 176 p.
[12] Asinovskii E. I., Pakhomov E. P. Analysis of temperature field in axisymetric electric arc (in Russian) , Teplofizika Vysokikh Temperatur (High Temperature) 1968. V. 6. ¹ 2. P.333-336.

Authors' address:

A. V. Gerasimov, A. P. Kirpichnikov
Kazan State University of Technology
Kazan, 420015, Russian Federation

Paper submited: March 20, 2003
Paper revised: May 28, 2003
Paper accepted: July 10, 2003